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OF

CIVIL ENGINEERS.

(INSTITUTED 1852.)

VOL. XLV.

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No. 888.

THE THEORY AND PRACTICE OF PRECISE SPIRIT LEVELING.

By DAVID A. MOLITOR, M. Am. Soc. C. E.

PRESENTED OCTOBER 3D, 1900.

WITH DISCUSSION.

INTRODUCTORY.

In presenting this paper to the Society, the writer is actuated by the conviction that the subject of precise spirit leveling has not received the exhaustive treatment which it manifestly deserves, and that a clear and comprehensive exposition of the latest developments and methods would be, at this time, of great value to the profession. A further object is to elevate the standards of spirit leveling to the point of truly scientific work.

Leveling, as commonly conducted, belongs to the most primitive surveying operations. However, when there is presented the problem of spanning the continent by lines of levels which shall be as accurate as the highest skill and most perfect instruments will permit, then it may be truly said that the primitive art has advanced to the status of a science.

Leveling is simple only when it is permissible to neglect all small errors; it becomes more complicated and requires more painstaking in proportion to the accuracy required.

The ultimate aim in precise leveling is to perform level work which is absolutely free from error, so far as this is attainable with the highest human skill, ingenuity and perseverance, combined with the use of the best instruments.

To manipulate a high-grade instrument with celerity and accuracy is artistic; to observe correctly, determinine all possible sources of error and eliminate these errors by proper methods of observing and by constant vigilance, is truly scientific. When all this is to be accomplished in the fields or on the public highways, exposed to the elements and many interfering annoyances, it seems almost miraculous to attain the results which some observers have obtained. Despite all these hindrances the subject demands a delicacy of instruments and an accuracy of results quite equal, or even superior, to the highest grade of geodetic work.

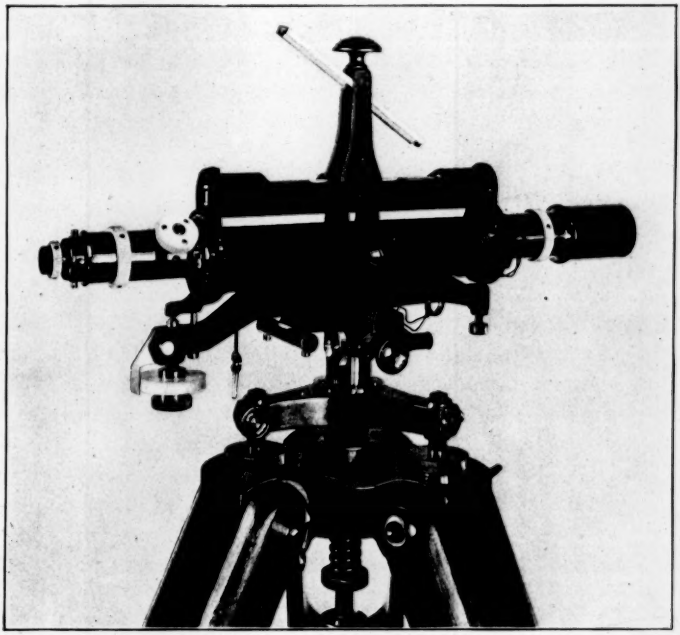
There are very few men who have made a success of precise leveling, and, of these, less than a half-dozen have given to the profession the results of their experience and study. Hence, the subject has advanced very little during the past fifteen years, and anyone detailed to such work is confined to very limited resources aside from his own. Also, the inducements offered are generally such that first-class men will not follow up precise leveling as a specialty, although the subject demands a specialist in the highest sense.

For this reason the writer hopes to add some new information to this important branch of geodetic work, by giving a complete treatise on precise leveling according to his own experience and research.

The historical side of the subject is so well treated in the report of the Geodetic Conference,* February 28th, 1894, that no attempt is here made in that direction.

In view of the many diverse opinions, theories and methods hitherto advocated, the present paper will be confined to the most practical and approved methods, and doubtful features will be discussed only when it is deemed necessary. The system of leveling here outlined is essentially that used by the United States Lake Survey and the Mississippi River Commission, with many changes which suggested themselves to the writer while engaged by the United States Board of Engineers on Deep Waterways, in running a duplicate line of precise levels along the St. Lawrence River from Tibbett's Point, N. Y., on

* "Report of U. S. Coast and Geodetic Survey," 1893, Pt. 2, p. 304.



BUFF AND BERGER PRECISE LEVEL, No. 2768.

Lake Ontario, to the Canadian Boundary Line at St. Regis, N. Y.; also by the United States Lake Survey in running a duplicate line around Lake St. Clair, from Grossepoint to New Baltimore, Mich.

It is not deemed necessary to state the fundamental principles and definitions of terms involved in common leveling. The reader is supposed to be familiar with the definitions given in textbooks on surveying, and these will be used so far as they are found applicable.

In common levels it is customary to neglect all refinements which are supposed to prevent only small errors, but not to affect the material accuracy of results. It is not generally considered necessary even to equalize back and fore sights, or to shade the instrument from the sun, or to plumb the rod with a spirit level or plumb line. However, this is a gross neglect, for most of these things can be done without additional expense, and, as will be shown later, are much in the interest of good work. The question always confronts us: What is the limiting magnitude of a so-called small error? Some of these neglected errors are likely to be surprisingly large.

The method of differential spirit leveling, as executed with the wye level, is, in principle, circumscribing a polygon around the earth's surface by a succession of tangential lines. If the earth were a perfect sphere, of uniform radial attraction at all points of its surface, then no error would be introduced into a line of spirit levels so long as back and fore sights are made equal, assuming that no deleterious atmospheric influences exist.

However, the earth is not a sphere, but an ellipsoid of revolution, thus making the theory of precise spirit leveling quite complicated. This it is proposed to discuss in the following under the head of errors.

In all following discussions and data, the metric system of weights and measures will be used exclusively, that system being almost indispensible to the class of work here considered.

Precise Leveling Instruments.

1. General Remarks.

There are a number of high-grade levels manufactured in different parts of the world. All makers are striving to attain the greatest perfection, yet the majority fall far short of the mark. The difficulty seems to be that the makers are not precise levelers and hence do not

fully know and appreciate the fine points required in such instruments. Also, the various methods of observing demand a variety in the construction of certain details which will not be harmonized until all observers have accepted a generally approved system of leveling.

Another difficulty in the way of progress is that the makers are peculiarly obstinate, and show a marked indisposition to alter their designs of instruments to better suit the requirements of the engineer. A battle of words is generally necessary to convince the maker of a desirable improvement, and, when convinced, he will adopt the improvement only when specially ordered. This applies more strictly to American makers, probably because American engineers, generally, are more easily satisfied and seem to entertain a high degree of confidence in the ability of the maker to produce the best possible instrument.

There is, for example, no need for the ordinary wye-level, this instrument being more than replaced by a transit with a few slight modifications which could be made without material increase in cost. Yet the makers (with few exceptions) will not adopt the progressive move, probably because it would prevent the sale of common levels.

The ordinary wye-level, no matter how well made, is very imperfect in principle and does not permit of quick and reliable work, although intended for that very purpose. The reasons for this will be brought out more clearly in the following.

No instrument has ever been made of which it could be said that it was free from error or could be made so by the most careful adjustment of its parts. Hence, the observer can place absolutely no reliance on any instrument for permitting perfect adjustment or for holding such adjustment longer than a few minutes at a time. His confidence and assurance of perfect elimination of error must rest on his own ability to accomplish this end, and the instrument which best permits this difficult achievement necessarily deserves the credit of being the most perfect.

It is by no means intended to convey the impression that the most perfect level is necessarily the most complicated. On the contrary, simplicity is a prime necessity to the usefulness of a level, and its peculiar purpose demands strength, durability, compactness and stability of all its parts. Being subject to somewhat severe treatment, for an instrument of such fineness and delicacy, it follows that to fill

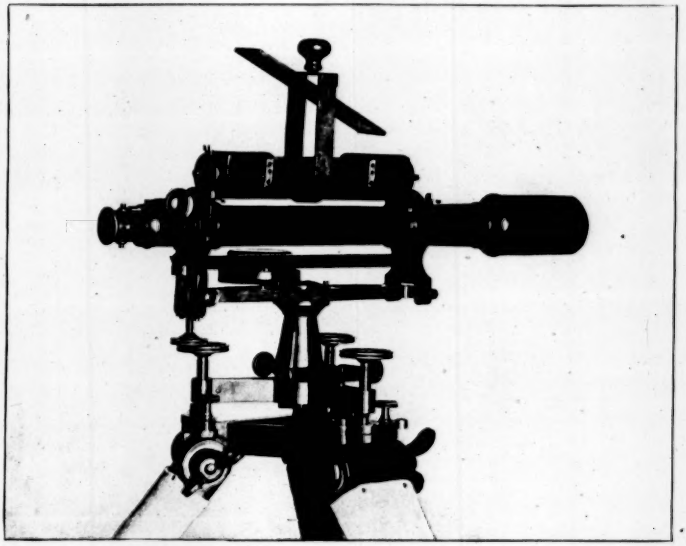


FIG. 1.—KERN PRECISE LEVEL.

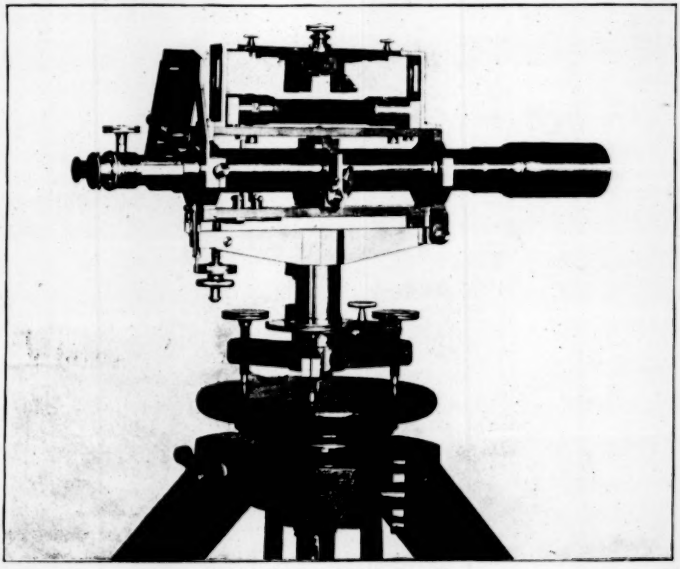


FIG. 2.—FRENCH GOVERNMENT LEVEL.

the requirements imposed, and especially to permit of easy, certain and rapid manipulations in the field, a perfect precise level becomes one of the most painstaking productions of the maker. To add to the difficulty of perfecting the design of levels, the maker has not the practical experience necessary to know the exact purpose to which he must adapt his instrument, but must base his ideas of purpose on a very meager, often conflicting, collection of requirements.

2. Description of Various Precise Levels.

a. Buff and Berger Precise Level No. 2768.—This level, shown on Plate I, was made in 1897, essentially according to the design of Professor T. E. Mendenhall, President of the Worcester Polytechnic Institute. It combines simplicity of construction with practical and substantial design and excellent workmanship, and is believed by the writer to be the most perfect precise level yet produced.

As previously stated, that instrument is the most perfect which best permits the elimination of all errors directly in the observations themselves, even though the adjustments of the instrument are somewhat imperfect and variable, a defect which can never be entirely avoided. This instrument may be said to permit this in a higher degree than any other known to the writer.

Perhaps the most important feature of a precise level is a good telescope, of high power, comparatively large aperture, perfect definition within limits of the telemeter threads, careful mounting of the lenses, especially of the objective, and true movement of the eyepiece in the telescope tube without a trace of lateral looseness.

In this level the telescope is supplied with two carefully ground steel collars upon which it is supported in any position by four agate points,* in the wye cradle. The striding level tube rests on these steel collars, forming four points of contact. The cross-threads and eyepiece are readily adjustable and are mounted in the end of a tube of almost the same length as the telescope, thus absolutely preventing all irregularity of motion. The material is generally phosphor- or aluminum-bronze, which is probably as good as any, though steel would be preferable were it not so subject to corrosion.

The striding level tube is of quite equal importance to the telescope, and a certain necessary relationship should exist between the

*Note (added by author after reading discussions).—Soft brass points were afterward substituted, though it is questionable whether or not this was an improvement.

two in order that they may be equally efficient in obtaining accurate readings. That is, for the maximum allowable length of sight, the error of pointing, due to the delicacy of the level tube, must be far within the attainable accuracy of a single rod reading with a telescope of given magnifying power.

The striding level, therefore, contains a most accurate and sensitive tube, which is mounted carefully in a bronze case, permitting vertical and lateral adjustments. This level tube has an air chamber at one end, thus making the length of the bubble adjustable for all ordinary temperatures. An adjustable revolving mirror reflects the bubble image to the eye of the observer in either position of the tube. A level tube should never be mounted rigidly in cement or plaster of Paris, as was done originally in the above instrument. A thin strip of cork or blotting paper, wrapped around one end of the glass tube and held by screw pressure within the case, is all that is required. The other end may be carried on two points and held in position by spring pressure.

The level tube may be reversed simply by lifting it out of the guides and replacing it in a reversed position. The telescope may be revolved on its collars through 180°, and its horizontal thread is made horizontal by adjustable stops on each side of the cradle at the eye end of the telescope. The striding level must always be removed when the instrument is being carried and while revolving the telescope.

The foregoing features constitute the vital points of a precise level, and the remainder of the instrument is of minor importance, though many of the details are extremely valuable in facilitating quick and accurate handling of the instrument. Perhaps the best and most practicable mounting of the telescope and level tube yet devised is represented in this instrument.

The cradle, taking the place of the wyes in a level of ordinary construction, is centrally mounted on a horizontal axis intersecting the vertical axis of the instrument. Hence, any tilting of the cradle that may be desirable to bring the bubble to the center of the level tube, does not in the least affect the height of instrument, so that the instrument need only be made approximately level by the small tubular spirit levels attached to the rigid frame, and the fine leveling is then done by means of the large milled-head micrometer at the eye end.

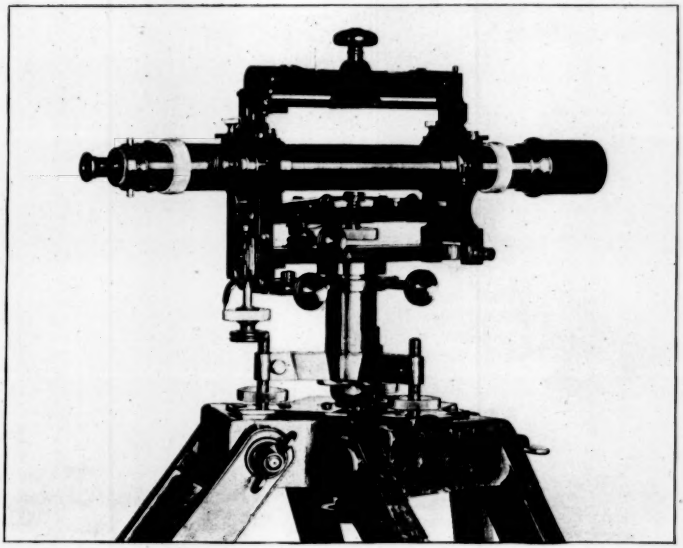
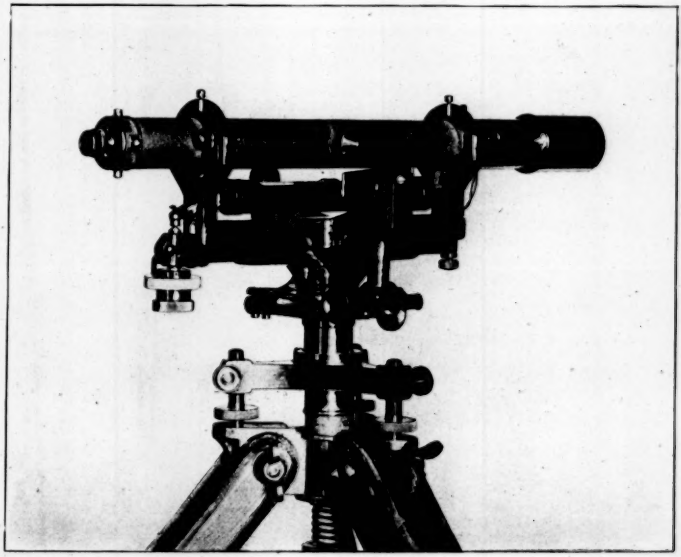


FIG. 1.—U. S. COAST AND GEODETIC SURVEY LEVEL.



NO. 2.—MASSACHUSETTS STATE SURVEY LEVEL.

The rigid wye of the common level is thus replaced by a swinging cradle which is adjusted to the level tube at every observation, thus transferring the movement of the wyes from the coarse leveling screws to a single delicate micrometer screw.

The cradle may be clamped rigidly by the screw at the right, but this is not considered useful or desirable.

The celluloid trimmings are intended to prevent temperature changes caused by handling the instrument.

The three leveling screws are admirably well adapted to this instrument, and permit of a generally low spindle and a very broad base on the tripod head, which makes the instrument very steady, even for high wind, and the numerous other disturbing elements with which the observer is constantly molested. The tripod legs are also very rigid and serve their purpose well.

b. The Swiss Precise Level.—This level, shown in Fig. 1, Plate II, was manufactured by Messrs. Kern and Company, of Aarau, Switzerland. It embodies essentially all the features described in the Buff and Berger level, though, in general, it is not so well constructed, and the level tube cannot be reversed on the telescope except by unclamping the lock device at the eye end of the instrument, which is somewhat annoying. The circular watchglass level for approximate setting of the instrument is a little better than the two small levels on the Buff and Berger level, which latter are partially covered by the lower frame. The level tube and telescope are much better than the remainder of the instrument.

The only objections to this instrument are: The short eyepiece tube, which frequently shifts laterally, thus introducing thread errors; the cradle joint not being in the vertical axis introduces errors from releveling; and the screws and other small parts generally are too light, and hence do not wear well.

The Kern level was first used in this country by the United States Lake Survey in 1876, and was retained on all precise leveling under the United States Engineer Corps prior to 1899. Fig. 1, Plate II, shows the most improved form, though the older levels were almost the same except that the mirror was fastened at one end of the level tube case, and did not permit of reading the level tube direct and reversed.

The Kern level is undoubtedly the oldest high-grade instrument

made, and has maintained its excellence to the present day, being exceeded by none in quality of material and workmanship, though being somewhat superseded in details by the Buff and Berger level designed by Professor Mendenhall.

c. The French Government Level.—This level, shown in Fig. 2, Plate II, was made by A. Barthelemy, of Paris, and used on the *Nivellement Général* of France. It is an instrument of somewhat different type from those just described. The illustration is taken from a photograph of the level at the College of Civil Engineering, Cornell University.

With the exception of the level tube, this instrument is almost exactly like the Kern level, and all criticisms offered regarding the latter level apply equally to the French level. Hence, it will be sufficient to describe only the exceptional construction of the level tube, which, in a general way, may be said to represent a most complicated solution of one of the least troublesome features of a precise level.

The idea in this level tube is to produce an image of the bubble which may be seen from, or near, the eyepiece of the telescope without parallactic error. To accomplish this, the image of each end of the bubble is carried by adjustable prisms, attached to the horizontal frame of the level tube, to a trough near the eye end of the telescope, in which four other prisms carry these images to an eyepiece, seen to one side of the telescope eyepiece. This is certainly a complicated solution of a small difficulty so readily solvable by an ordinary mirror, as is done in the two levels previously described. That the parallax of the bubble image in an inclined mirror can be easily prevented, and that even when allowed to exist it eliminates itself from the work when back and fore sights are made equal, will be shown later.

However, granting, for the present, that such a complicated device were necessary, it is readily seen that the manner in which it is here applied is not at all satisfactory, because the two eyepieces are about 25 mm. or 1 in. apart, and hence the bubble cannot be observed at the instant of taking the rod reading, which fact is far more objectionable than a slight parallax. Also, in reversing the level tube the prisms over the level must be reversed and refocused, which consumes time and is apt to disturb the instrument.

As will be seen from Table No. 1, this instrument is not nearly as

TABLE No. 1.—CONSTANTS OF VARIOUS PRECISE LEVELING INSTRUMENTS.

INSTRUMENT.	Material.	TELESCOPE.					LEVEL TUBE.			WEIGHT.			Quick Leveling.	Wye Cradle Pivoted.	Leveling Screws.
		Focal Length.	Diameter of Obj.	Focusing.	Magnifying Power.	Thread Distance.	Type of Level.	Level Reading Device.	Curvature per 2-mm. Division. Seconds.	Instrument.	Stand.	Total.			
		mm	mm		dia.	ratio.				Kgr.	Kgr.	Kgr.			
a. Buff & Berger Level No. 2768. Pattern of Professor Mendenhall	Steel and Bronze.	410	38	eyepiece	50	1:200	Reversible striding.	Revers. mirror over level.	5.0 changed to 2.23.	5.3	7.6	12.9	Two tubular plate levels.	on center	3
b. Swiss Level, Kern & Co., No. 2	Steel and Brass.	370	37	"	50	1:231	"	"	1.7 to 3.4	5.0	5.8	10.8	Circular watchglass level.	on end	
c. French Government Level. By A. Barthelemy, Paris	"	360	36	"	25	.....	"	Prismatic eyepiece.	8.26	5.4	6.3	11.7	"	"	3
d. U. S. Coast and Geodetic Survey. Level used prior to 1899	"	348	29	"	28	1:180	"	None.	2.1	4.5	6.2	10.7	None.	"	3
e. Massachusetts State Survey. By Buff & Berger, Boston	Steel and Bronze.	380	38	objective	35	1:100	Fixed.	Metallic side mirror.	6.4 to 8.0	5.6	4.7	10.3	Two tubular plate levels.	on center	3

delicate, in point of magnifying power and level tube curvature, as are those previously described.

d. The United States Coast and Geodetic Survey Level.—This instrument, shown in Fig. 1, Plate III, was designed and made in the shops of the Survey, and represents the type of level used by this bureau of the Government prior to 1899. Some changes in this instrument, as also in the field methods used by the Survey, have been made recently, but these have not yet been made public, and hence only a passing reference will suffice to point out a few of the most striking features which are different from other high-grade levels.

The general design does not differ materially from the Swiss instrument. There is no means, either by mirror or otherwise, of observing the bubble simultaneously with the rod, which is a very serious omission. Also, there is no coarse level for approximate setting of the instrument, and hence this must be done with the striding level in two positions, which is a slow process. A novelty (supposedly) is introduced by a system of bearings or wyes, inside the principal wyes, for the purpose of raising the telescope off its supports while it is being revolved on its horizontal axis, to prevent the wear usually attending this manipulation. However, as has been intimated, and as will be brought out more clearly in another place, this is not necessary, because the wear of the collars does not influence the relation existing between the level tube and the telescope, except when the tube contact and the wye points of support are on the same circles of the telescope collars, which is seldom the case and should be avoided in all instrument designs.

e. The Massachusetts State Survey Level.—This instrument, shown in Fig. 2, Plate III, was constructed by Messrs. Buff and Berger, of Boston, Mass., and used also by the United States Geological Survey. It is not really a precise level, but merely a good common level.

This instrument, in principle very much like that shown on Plate II, embodies most of the features of a precise level, but, without a striding level tube, it cannot be considered as such. The focusing is accomplished by motion of the objective, which is somewhat inconvenient to the observer. This instrument could readily be altered to become a very good level for precise or common work merely by removing the fixed level and replacing it with one that can be used as a combined striding and fixed level, as is done on the Buff and Berger

plane table alidade. However, the mirror should not be omitted, and the form used in Plate II would be preferable to that here used.

The properties of the five instruments just described are given in Table No. 1.

3. Salient Features of a Precise Level.

For a general criterion or standard of excellence of a precise level the following will serve as a guide:

The telescope should be about 420 mm. long, have an aperture of about 38 mm., and a magnifying power of about 50 diameters. It should be readily reversible and invertible in its cradle supports. The focusing should always be accomplished by movement of the eye-piece, and the latter should be absolutely free from all lost motion and wobbling. The position of the telescope in the cradle should be regulated by adjustable stops (but no clamps), so that the threads may be made exactly horizontal when the telescope is normal or inverted. The thread distance of extreme wires should be 1:200, or 5 mm. per meter of distance. There should be one vertical and three horizontal threads, and the two half intervals should be as nearly equal as possible.

The level tube should have a curvature of about 2 seconds per division of 2 mm., have an air chamber, and be adjustable vertically and laterally. The bubble must be surmounted by an adjustable mirror, so that parallax is eliminated in both positions of the level and mirror.

The level tube should rest on the telescope collars on four points which are not on the circles passing through the points of contact between these collars and the cradle supports. The tube should be guided laterally and horizontally, so that the contact points between it and the collars shall remain the same, but it shall be otherwise free so that reversals may be readily made without any unclamping, etc.

The cradle support, taking the place of the wyes of an ordinary level, should swing vertically about a center in the vertical axis of the instrument, so that releveling will not change the elevation of the telescope. The cradle should be movable by means of a large micrometer screw working against a counter spring. The instrument should be supplied with a small watchglass level for approximate and quick leveling (not for taking readings), thus leaving the micrometer only for fine setting for each individual rod reading.

4. Precise Level Rods.

Like the levels, the style of rod has passed through many stages of evolution, and, even at this time, widely different opinions exist in regard to the best material, cross-section, graduation, etc., to be used.

a. The Material.—This has generally been wood, and clear, white pine seems to have received the preference. Various methods of treating the wood have been used and advised, and the whole matter appears to be resting more or less on individual opinions.

The usual treatment consists of several coats of linseed oil covered by several coats of white zinc paint. This treatment has given good satisfaction, generally, and the old Lake Survey rods, made by Kern & Company, thus prepared, have given excellent results and withstood hard usage for the past 24 years. The United States Coast and Geodetic Survey, in 1895, prepared two pine rods by immersion in hot paraffin until they had increased from 70 to 95% in weight. Similar rods, recently made for the United States Geological Survey by Messrs. W. and L. E. Gurley, of Troy, N. Y., were treated with paraffin to a penetration of about 3 mm.

However, the experience with paraffin rods would go to show that this treatment causes the wood to become soft, and prevents the perfect adhesion of paint. A partial saturation with linseed oil would not injure the wood, as the oil soon hardens and thus lessens the hygroscopic property of the wood. The oiled wood is also well prepared to hold the white paint, which latter is claimed, by Colonel Goulier, to assist materially in lessening the effect due to moisture. According to the determinations of the United States Coast and Geodetic Survey, the paraffin treatment would appear to increase the coefficient of expansion of pine wood about 50%,* which would be another undesirable feature of this process.

b. Cross-Sections.—Various cross-sections have been used for leveling rods, but the general consensus of opinion tends toward a T-section, unless the graduation, or other appliances, like targets, influence the choice.

c. Graduation.—The style of graduation depends on whether a target rod is used or a self-reading rod. For the former, any common, easily

*NOTE (added by author after reading discussions.)—This is an error; there is no appreciable change.

readable scale, with vernier, will answer. Perhaps the most elaborate target rods ever constructed are those designed by the United States Coast and Geodetic Survey.*

The rod used on the general levels of France, and designed by Col. Goulier, is more on the order of a self-reading rod, with the smallest graduation of 2 mm. However, the graduations are not well adapted to close estimation, being more on the principle of line graduation. This rod has a very ingenious device for making temperature corrections, which consists of a metallic thermometer of iron and brass bars attached to the bottom of the rod and allowed to expand upward. Such a rod forms part of the collection of the Department of Civil Engineering of Cornell University.

The Kern level rods, previously referred to, are graduated to centimeters by a double system of black and white figures, upon which it is intended to estimate at least to millimeters. After much practice and study of the subject of estimating, a careful observer can estimate to from 0.2 to 0.5 mm. for sights up to 80 m. with a magnifying power of 50 diameters, though this is a difficult task, requiring exceptional eyesight and long practice. Few observers ever reach such precision. However, these rods† were used on all the work done under the Corps of Engineers, United States Army.

Without stopping to quote numerous opinions of others, the following brief argument is presented to show the superior practicability of self-reading rods over target rods. In general, with a good instrument and well adapted rod graduation, the position of a thread on the rod can be as sharply read as it is possible to set a target, because the space covered on either rod or target by the thread itself vitiates a target setting more than a direct estimation. The target, wherever set, can be read with a vernier or other fine device, but such accuracy will be farcical, not real. Hence, if there is to be an error in pointing it might as well be made in the rod reading as in the target setting.

Further, the accuracy of direct reading, so far, has been found to be well within the limits of attainable accuracy, owing to many conditions of atmosphere and instruments used.

With a suitable graduation, the probable error of repeated point-

* Described and illustrated on page 381 of the Survey report for 1895.

† Further description and illustration of this rod may be found in *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 421.

ings made on a target and duplicated on a self-reading rod, show no superiority of the former even when only one thread is read on the latter. With precise methods, however, it is necessary, for other reasons, to read three threads, and then the accuracy attainable with the self-reading rod is far higher.

The most important feature of the self-reading rod, however, is the rapidity and absolute control of the instrument which the observer enjoys when he is independent of all actions of the rodman, save that the latter shall hold his rod plumb and quiet. With this system alone is it possible for the observer to take the responsibility of his work and follow the now generally accepted method of observing with the bubble in the center.

It may be added that ordinary work can be done with greater accuracy with a self-reading rod than with a target rod, and usually the readings can be taken in less time than is required to set the target only approximately correct. Hence, the self-reading rod deserves decided preference for all classes of work, from the coarsest to the finest. There is only one exception known to the writer, and that is on construction work, where, for instance, a number of piles should be sawed off at the same level, or such similar cases, when a target rod would come handy, although many errors are attributed in such cases to slipping or erroneous setting by the rodman.

d. New Self-Reading Rod.—A new self-reading rod, designed by the writer in 1899, is illustrated in Figs. 1, 2 and 3. This design is the result of much experience and careful study, and involves the simplest figures which are discernible at the greatest distance.

The design is based strictly on the self-reading principle, and thus enables the observer to read, practically without counting anything, from the coarsest down to the finest graduation. The fraction of the smallest graduation (2 mm.) is then estimated with a precision proportional to the distance and the quality of the instrument used. This rod is, therefore, well adapted to the coarsest as well as the finest kind of work.

By exercising great care, good readings may be obtained from 1-cm. graduations, but the faculty for close estimating soon tires. Hence, to increase the general accuracy of the work, lessen the strain on the observer, and facilitate progress, a self-reading rod with 2-mm. unit graduations is considered necessary.

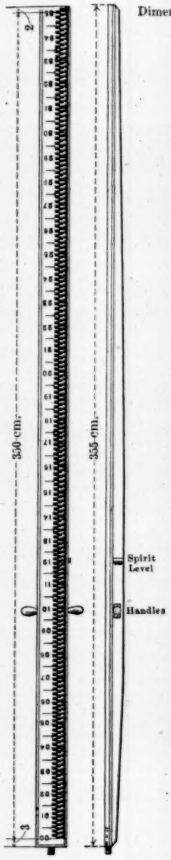


FIG. 1.

Dimensions are in centimeters

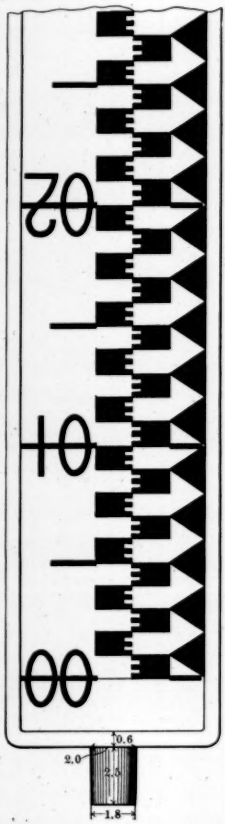


FIG. 2.

SELF-READING
PRECISE LEVEL ROD,
DESIGNED BY
DAVID MOLITOR, M. AM. SOC. C.E.
JAN. 16TH, 1899.

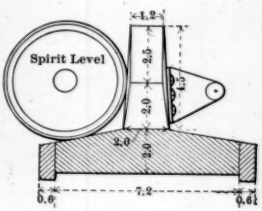


FIG. 3.

The following specifications for a pair of such rods will suffice to give a good description of the rod and its appurtenances:

"Two level rods complete, each with handles, spirit level, plumb-bob and thermometer, shall be furnished, packed in a light pine box provided with hinged cover and locks. The size of rods and style of graduation is shown on the sketch hereto attached (see Figs. 1, 2 and 3).

"A flexible steel scale, 4 m. long, divided into centimeters, except the first 10 decimeters, which shall be divided into millimeters, shall be furnished on a reel 8 ins. in diameter and mounted in a neat carrying case, provided with a handle and lock cover. The scale shall be numbered by decimeters from 0 to 40. The reel shall be so mounted in the case as to permit of drawing out and winding up the scale without removing the reel from the case. A thermometer, as specified below, shall be furnished with this scale, and be attached to the case.

"The rods shall be made of the best seasoned white pine wood, well impregnated with linseed oil and then dried, preferably by application of slight heat. The beaded edges of the rod shall be made of clear hard maple or oak, similarly treated. After the rods have been thoroughly sandpapered, especially the faces to be graduated, they shall be painted a sufficient number of coats of best white zinc paint and thoroughly dried, after which they may be graduated, as shown on the drawing.

"The 2-mm. graduations shall be accurately marked on the painted rods by a graduating machine, and the figures afterward constructed on this graduation shall be painted black.

"Each rod shall be provided with a pair of detachable handles and a watchglass level capable of easy adjustment. These shall be securely fastened to the rod, where indicated on the sketch, by a very substantial fastening, using small brass bolts instead of wood screws. All metal parts and trimmings shall be of brass.

"The pin at the foot of each rod, together with its yoke, shall be cast from best phosphor-bronze, free from lead and very hard. This pin shall be truly faced, making its bottom surface perpendicular to the axis of the rod. Otherwise the pin shall be dressed on an emery wheel and be rigidly connected to the foot of the rod by brass bolts. The distance between the bottom of this pin and the zero of graduation must be made exactly equal in the two rods.

"A mercurial thermometer, registering half degrees Centigrade between + 50° and — 15° (No. 245, Catalogue, H. J. Green) shall be inlaid in the wood of each rod and be covered by a small glass plate. This thermometer shall be placed near and over the spirit level. A similar thermometer shall be provided with the flexible steel scale and be fastened inside the case.

"A small plumb-bob, not larger than 1 in. in diameter, shall be

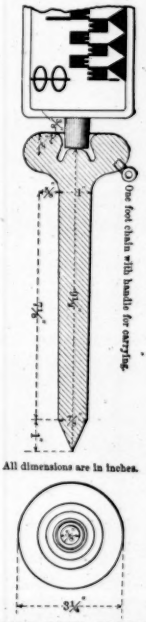
furnished with each rod, and the rods shall be provided with small brass points and hooks between which the plumb-bob may be swung when the spirit level is being adjusted. Such a pin point is shown in plan, in Fig. 3.

"The two rods, with handles, spirit levels, plumb-bobs, etc., shall be packed neatly in a light but durable pine box, in which they may be transported and preserved. Care should be taken to maintain the smallest possible dimensions for this box, as it is not desirable to have this part of the outfit unnecessarily heavy."

Readings taken on this graduation, with Buff and Berger Precise Level No. 2768, showed that the greatest error in a single reading would vary from 0.2 to 0.4 mm. for distances of from 10 to 85 m., respectively. The steel scale is intended for tri-daily rod comparisons to furnish the error of rod length and temperature correction to be applied to the work. The same scale is also intended to replace the rod when reading on bench-marks on which the rod cannot be held.

e. Precise Level Rod Supports.—The supports which have given the best satisfaction consist of steel pins, as shown in Fig. 4. These pins are driven into the ground with a wooden mallet, and, with ordinary care, furnish a support which is as safe as a temporary bench-mark. There has never been the slightest indication of heaving or settling when these pins were used.

The general idea of using a steel pin was originated in France and Germany many years ago, but the form here illustrated, with a slightly different head, was first used by J. B. Johnson, M. Am. Soc. C. E., United States Assistant Engineer, in 1881.



FORGED STEEL TURNING POINT FOR PRECISE LEVELING.

FIG. 4.

It is needless to take up the discussion of pins *versus* foot-plates, as the latter are considered entirely out of date.

BENCH-MARKS.

Two classes of bench-marks are generally used in precise leveling, the permanent and the temporary. The former are designed for permanent and lasting record, while the latter are necessary only in carry-

ing the work between the permanent benches, and thus serve only a temporary purpose.

According to American practice, the temporary benches are made by driving a 60-penny wire spike in the root of a large tree, the root being first dressed off with a hatchet. Frequently, large boulders are found which are preferable to trees. With a cold chisel, a smooth, rounded surface is worked on a suitable part of the boulder, and this point is enclosed by a square, likewise cut into the stone. These benches are always numbered so as to preserve their identity for some time.

The permanent bench-marks are placed either on substantial permanent public or private buildings, on bed-rock, or subterraneous monuments of stone or concrete. The mark itself generally consists of a $\frac{1}{4}$-in. copper or brass bolt cemented into the structure or monument, and is placed horizontally or vertically as each case requires.

The horizontal bolts are made square ended, cemented flush with the masonry and marked with a fine center punch mark for the elevation point. The vertical bolts are round headed and made to project about $\frac{1}{8}$ in. above the stone. The name and number of each bench is chiseled into the stone near the mark.

The American public has not yet reached the generally high regard and appreciation for scientific work found to exist in Germany and France, and, for this reason, it is not always prudent to place marks, intended for permanence and security, in plain sight. Hence, the subterraneous monuments, while they are more difficult to describe and find, are by far the most permanent.

In the more advanced European countries the permanent benches are always placed in plain sight and provided with a cast-iron or bronze name plate, giving the name of the department by which established, the name of the bench and its elevation in meters above mean sea level or normal datum plane. For Germany, this datum is the permanent bench-mark of the Berlin Astronomical Observatory,* at elevation 37 m.

The United States Geological Survey has adopted a uniform system of permanent bench-marks† which is similar to some forms used in France and Germany. While it is sought to overcome the many

* "Handbuch der Vermessungskunde," by Dr. W. Jordan, Vol. 2, p. 456.

† *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 342.

destructive tendencies of our population by specifying the fine for disturbing these marks, it is needless to add that the offence must be proven before a fine or other punishment can be imposed, and to this extent little is accomplished by the law. The safe plan is to conceal bench-marks in such a manner as not to invite the maliciously inclined to do mischief. Yet, this condition will probably be improved in the course of time, and it may become safe to adopt such a form of bench for general use.

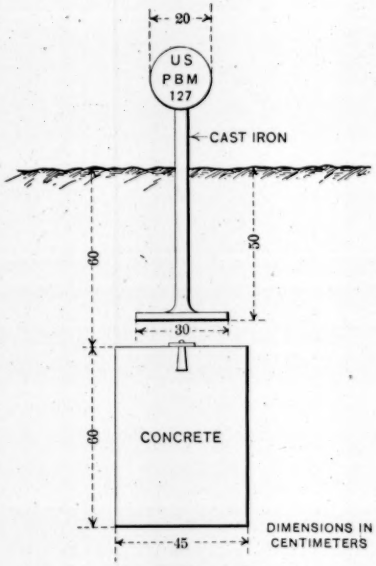


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FIG. 5.

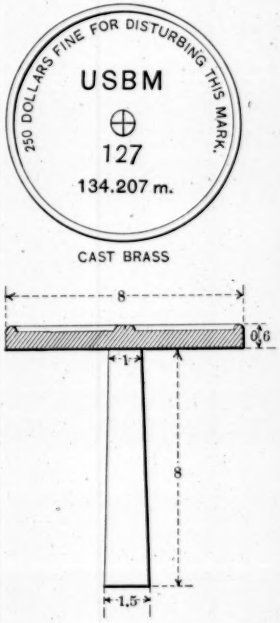

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FIG. 6.

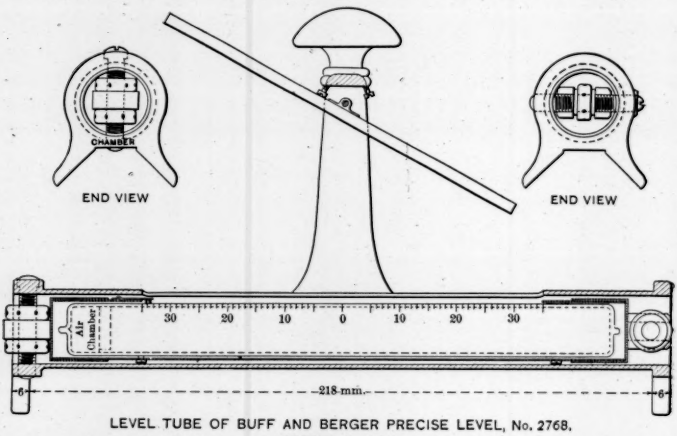
There are many instances where bed-rock or permanent buildings are not available, and then the subterraneous monument forms the best and safest bench-mark. A cast-iron surface marker should be used, and be placed in some definite position with reference to the hidden monument, as, for instance, 5 ft. south. Such a bench is illustrated in Fig. 5. A name-plate suitable for this or for buildings is shown in Fig. 6, and is quite similar to the Geological Survey

mark. When the name-plate is placed in a vertical wall, the point held on should be marked by a fine cross, cut as shown, though this is not necessary when used in a concrete bench.

The number can be stamped on the name-plate at the time of placing the bench-mark, and the elevation would best be supplied on a separate plate, which could be fastened with two small screws.

The concrete base may be made by carefully digging a hole of the required diameter and depth, filling it to the desired level with cement concrete, and finally placing the name-plate before the concrete has set.

There are some features about very sensitive spirit levels, as used on precise leveling instruments, which deserve special consideration.



LEVEL TUBE OF BUFF AND BERGER PRECISE LEVEL, No. 2768.
SECTIONAL VIEW.

FIG. 7.

THE SPIRIT LEVEL.

a. Level Tube.—The level tube is made cylindrical, of extremely hard glass, and is then ground, generally on one side only, to the arc of a circle, whose radius varies from about 50 m. to several hundred meters, depending on the desired sensitiveness. This tube is filled with absolute sulphuric ether, with the exception of a small air bubble, the length of which may be read on a scale graduated on the glass tube. The ends of the tube are closed by fusing (never by cement), and

near one end a ground glass disc with a small notch is inserted in the tube to form an air chamber by which the length of the bubble may be regulated (see Fig. 7). The length of the level tube should be about 200 mm., and its external diameter about 19 mm.

The numbering of the graduations of the level is generally from the center toward the ends, though some makers have adopted a continuous numbering from one end to the other. The writer has used both styles and has reached the conclusion that for spirit leveling the graduation from the center is far more preferable, and does away with all reductions to the center necessary with the new style. The chances of misreading or erroneous centering are also much greater for the continuous graduation. The only argument in favor of the latter is that the reading identifies the end of the level, but in spirit leveling this is not important except when reading bubble displacements, which is done only in connection with determinations of instrument constants.

The level tube is encased in a metal tube and is held in this tube at one end by slight screw pressure against blotting paper or cork wrapping on the glass tube, and the other end is left free to slide on two points against which the glass tube is pressed by a spring. In no instance should the glass tube be cemented into the metal tube at either end. Such rigid fastening is likely to break the tube, and prevents its keeping in adjustment.

This metal case is carried in the casing proper, or frame, in which the level tube may be adjusted laterally at one end and horizontally at the other end.

b. Sensitiveness of the Level Tube.—The sensitiveness of the level tube depends on the curvature of the tube and the length of the bubble. There may also be irregularities due to improper grinding. Grit, not properly removed from the tube after grinding, or corrosion of the glass by the ether, may cause unevenness in the movements of the bubble.

The angular value corresponding to a movement of the bubble over a distance of one graduation (generally 2 mm.) may be found by the use of a level trier, or by rod readings taken with different positions of the bubble. The level trier is very convenient, and especially well adapted to test the uniformity in grinding over the entire length of the tube. However, for field use the rod readings are preferable

and quite as accurate, and the curvature thus determined represents more nearly the actual condition, compared with indoor measurements on a level trier.

The bubble should be retained at an almost constant length of about half the length of the ground portion of the level tube. This makes the air chamber of prime necessity.

A short bubble has a very sluggish movement even in the most sensitive tubes. This is caused by the high friction along the glass compared to the slight lifting power of the small bubble. There is no danger from excessively long bubbles except the practical limit of the graduation.

According to Reinhertz, a level tube of 10.8 seconds per division of 2 mm. gave the mean error of position of bubble for different lengths of bubble as follows:

Length of bubble.....	4.9	13.3	22.2	28.9 divisions;
Mean error of position....	1.0	0.5	0.3	0.3 seconds of arc.

The sensitiveness best suited to high-grade work is between 2 and 5 seconds per 2-mm. division, ranging with the rigidity of the instrument and tripod, the ground traveled over and the delicacy of manipulation of which the observer is capable. The writer has found no difficulty in using a 2-second level tube on Buff and Berger Level No. 2768, even in windy weather, with no other shelter than a large wagon umbrella.

c. Temperature Effects on Level Tubes.—The sensitiveness of a level changes with the temperature. Hence, all level readings, used in such a manner as to involve the value of the level graduation, will have a temperature correction which must be determined previously, and the value of a graduation should be expressed as a function of temperature. To illustrate this effect, the following figures are quoted from experiments by Reinhertz:

(t) temperature, degrees Centigrade,	−2.2°	+9.0°	+ 19.7°	+27.7°
(v) seconds per one division of level,	8.46″	7.65″	6.12″	5.67″

The equation representing the value of one division of this level for any temperature within the observed range is $v = 8.2 - \frac{t}{10}$ seconds.

Rapidly changing temperature causes much annoyance, and introduces an uncertainty which prohibits the use of sensitive levels, even

when shaded by an umbrella. The latter should always be used on precise level work.

The length of the bubble changes materially with the temperature, and the rate of change depends on the sensitiveness of the level. The data in Table No. 2, taken from the above-named experiments, will illustrate this. The length of the bubble is thus inversely proportional to the temperature.

TABLE No. 2.

Value of level per division. Seconds.	Length of Bubble in Divisions.			Change per 1° C.
	0° C.	15° C.	30° C.	
5	32	25	18	0.45 div.
10	30	24	18	0.35
15	26	22	18	0.28
20	24	20	17	0.24
30	23	20	18	0.18

The Telescope.

A brief description of the telescope and its vital parts is considered necessary to a clear understanding of many of the points treated in the following.

a. The Laws of the Lens.—The only forms of lenses used in telescopes for surveying purposes are the double convex and its special case of plano-convex.

The axis of a lens is the line joining the centers of curvature of its two surfaces, and in a plano-convex lens the axis is perpendicular to the plane surface and passes through the center of curvature of the curved surface.

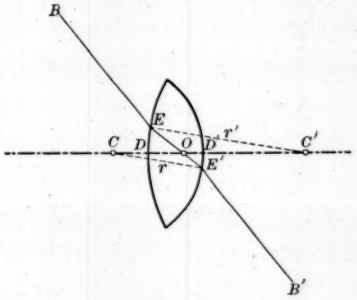


Fig. 8.

The optic center of a lens is the point, any ray passing through which suffers no angular deviation. The only point satisfying this condition is on the optic axis, and divides the thickness of the lens proportionally to the radii of curvature. Hence the simple construction in Fig. 8, for finding the optic center.

The radii r and r' are drawn parallel, and the optic center O is at the intersection of $E E'$ with $C C'$. The ray $C\ C'$ passes through the optic center and is not bent by refraction, while the ray $B\ B'$, not through O, is refracted, making $B E \parallel B' E'$. It is readily seen that in a plano-convex lens the optic center is on the curved surface at the intersection with the optic axis.

The focal length of a lens is found from the following formulas:

For double convex lenses

$$f = \frac{1}{(n-1)\left(\frac{1}{r}+\frac{1}{r'}\right)}$$

For plano-convex lenses

$$f = \frac{r}{n-1}$$

in which n is the refractive index of the glass, and r and r' are the radii of curvature of the lens surfaces.

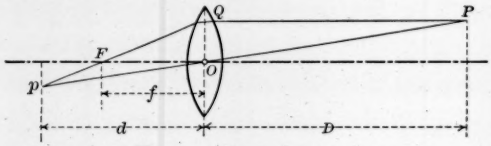


FIG. 9.

The rays from any point of an object distant D, from the optic center of a lens, converge to a focus on the opposite side of the lens, forming an image at a distance d, from the optic center, according to the general law expressed by

$$\frac{1}{D}+\frac{1}{d}=\frac{1}{f}$$

Such a pair of points are called conjugate points, and all rays parallel to the optic axis of a lens, or passing through the optic center of a lens, are called principal rays.

In the above equation when $D = \infty$, $f = d$, giving a ready means of determining the focal length of a lens by direct measurement. When D varies between infinity and $2f$, $d > f$, and the image is always inverted and demagnified, which is the condition of an objective lens. For values of D between zero and f, d becomes negative and the image is magnified but not inverted, which is the case of a magnifying lens or eyepiece.

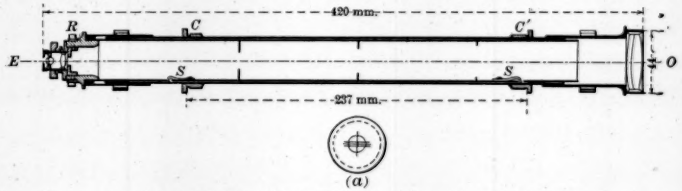
When the position of a point or object is known with reference to a certain lens, the conjugate point or image may be found (Fig. 9) by

drawing the two principal rays through this object; thus $P\ Q$ parallel to the axis of the lens and through the focus F, and $P\ O$ through the optic center of the lens. The intersection, p, of $Q\ F$ produced and $P\ O$, is the image of P. This construction can be readily made when f is known, and applies as well to the case when d is negative.

To correct spherical aberration good lenses are generally made with slightly unequal curvature on the two surfaces, and the flatter surface should face the object.

The foregoing general principles will suffice to make the construction of a telescope understandable.

b. The Telescope.—The telescope consists of a lens forming the objective at O, Fig. 10, mounted in a metal tube into which slides a second metal tube carrying the eyepiece lenses at E and the reticule at R, which latter must be at the focus of the objective.



TELESCOPE OF BUFF AND BERGER PRECISE LEVEL, No. 2768.

FIG. 10.

The eyepiece lenses are arranged so that they may be focused on the threads of the reticule, and both are made to move together, by rack and pinion motion, to bring the reticule to the focus of the objective. Thus the reticule is made to coincide with the inverted and demagnified image of the objective at R, and both are seen through the magnifying lens (or lens combination) of the eyepiece. Hence, the image in such a telescope is always inverted unless a special set of erecting lenses is added to the eyepiece, which is not advisable, owing to loss of distinctness and loss of light caused thereby.

The tube of the eyepiece must slide in the telescope tube without lateral play and in a straight line. To insure this, the eyepiece slide should be very nearly the full length of the telescope and be pressed against one side of the telescope tube by four springs S, placed in pairs 120° apart.

The entire telescope is supported on agate points of a cradle

against steel collars C and C' of exactly equal diameters, and rigidly connected with the tube of the objective forming the body of the telescope. The entire telescope can thus be revolved about an axis passing through the centers of the two collars. This axis is the axis of the telescope or axis of collimation, and, in a perfect instrument, the optic centers of all the lenses and the center of the reticule should be on this axis.

The line joining the optic centers of the eyepiece and objective is termed the optic axis. The line of sight is the line joining the optic center of the objective with the center of the reticule, and is independent of the position of the eyepiece, and not coincident with the axis of the telescope, unless made so by the maker and by adjustment. The extent to which this may be accomplished will be shown later.

c. The Objective.—The objective consists of a double convex lens of crown glass cemented to a concavo-convex or plano-concave lens of flint glass, supplying the very essential property of achromatism. This combination may be regarded as a double convex lens, for all practical purposes, and should have a long focal length so as to give the largest possible inverted image. The laws of the simple lens apply also to this combination.

The diameter of the objective governs the illuminating power of the telescope, and for this reason the objective lens should be as large as possible, consistent with other details of construction. Neglecting loss of light from obstructions in the lens itself and color effect, the relative illuminating power of a telescope is approximately represented by $i = \frac{\delta^2}{5\,m^2}$, where δ = diameter of objective in millimeters, and m = magnifying power of the telescope in diameters. The natural illumination is taken as unity.

d. The Eyepiece.—The eyepiece may consist of a single lens or a combination of lenses. The single lens is subject to spherical and chromatic aberration, and is, therefore, inferior to the combinations which are designed to avoid either or both of these defects. The Ramsden eyepiece, consisting of two plano-convex lenses with the convex surfaces toward each other, corrects spherical aberration, but is not achromatic, while the Kellner and Steinheil combinations are both achromatic and free from aberration.

The eyepiece is simply a magnifier of the image formed by the objective, and, as such, does not influence the errors of the telescope. Very bad mounting would interfere with the sharpness of definition in parts of the field of view, and should be avoided, to the extent of obtaining sharp definition.

The focal length of a compound eyepiece is expressed in terms of the focal lengths of its respective lenses and the distance, a, between their optic centers, by the following formula:

$$f = \frac{f' f''}{f' + f'' - a}$$

e. The Reticule.—This was invented by William Gascoigne in 1640, and consists of a ring interposed in the eyepiece slide of the telescope perpendicular to the telescope axis. It is made adjustable horizontally and vertically by four capstan screws, Fig. 10 (*a.*) Four spider threads are mounted on this ring, one vertical and three horizontal. The two extreme horizontal threads are used for distance determinations, and incidentally give a means of increasing the number of readings to eliminate errors of estimation. They serve an almost indispensable purpose in avoiding misreadings. The intersection of the mean horizontal and the vertical threads marks the aiming point on the line of sight for the eye-end of the telescope.

The interval between the extreme horizontal threads is so chosen that the intercept formed by them on a vertical scale or rod is some even fraction of the distance of such rod from the instrument. Owing to the high magnifying power of the eyepiece generally used on precise levels, this interval should be such that the ratio between rod intercept and distance should not exceed the ratio 1:200, and may be still smaller, up to 1:500, though 1:200 does not approach the limits of sharp definition of the eyepiece, and yet is very convenient for converting rod intervals into actual distance. Smaller ratios, of course, give less accurate distance determinations.

The actual horizontal distance, l, is determined from the rod intercept, i, or wire-interval, by the well-known stadia formula:

$$l = k\, i + c + f, \text{ for horizontal sights,}$$

in which k is the stadia constant, preferably 200 for levels, f is the focal length of the objective, and c is the distance from the center of the instrument to the optic center of the objective.

Hence, for any number, n, of successive settings of an instrument, the continuous distance for a stretch of levels would be

$$L = \Sigma_o^n l = k\, I + 2n\,(c + f)$$

where $\Sigma_o^n\, l$ is the sum of all back and fore sights of the stretch, and $I = \Sigma_o^n\, i$ is the sum of the thread distances, or rod intercepts, for all back and fore sights. This formula seems complicated, but, in practice, its solution is made very simple by tabulating the function $k\, I$ for values of I from 1 to 100. However, when k is exactly 200, a condition which the maker can supply, then no tabulation is necessary.

f. The Steel Collars.—These form a very important feature of the telescope, and, as might be inferred from what has already been said, the direction of pointing of the telescope depends on the condition that these collars are exactly equal and that they remain equal. This condition is never realized in practice, but no serious results need follow from this source, as will be shown later.

Some makers prefer to make the collars of bronze instead of steel, but, with ordinary care, steel collars will not rust, and this is their only objectionable feature, while bronze is very subject to corrosion, especially in contact with grease or oil, and, being generally quite soft, is not nearly as desirable as high steel.

The Sources of Error in Precise Leveling.

1. Introductory.

It is proposed here to show the many sources of error to which the process of leveling is subject, and to demonstrate methods of determining them when they are sufficiently constant to make such determination possible.

As is readily seen, these errors are not all of an accidental nature, and some of them do not depend either on the instrument or the observer.

The variations in gravity at different points and elevations on the earth's surface and the attraction of the sun and moon are constant sources of error, but these, while they may be considered measurable, and should enter into a discussion of this kind, are, nevertheless, of minor importance to the instrumental errors. These latter, as will be shown later, may be eliminated by proper methods and careful work.

leaving as the most dangerous errors those which are caused by rapidly changing temperature and atmospheric conditions.

All the foregoing sources of error have been classified in a manner appearing most logical and natural, and while much that is said is not new, yet a great deal of this work is original and is considered as valuable in throwing light on many difficulties previously attributed to erroneous causes. The writer has aimed to discover, by careful study and research, all the errors to which the process of leveling is subject. Whether or not this aim has been materialized remains for future investigation to prove.

2. Effect of the Spheroidal Shape of the Earth on Long Level Lines.

a. Force of Gravity.—The direction of the plumb line at a point on the earth's surface is the resultant of the force of gravitation and the centrifugal force caused by the earth's rotation. Since the earth is an ellipsoid it follows that both gravitation and centrifugal force change their relative values for different points on the earth, and thus affect the resultant which governs the direction of the plumb line.

"The attraction which the earth exerts upon a body at its surface is the sum of the partial attractions which each part of the earth exerts upon the body, and the resultant of all these attractions may be considered to act from a single point—the center. Hence, if the earth were a perfect sphere, a given body would be equally attracted at any point of the earth's surface. The attraction would, however, vary with the height above the surface."*

In order to compare the force of gravity for different points and different elevations, it should always be corrected to correspond to sea level.

Since the force of gravity is inversely proportional to the square of the distance from the earth's center, and calling g_o this force at mean sea level, g' the force of gravity at an elevation h in free air above sea level, and R the radius of the earth, then

$$\frac{g'}{g_o} = \frac{R^2}{(R+h)^2} = 1 - \frac{2h}{R} + \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For a point on a high plateau of elevation h above mean sea level, the force of gravity is affected by the mass of this plateau and becomes g''. If δ denotes the density of the material of the plateau, δ_m the

* Ganot's Physics, 1883, Art. 82, p. 66.

mean density of the earth, Δg the effect produced by the mass of the plateau on the point h, exclusive of the attraction produced by the earth, and k^2 a constant, then $\Delta g = 2 \pi k^2 \delta h$ (nearly).*

The attraction of the earth alone (using the same constant k^2) is

$$g_o = \frac{4}{3} k^2 \frac{\pi \delta_m R^1}{R^2} = \frac{4}{3} k^2 \pi R \delta_m \quad \ldots\ldots (2)$$

The ratio $\frac{\delta}{\delta_m}$ is generally about $\frac{1}{2}$, as the density of the interior earth is about twice that at the surface, hence

$$\frac{\Delta g}{g_o} = \frac{3 \delta h}{2 \delta_m R} = \frac{3 h}{4 R} \quad \ldots\ldots (3)$$

But the force g_o corrected for the elevation h, without regard to the mass of the plateau, would be, from Equation (1),

$$g' = g_o \left(1 - \frac{2 h}{R}\right) \quad \ldots\ldots (4)$$

Hence, since $g'' = g_o + \Delta g + (g' - g_o)$, the actual attraction g'' found from Equations (3) and (4) is

$$g'' = g_o \left(1 + \frac{3 h}{4 R} - \frac{2 h}{R}\right) = g_o \left(1 - \frac{5 h}{4 R}\right) \quad \ldots\ldots (5)$$

In applying the foregoing to a revolving ellipsoid like our earth, other theories are involved which are considered in the following.

The foregoing equations apply, strictly speaking, only to a sphere, but, within knowable limits, they also apply to an ellipsoid, provided allowance is made for the change in direction of the gravity forces resulting from the latter shape, in which the forces no longer intersect in the center of the mass, and, for a general assumption regarding distribution of mass, these forces cannot be assumed normal to the surface of the ellipsoid.

Since, according to the Kant-Laplace theory, the earth is an ellipsoid of revolution, produced from a fluid or semi-fluid mass, by the influence of attraction and centrifugal force, it follows that the resultant of all the forces acting at a given point of the earth's surface must be normal to the surface at the given point.

* See Helmert, Höhere Geodesie, II, pp. 142 and 164.

The mass attraction g', of the ellipsoid Fig. 11, on a point p of its surface, may be resolved into two components G and Q; the former, being the larger component, is directed toward the center of the earth at O, and the latter is normal to G at p.

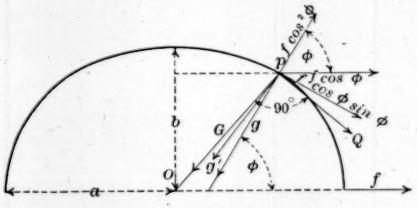


FIG. 11.

The centrifugal force at the equator is $f = a\,v^2$, in which v is angular velocity. For a latitude ϕ this force becomes $f_\phi = a \cos \phi\, v^2 = f \cos \phi$. This centrifugal force may also be resolved into the component $f \cos^2 \phi$, normal to the surface at p, and the tangential component $f \cos \phi \sin \phi$, the latter coinciding very nearly in direction with Q. The resultant of the gravity and centrifugal forces at p will then be g, which makes a small angle with g' and the angle ϕ with the equator. In other words, g is normal to the ellipsoid at p and makes the angle of the latitude with the equator.

Hence, it follows that g, the force or acceleration of gravity, at p, is proportional to $\sin^2 \phi$* as expressed by the formula

$$g = 9.7800\ (1 + 0.005310 \sin^2 \phi) \text{ m. per second} \ldots\ldots\ldots (6)$$

For latitude 45° this becomes

$$g_o^{45} = 9.7800\ (1 + 0.002655) = 9.80597 \text{ m. per second at sea level} \ldots (7)$$

By substitution in (6) for $\sin^2 \phi = \frac{1}{2}\ (1 - \cos 2\,\phi)$ and combining (6) and (7) then

$$g = g_o^{45}\ (1 - \beta \cos 2\,\phi) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which g_o^{45} at sea level = 9.80597 m. and $\beta = 0.002655$, a constant.

For any other point in free air and at an elevation h, Equations (1) and (8) give :

$$g = g_o^{45} \left(1 - \beta \cos 2\,\phi - \frac{2\,h}{R}\right) \ldots\ldots\ldots\ldots (9)$$

and for a point on a high plateau of elevation h, Equations (5) and (8) give :

$$g = g_o^{45} \left(1 - \beta \cos 2\,\phi - \frac{5\,h}{4\,R}\right) \ldots\ldots\ldots\ldots (10)$$

* See Helmert, Höhere Geodesie, II, pp. 241 and 609.

b. Dynamic and Orthometric Heights on the Earth's Surface.—The elevation of a point h above sea level, determined by direct measurement on a vertical line, gives the orthometric height of that point.

The dynamic height of this point is represented by the work required to raise a unit mass through the height h.

Since the force of gravity varies at different points on a meridional plane, as represented by Equations (9) and (10), it follows that the amount of work required to raise a unit mass through the orthometric height h, at different points on a meridian, will vary inversely as the force of gravity at these points. Calling g_1 and g_2 the forces of gravity for two points on the ellipsoid in latitudes ϕ_1 and ϕ_2, respectively, and h_1 and h_2 the orthometric heights corresponding to equal dynamic effects at the two points, respectively, then the following relations must exist:

$$g_1 h_1 = g_2 h_2 \text{ or } \frac{h_1}{h_2} = \frac{g_2}{g_1} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Now, from Equation (10),

$$g_1 = g_o^{45} \left(1 - \beta \cos 2\phi_1 - \frac{5 h_1}{4 R}\right) \text{ and } g_2 = g_o^{45} \left(1 - \beta \cos 2\phi_2 - \frac{5 h_2}{4 R}\right)$$

from which

$$\frac{g_2}{g_1} = \frac{h_1}{h_2} = \frac{1 - \beta \cos 2\phi_2 - \frac{5 h_2}{4 R}}{1 - \beta \cos 2\phi_1 - \frac{5 h_1}{4 R}} = 1 - \beta (\cos 2\phi_2 - \cos 2\phi_1) \text{ nearly.}$$

$$= 1 + 2\beta \sin (\phi_1 + \phi_2) \sin (\phi_2 - \phi_1).$$

Calling ϕ the mean latitude between the two places and L the meridional distance or difference in latitude measured in kilometers, then

$$\sin (\phi_1 + \phi_2) = \sin 2\phi$$

and $\sin (\phi_2 - \phi_1) = \frac{L}{R} \cos \left(\frac{\phi_2 - \phi_1}{2}\right) = \frac{L}{R}$ nearly, since $\frac{\phi_2 - \phi_1}{2}$

is generally a very small angle and its cosine nearly equal to one. Hence

$$\frac{h_1}{h_2} = 1 + 2\beta \sin 2\phi \frac{L}{R} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$\text{or } h_1 - h_2 = 2\beta \sin 2\phi \frac{HL}{R} \text{ nearly} \ldots\ldots\ldots\ldots\ldots (13)$$

in which H = mean elevation of the two points above sea level. The factor HL (see Fig. 12) then becomes equal to the projected area of the elevation or plateau (upon which the two points h_1 and h_2 are

situated) upon a middle meridional plane. Calling this area $HL = A$ and substituting for $\beta = 0.00265$ and $R = 6\,370\,000$ m. in Equation (13), then

$$h_1 - h_2 = 0.832\ A \sin 2\phi \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

If A be given in square kilometers, in Equation (14), $h_1 - h_2$ will be in millimeters.

Equation (14) then represents the orthometric error produced by the dynamic effect of the earth on the plumb line in passing between points ϕ_1 and ϕ_2 at a mean level H above sea level. It will be seen that this error is confined to meridional distance alone, since a section of the earth parallel to the equator is always a circle and accordingly the force of gravity is constant for a given latitude. This would be true for any imaginary sea level above the present level.

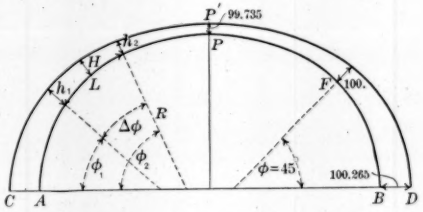


FIG. 12.

Considering $A\ P\ B$ (Fig. 12) the present sea level, and then supposing that this level were raised to a spheroidal surface $C\ P^1\ D$, as in the case of a flood, the following conditions will be found to exist:

From Equations (8) and (11) it is seen that equal dynamic heights at the points B (equator), F (for $\phi = 45°$) and P' (at pole), will bear the ratio of

$$B : F : P' = 100.265 \text{ m.} : 100.000 \text{ m.} : 99.735 \text{m.}$$

Hence such a new water level would not form a concentric curve with the present spheroid, but would have the shape indicated in Fig. 12. This leads to the following conclusion as regards the gravity effect on absolute elevations as referred to mean sea level.

It is readily understood that in a line of precise levels run along the earth's surface on a meridian, and at sea level, from the equator to the pole, the closure with sea level at the pole would be zero. But if a return line were run from the pole to the equator, taking the path indicated by the line $P'\ D$ (Fig. 12), and then closing back on the point B, the complete loop, even if free from all error, would close with a spheroidal error of $99.735 - 100.265 = -0.530$ m.

In a similar manner Equation (14) then represents the error introduced by gravity into any line of levels run between points on latitudes ϕ_1 and ϕ_2, or through a meridional arc ϕ (in degrees), in which A represents the projected area of the level profile upon a middle meridional plane, taking the mean sea level for the axis of ordinates. Also, $h_1 - h_2$ represents the gravity error, and for a point north of the starting point this error is positive or additive to the elevation of the terminal point to give true sea level. The error is negative for a point south from the starting point.

While the above derivation is not rigid and the resulting Equation (14) is only approximate, it is believed that the accuracy attained by its use is quite within practical limits.

It should be noted that the local deflection of the plumb line, as might be caused by a mountain, when leveling up on one side, is not considered in the foregoing.

3. Disturbing Influence of Differential Attraction of the Sun and Moon on the Direction of Gravity, and Its Effect on Spirit Leveling.

The discussion of this subject is abstracted from the Report of the Coast and Geodetic Survey, 1887, p. 196, from the article of the above title by Charles A. Schott, Assistant.

The following formulas for the disturbance in direction of gravity by the action of the sun or moon as a result of the diurnal variations in position of these luminaries with respect to any point of the earth's surface, were the result of an investigation by Dr. F. R. Helmert*:

Let M = Mass of the sun or moon;
$M_\odot$ = Mass of the earth;
p = Horizontal parallax of the luminary;
z = Zenith distance of the luminary;
A = Azimuth of the luminary, counting from south around by west;
α = Azimuth of the level line, counting from south around by west;
l = Length of a single shot, in meters;
P = The disturbing effect on the direction of gravity;

then $$P = \frac{3\,M \sin^3 p}{2\,M_\odot \sin 1''}$$

* Höhere Geodesie, Vol. 2, Chapters 5 and 7; Leipzig, 1880-84.

and the southerly deviation of the disturbed zenith will be

$$S = -P \sin 2z \cos A,$$

and the westerly deviation will be

$$W = -P \sin 2z \sin A.$$

In consequence of the disturbed direction of gravity, the position of the level surface at any point on the earth will be changed, and this will affect the pointing of a level.

If, for instance, a level line is run in a southwesterly direction the fore sights will be depressed in consequence of the disturbed vertical by the angle

$$\gamma = S \cos \alpha + W \sin \alpha = -P \sin 2z \cos (A - \alpha) \ldots\ldots (1)$$

and the correction to the rod readings will be $+ l\gamma$ on the fore sight and $- l\gamma$ on the back sight. Hence, the correction to difference of elevation will be

$$2 l P \sin 2z \cos (A - \alpha) \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Introducing numerical values in the above expression for P, the maximum effect for the moon becomes $P = 0.0174'' = 0.000\ 000\ 083$ in arc, and for the sun $P = 0.0080'' = 0.000\ 000\ 043$ in arc. Thus, for any one instrumental setting this effect is altogether insensible, but it may become sensible by accumulation in extended lines of levels.

Let it be supposed that the sun and moon are in the equator, or have zero declination, and that the leveling is done in the northern hemisphere, then, in the expression (2), z in the first quadrant will be accompanied by A in the first or fourth quadrant, and z in the second quadrant will be accompanied by A in the second or third quadrant. Hence, in leveling from north to south, or for $\alpha = 0$, this expression will always be positive, whereas for an east and west line ($\alpha = 90°$) a change in sign may take place and compensation will, to some extent, diminish the effect under consideration. The circumstance that the luminaries assume different declinations does not, in the main, alter the general effect.

For level lines in a north and south direction there will remain some average effect on the resulting difference of elevation which cannot be eliminated by a repetition of lines. However, for an east and west line, the lunar effect may be eliminated and the solar effect be considerably diminished. The actual effect on a line may not exceed two-thirds the maximum value, and will generally be less than one-half of the same.

A line of levels, 100 km. long, running due south, might incur a maximum producible error from the moon of + 8.3 mm., though the actual error would probably not exceed 4 or 5 mm. The sun alone would produce about half as much as that estimated for the moon.

For a duplicate (direct and reverse) line of levels, 1 000 km. long, in an east and west direction, during the summer months, running forenoons always westerly and afternoons always easterly, Dr. Helmert estimates that the results of the two levelings may differ, in consequence of differential solar attraction, by as much as 87 mm., although the mean result would not be affected.

Whether or not such an effect could be demonstrated with the present methods and instruments seems somewhat doubtful, as it would appear that so small an error as 0.08 mm. per kilometer would be entirely concealed by the existing probable uncertainties of leveling.

However, the following observations made by the writer in 1898–99, while running a duplicate line of precise levels along the St. Lawrence River, show something of this kind.

The general direction of the line was from northeast to southwest, over a distance of about 207 km., run in 167 separate stretches, during August, September, October and November, 1898, and May, 1899.

Table No. 3 shows the number of closures of each sign for the four combinations of direct and reverse lines run forenoons and afternoons.

TABLE No. 3.

	Stretches Run.							
	Direct, A. M. Reverse, P. M.		Direct, P. M. Reverse, A. M.		Direct, A. M. Reverse, A. M.		Direct, P. M. Reverse, P. M.	
No. of stretches.....	30	25	17	23	19	14	18	21
Sign of closure.....	+	−	+	−	+	−	+	−
Percentage	82% minus.		74% plus.		73% minus.		85% plus.	

Of the 167 stretches, 84 closed with a plus error and 83 with a minus error, showing that this effect was practically eliminated in the entire work by properly dividing up the work between forenoons and afternoons. This division was as follows: 88 direct lines run A. M.; 79 direct lines run P. M.; 73 reverse lines run A. M.; and 94 reverse lines run P. M.

The stretches tabulated in the third and fourth combination, in which direct and reverse lines were run in the same half of the day, vitiate to some extent the law sought, because it there depends on which one of a pair of lines was run first. As, for instance, of the 33 lines run direct and reverse in the forenoon, if 16 were run direct in the earlier part of the forenoon, and 16 were run reverse in the earlier part, then there should be an equal number of errors of each sign. The same would apply to lines run in both directions in the afternoon.

Again, the work done in clear weather, which generally prevailed, involves a much larger accidental error than is found in work done on cloudy days, and this again vitiates the distinct manifestation of a definite law. However, the general result of the first and second combination in Table No. 3 exhibits a fair indication of the effect here considered.

Hence, it might be concluded that, for an east and west line, this error can be eliminated by dividing the work equally between the two halves of the day, while for a north and south line the error would not be manifested by the work, although it might exist to the same extent.

However, it is quite likely that this is merely a seeming detection of errors from the source here considered, and that the real error is one due to temperature, as discussed further on.

4. Errors of the Level Tube.

a. Parallelism of the Level Tube with the Vertical Plane through the Telescope Axis.—When the level tube is not parallel to the vertical plane through the telescope axis a slight lateral tilting of the striding level will cause the bubble to run to one end of the tube, and this would introduce an error of pointing unless the level tube were always placed with its axis in a horizontal plane, which is impracticable. There must always be a slight play laterally to permit of readily removing the tube from its supports and reversing it. However, the play necessary for this purpose is very small.

Parallelism of the level tube with the vertical plane through the telescope axis can be readily established by adjustment with the horizontal adjusting screw, and should be continued until a lateral tilt of the level tube no longer displaces the bubble. This adjustment is called the lateral adjustment of the level tube, and is not likely to

change much in a well-constructed instrument, but the adjustment should be tested occasionally. It is very important that a level should be free from error of this kind, because the observations cannot be made to eliminate such errors, and it is one of the few instances in which the accuracy of the work depends absolutely on the adjustment of the instrument.

b. The Horizontality of the Level Tube.—This is indicated by the position of the bubble with respect to the graduation on the glass tube of the spirit level. This graduation may be continuous from one end of the tube to the other, or it may extend from the center of the tube toward its ends, which latter style is generally more convenient, although the first always identifies the end of the tube read. Still, this is quite unnecessary, since the bubble ends can be identified in other ways, and readings taken from the center outward require no reduction, and are, therefore, preferable.

The striding level, which should always accompany a precise leveling instrument (see Fig. 7), is so arranged that it may be read either in a direct or reversed position, and the construction permits of adjusting the inclination of the level tube to the points of support on the telescope collars. This may be done by clamping the instrument, after it has been made approximately level by the small spirit levels, and then reading the striding level in its direct and reversed position. One-half the discrepancy thus obtained is then taken up on the vertical adjusting screw at the chamber end, and the other half is corrected by the micrometer on the wye-cradle. This operation should be repeated until the bubble remains in the center for both direct and reversed positions. When this has been accomplished, the level tube is parallel to the axis of the telescope, the lateral adjustment having first been made and the telescope collars being exactly equal. The effect of unequal collars will be treated later.

While this adjustment would generally eliminate the error of horizontality of the level tube, it is found that no very sensitive level tube will hold this adjustment very long, and it often happens that the first two or three reversals of a level may show perfect adjustment, and a fourth reversal may indicate a considerable error. The converse also frequently occurs, especially if the adjustment is tested soon after removing the level from the box.

Many disturbances of the level tube will be caused by fine particles

of grit on the telescope collars, sudden changes in temperature and wind, cooling effect of rain drops, should any fall on the level, and passing clouds which change the direct and reflected rays from the sun. All these are ever present to affect the horizontal adjustment of the level tube, and it is altogether out of the question to place any reliance on a level holding this adjustment. It is not a question of quality and fineness of the instrument; on the contrary, the more sensitive the level the more readily will its adjustment be affected, and this is no detriment, as many have claimed, but is a positive safeguard, and keeps the observer constantly informed of all deleterious influences that may exist.

The effects of temperature and length of bubble have already been discussed. Suffice it to say here that the variations in curvature of a level tube from temperature effect vitiates the use of the level for any measurements in which the value of this curvature is involved, while it does not in any way affect the utility of the level in the determination of the horizontal when the bubble is always brought to the center. This is one of the prime reasons why the older method of reading the bubble in any position was less accurate than the method of taking readings with the bubble in the center.

It follows, then, that no reliance should be placed on the capability of a level tube for holding its horizontal adjustment, and hence there is no possibility of measuring this error and applying a correction therefor to the observations. The only rational elimination of this error can be accomplished by taking observations for each sight with the level tube direct and reversed. The mean of such a pair of sights, taken in quick succession, always eliminates the bubble error, whatever its amount may be. Hence, the observer need only keep the level adjustment within reasonably small limits, and carefully avoid dust between the contact points of the level tube and the telescope collars; then, repeated pointings, with level tube direct and reversed, will always eliminate the bubble error.

c. The Parallax between the Reflected Images of the Bubble and the Level Tube Graduation.—This, as seen in the mirror, constitutes an error in level pointings which is apparent from the diagram, Fig. 13.

The bubble a—b, shown in the center of the level tube and reading 15 at each end, is seen at a'—b' in the mirror m—n, while the image of the graduation is seen projected upon a different plane, thus giving

rise to a parallactic error. In the diagram, the mirror image would show the eye end of the bubble at 14.1 divisions and the object end at 15.7 divisions from the center of graduation. The mean error due to parallax would then be 0.8 division.

Thus, while there may be considerable error in the apparent position of the bubble as seen in the mirror, this error remains constant so long as the inclination of the mirror remains the same and the eye is always in the same position. Both these requirements are practically fulfilled. Hence, if back sights and fore sights are equal, this error is eliminated. It is this difficulty which is solved in such a complicated manner by the prismatic combination attached to the French precise level, Fig. 2, Plate II, without any urgent necessity.

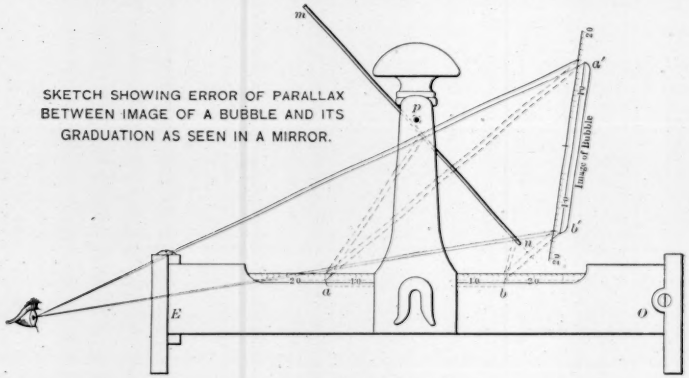


FIG. 13.

With a little study of the diagram, Fig. 13, it is readily seen that for a constant position of the eye, a certain inclination of the mirror will make the parallax of the bubble ends equal and opposite, thus affecting only the apparent length of the bubble, as seen in the mirror, but not its horizontalism. This angle is readily found, and small set-screws provided on the mirror or mirror-frame will limit the motion of the mirror to the desired angle (see Fig. 7). The eye can, with little practice, be placed near enough to the same position to reduce this error, due to parallax, to insignificance, and thus make the mirror the simplest and best device for viewing the bubble while taking the rod reading. With little practice the observer can learn to

observe both the rod and the bubble simultaneously, an advantage which cannot be overestimated.

Another feature in favor of the mirror, as applied to high-grade levels, is that the less the curvature of a level tube, the less will be the error due to parallax, supposing that the mirror is not adjusted. Thus, suppose the parallax for a given case to amount to 0.5 bubble division, then if the level have a curvature of 10 seconds per division this would cause an error of 5 seconds in pointing, while for a 2-second level tube the error in pointing would be only 1 second. Hence, a poorly adjusted mirror might prove serious on a common coarse level, while on a very sensitive level the error might be inappreciable.

d. The Error of Pointing.—As influenced by the sensitiveness of the level, this is an important question in precise leveling, and many are of the opinion that the accuracy of work attained with a level of 6 to 10 seconds curvature is equal or superior to that attainable with a 2-second level. However, this assertion is not sustained by practice or theory, and is largely the result of prejudice and lack of experience on the part of the person holding such views.

An illustration of the relation between curvature and error of pointing of level tubes is given by the experiments of Reinherz in Table No. 4, in which the length of one division of the bubble is 2 mm. This applies to a large number of readings, while a single reading may readily be in error by ± 0.1 division.

TABLE No. 4.

Sensitiveness of tube.	Mean Error	
	of centering bubble.	of reading bubble.
2.7 seconds of arc.	± 0.135 seconds = ± 0.045 div....	± 0.27 seconds = ± 0.09 div.
4.5 "	0.18 " 0.036 "	0.315 " 0.063 "
9.0 "	0.27 " 0.027 "	0.45 " 0.045 "
18.0 "	0.36 " 0.018 "	0.765 " 0.036 "
27.0 "	0.45 " 0.013 "	1.08 " 0.036 "
54.0 "	0.63 " 0.009 "	1.44 " 0.027 "

From these figures it is seen that the accuracy attainable in centering the bubble is far greater than that attainable by reading the bubble in any position and making the reduction to the center. This again proves the desirability of observing always with the bubble in the center, in preference to reading the bubble and reducing to the center.

Accordingly, for a 9-second level, the mean* error of centering would affect a rod reading for a distance of 100 m. by ± 0.5 mm. which is greater than the permissible error of closure for a stretch of 100 m. For a level tube of 2.7 seconds curvature, the mean* error for the same length of shot would be ± 0.1 mm., which is about equal to the error of estimation. Hence, while the coarser level might enable one to close a stretch with a comparatively small error of closure, such fact would only indicate an accidental compensation of errors, but would not be a measure of the excellence of work equally producible with such a coarse level. With the finer level, an accidental cumulation of errors might cause a comparatively wide closure on a given stretch, but this would then be the most unfavorable result possible and would probably still remain within the assigned limits for good work.

The length of the bubble is also a factor in the accuracy of pointing, as was previously shown, and should be considered in this connection. A short bubble creeps very slowly and does not indicate the slight and sudden changes which immediately start a long bubble. A similar effect is produced by certain conditions of damp, cold weather and by levels of about 6 seconds curvature and over. A very sensitive level can be forced to the center by the micrometer and a reading may be taken at once, while in a coarser level the bubble must be allowed sufficient time to stop creeping before the reading can be taken. Hence wind and other disturbing influences produce comparatively more annoyance when working with a coarse level.

5. Errors Pertaining to the Telescope.

a. The Reticule Error.—In a properly adjusted level, the reticule should be sharply seen through the eyepiece. There should be no blurred effect at any point of the threads within the field of distinct vision, and the threads should show no parallax when the eye is moved slowly up and down in front of the eyepiece. When the eyepiece lenses and the reticule are all perpendicular to the telescope axis, there will be no difficulty in preventing all errors from this source by properly focusing the eyepiece on the threads of the reticule.

When the horizontal threads of the reticule are not absolutely horizontal, the readings taken at different points of these threads may

* Note (added by author after reading discussions).—The word "mean" should be omitted. The comparison is made for a single reading where the bubble may be ± 0.1 division out of center.

differ materially, and, to prevent errors from this source, the horizontal threads must always be read exactly at the intersection points with the vertical thread. As an additional safeguard, the horizontality of the threads should be made adjustable by a simple adjustable stop device to check the movement of the telescope in its cradle a exactly the desired horizontal position when the telescope is moved in azimuth. This adjustment, properly made for the normal and inverted position of the telescope, will prevent all errors from lack of horizontality of the threads.

The thread interval should be as small as possible and yet permit of making distance determinations with an accuracy of at least one in a thousand. For a magnifying power of 50 diameters a thread interval of 1 in 200 will satisfy this requirement. When threads are widely separated, they will not permit of sharp focusing on all threads simultaneously, and the widely divergent rays will traverse strata of air of different temperature, density and humidity. Many serious errors may result from a slightly excessive thread interval.

The thread interval is not absolutely constant, and, strictly speaking, is dependent on the temperature of the reticule frame. If the reticule be of brass, the distance determination should be reduced to a standard temperature by using the coefficient of expansion for brass. However, the distance determination need only be relative between back and fore sights, and, as the change in temperature would not be sufficiently great to affect this relation, it may be neglected. It is valuable, however, in making repeated determinations of the value of a certain thread interval, to note the different results for different temperatures.

The most serious error in connection with the reticule is its position with reference to the axis of the telescope. The level tube is the starting point to which the direction of the telescope must be referred. Hence, the center of the reticule (formed by the intersection of the mean horizontal thread with the vertical thread) should be on the axis of the telescope, or axis of collimation, which can be accomplished by adjustment of the reticule, provided the optic center of the objective lens is also on this axis, a condition which is rarely ever fulfilled by the maker. Since the objective is fixed in the telescope tube no adjustment is possible, hence what is ordinarily called collimation adjustment does not generally accomplish the object implied

by this term. If it were possible to bring the line of sight into coincidence with the telescope axis, as defined by the centers of the collars, by adjustment of the reticule alone, then the commonly applied term "collimation adjustment" would be correct, but this not being the case it is better to adopt the more precise term "thread or reticule adjustment," which will be used in the following.

When the line of sight does not coincide with the telescope axis, then the former is inclined to the position indicated by the bubble, and the error resulting from such inclination can only be eliminated by repeated readings taken with the telescope normal and inverted, always keeping the bubble in the center. This is true whether or not the collars of the telescope are equal.

The reticule might be so adjusted that for a certain length of sight the readings of the middle thread would be the same for telescope normal and inverted, but this adjustment would not hold good for any other length of sight unless the optic center of the objective lens be in the axis of the telescope. Hence, the thread adjustment alone will not generally eliminate errors of collimation, but repeated readings in normal and inverted positions of the telescope will effectually eliminate collimation errors of whatever magnitude.

With this method a very good line of levels can be run by reading only one of the extreme threads; the mean of the two readings will always agree closely with the mean of the mean thread readings. This subject will be more clearly represented in the following, in connection with the mounting of the lenses.

In closing this subject it should be noted that the thickness of the telemeter threads is a very important matter in the attainable accuracy of readings at long distances. The threads should be the finest possible, otherwise the length of sight must be limited to suit the quality of the threads.

b. Errors Due to Defective Mounting of the Lenses.—As has been stated previously, a slight eccentricity in the mounting of the eyepiece lenses produces no deflection in the line of sight, though it changes the optical axis of the telescope. If the eccentricity is bad, or the axis of the lens is not parallel to the line of sight, the definition may be poor, and thus affect the sharpness of the extreme threads, or even prevent obtaining a sharp image simultaneously on all threads. This is then a serious defect which must be remedied by the maker.

Displacements of the objective lens are far more serious, and affect the line of sight by the amount of such displacement. The line of sight is not affected by oblique mounting of the objective, but this fault will interfere with the definition, as in the case of eyepiece obliquity.

Any lateral displacements of the eyepiece or objective may be readily discovered by focusing on a distinct object and then carefully revolving the telescope on its horizontal axis. The image and the threads (even when the thread adjustment is perfect) will appear to move in the arc of a circle about the axis of the telescope.

It is easily seen that for a telescope, with laterally displaced objective, the thread adjustment, when performed, does not bring the line of sight into coincidence with the axis of the telescope (collimation axis). When the center of the reticule is so adjusted that it coincides with an image for both normal and inverted positions of the telescope, then the center of the reticule, the optic center of the objective and the object sighted on are in a straight line called the line of sight, but this line is neither parallel nor coincident with the telescope axis except when the thread adjustment was performed for an object at an infinite distance and the optic center of the objective is on the axis of the telescope. Hence, for thread adjustment on an object at a finite distance, D, and the objective not truly mounted, the line of sight will be inclined to the telescope axis by an angle ε, expressed in seconds by the formula,

$$\varepsilon = \frac{a}{D \sin 1''}$$

in which a is the lateral displacement of the optic center of the objective from the axis of the telescope. Thus, for a displacement $a = 1$ mm. and thread adjustment performed for the usual distance $D = 50$ m. the inclination ε becomes 4.1 seconds or 0.02 mm. per meter of distance.

While this displacement a is generally quite small, yet the effect produced thereby on the line of sight should not be overlooked.

This error should be eliminated in the observations in the manner indicated for thread errors, viz.: by reading with telescope normal and inverted.

c. Movement of the Eyepiece Slide.—This is frequently very defective, even in high-grade levels, and constitutes a serious detriment in any instrument, because any obliquity of motion of the eyepiece with

respect to the line of sight affects the thread adjustment, since the reticule moves with the eyepiece. The same difficulty is encountered with telescopes in which the objective is made to slide, as in Fig. 2, Plate III.

When the sliding tube (be it eyepiece or objective) is short, its motion may be very irregular, a circumstance rendering the telescope almost worthless for precise work. The sliding tube should be as long as is permissible by the telescope tube, and all lateral motion should be prevented by spring pressure, as shown in Fig. 10. With such an arrangement the direction of motion will be a straight line even if it does not coincide with the axis of the telescope, and the repetition of readings in normal and inverted positions of the telescope will again eliminate any error caused in the thread adjustment as a result of inclined motion of the eyepiece or objective.

A method for determining the regularity in the eyepiece movements will be given under the head of instrument constants.

d. Error Resulting from Inequality of the Telescope Collars.—Any existing inclination in the line of sight, not explained in the foregoing, and inherent in the instrument itself, is presumably caused by unequal telescope collars. The commonly accepted method of determining the inequality will be briefly given, while no further use will be made of this method, owing to its many imperfections, which will be presently demonstrated.

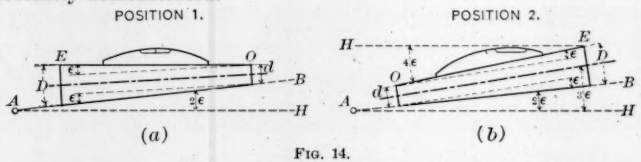


FIG. 14.

The common method of determining the inequality of the collars is illustrated in Fig. 14, and the operation consists in setting up the instrument on a rigid stone pier, and, after leveling up carefully, the limb of the instrument is tightly clamped and the bubble is read for the telescope pointing direct, as in Fig. 14*a*. The level and the telescope are then reversed together on the cradle supports, as in Fig. 14(*b*), taking care not to jar the instrument. When the bubble has become quiet a second reading is taken. This constitutes one determination, as indicated in the following example, where the bubble is read from the center and is in perfect adjustment.

The figure shows that the angular inclination measured by the two positions of the bubble (whether in adjustment or not)* is four times the angle between the axis of the collars and the elements of the cone enveloping the collars. The line $A\ B$ represents the position of the cradle supports, which remains constant during the experiment, and Position 1 makes the upper element of the cone horizontal, or at least with the bubble in the center, while in Position 2 the bubble indicates a displacement toward the larger collar equal to four times the error in axial pointing of the telescope, or twice the collar inequality.

Example.

Position 1.		*Position 2.*	
Bubble readings:		Bubble readings:	
right-hand end.	left-hand end.	right-hand end.	left-hand end.
16.5	16.5	13.1	20.1

0 .....center of bubble at..... —3.5

or $4\ \varepsilon = 0 - (-3.5) = +\ 3.5$ or $\varepsilon = +\ 0.875$ bubble divisions.

The value of one division of the bubble being 6.5 seconds, $\varepsilon = 0.875 \times 6.5 = 5.688$ seconds. The mean of twenty observations gave $\varepsilon = 5.90$ seconds $= 0.0276$ mm. per meter.

This method assumes that the collars are perfectly circular, that the level tube actually indicates the direction of the topmost element of the enveloping cone of the collars, and that the points of contact between each collar, the level tube and the cradle supports, are on the arc of the same circle; an assumption which is rarely true of a new instrument and never true of an old one.

The telescope does not rest on the lowest element of the collars, but on the points s, Fig. 15(a), because the two collars penetrate the angular aperture of the forked cradle supports unequally. Hence, the level tube pointing does not involve the true inclination of the upper elements of the cone formed by the collars, with respect to the points of support s. The same is true of the contact points c between the level tube and the collars, Fig. 15(b).

It would generally be assumed that this error is beyond the limits of measurement for the slight inequality usually existing between the collars, and could, therefore, be neglected. The case, however, demands investigation.

* NOTE (added by author after reading discussions.)—This should read "when in exact adjustment," as illustrated by the example. For bubble not in adjustment, it must be read direct and reverse for each position of the telescope.

Using the dimensions as indicated in Fig. 15(*a*), and calling ε the correction for collar inequality, as previously determined by the common method, w, the actual inclination of the telescope axis resulting from collar inequality, and t* the vertical distance between s' and s, then the following approximate relations may be used to determine w and t:

$$w = \varepsilon \sin \frac{\beta}{2} + t, \text{ and } w = \frac{t}{\cos^2 \frac{\beta}{2}}$$

Thus, for the Buff and Berger Level, No. 2768, $\varepsilon = 0.0276$ mm. per meter, and $\beta = 90°$; hence, by substituting and solving, $t = 0.0195$ and $w = 0.0390$ mm. per meter.

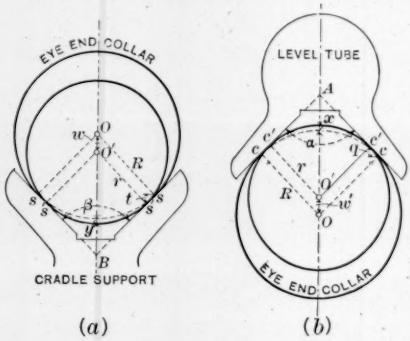


FIG. 15.

By making allowance for the fact that in the determination of ε by the ordinary method, this quantity is involved four times in the reversal of the telescope in the cradle supports, while t enters only twice, the inclination for telescope axis resulting from inequality of collars should be $\varepsilon + 2\,t = 0.0666$ mm. per meter.

The effect of the collar inequality on the contact points of the level tube is opposite to that produced by the cradle supports, and is eliminated when the level tube is read always in the same position relatively to the telescope. However, when the level tube is read direct and reverse the actual inclination of the telescope axis is smaller than the above value w by the amount $\frac{q}{2}$, found from Fig. 15(*b*), by a similar process of reasoning as applied to Fig. 15(*a*).

* NOTE (added by author after reading discussions).—The quantities t, q, w, w' and ϵ are all linear dimensions for collars 1 m. apart. Hence they also represent tangents for the angular deflections under consideration.

For $\alpha = 89° 30'$, t is practically equal to q, and $\frac{q}{2} = 0.0097$, which, subtracted from the last corrected value, gives the final resulting inclination $p = 0.0666 - 0.0097 = 0.0569$ mm. per meter.

This is all based on the assumption that the collars are perfectly circular, and that the four contact points of each telescope collar are in a plane normal to the telescope axis, all of which is not generally true. The worthlessness of the common method of finding the collar inequality is thus made apparent.

It is readily seen that the axial inclination of the telescope, resulting from collar inequality, is measured by the position of the level tube with respect to the collars, but bears no relation to the cradle supports s, except when the latter become involved by some method as the above, which is worthless because of this fact. Hence, the wear on the collars cannot be determined in any manner involving the contact on the cradle supports. Also, the accuracy required in the determination is beyond limits of calibration. To add to this difficulty, the cradle supports are never in the same planes with the level tube contacts of the respective collars, so that this question must be solved by another method.

The resultant axial inclination from unequal collars may be made constant by confining the wear of the collars on the cradle supports to a special path on each collar and never revolving the telescope on its axis without first removing the striding level, so that the wear between the level and collars is practically impossible and is independent of the cradle supports. The relation between the telescope axis and the striding level then remains constant, and any wear caused by the continual revolving of the telescope, whatever its amount, does not affect the telescope pointing as indicated by the level tube. All this is done simply by making the distance between the cradle supports different from that between the forked ends of the striding level, or by supplying two pairs of collars, one for the cradle contact and the other for the level tube. This is exactly in contradiction to past practice, because the axial inclination still existing, while it would thus remain practically constant, appeared indeterminate by any method of direct measurement with the bubble.

A very simple and extremely accurate method of determining the real inclination between the telescope axis and the striding level,

devised by the writer, and considered to be a most satisfactory solution of this and a number of other uncertainties connected with the level, will be given under the head of instrument constants.

The complexity of the axial error resulting from collar inequality is made apparent by the foregoing considerations, and the disparity shown to exist between the actual conditions and the commonly accepted method of determining the inequality in telescope collars is quite sufficient to explain many of the constant errors frequently encountered in leveling operations and attributed to a great variety of causes, principally personal equation and settlement or heaving of instrument or rods. This error may, however, be eliminated by a strict equalization of back and fore sights, and hence becomes quite insignificant when the nature of the error is properly understood.

6. Errors Pertaining to the Level Rods.

a. The Error in Verticality.—This generally receives little consideration, further than to prevent excessive inclination of the rods by using watchglass levels, which are frequently adjusted. However, the subject demands very careful attention.

If for some reason the rod levels are out of adjustment and the rods are not held plumb, all readings become too large, and this would produce errors when working over inclined ground. Such errors are of a somewhat constant nature and may work serious harm, hence the importance of frequent and careful tests of the rod adjustment.

b. The Errors of Temperature and Graduation.—These errors, though widely different in cause, are conveniently considered together, since both may be corrected in one operation.

Generally, the temperature effect is a constant change per degree of temperature within the limits during which work should be conducted. The same is true of errors of graduation, which are generally proportioned to the distance measured on the rod.

There are cases, however, where neither the temperature nor the graduation correction are constant, and for such rods the true length for each portion of the rod must be measured with a correct standard for all working temperatures, these values being then tabulated for use. This is the French system, wherein the rods are purposely graduated irregularly to prevent the observer from knowing what his

closures are until the rod lengths have all been reduced and corrected. It is, no doubt, better policy to simplify this matter by a uniform graduation and to place a little more confidence in the integrity of the observer.

The combined graduation and temperature correction may be applied in one of two ways: By determining, by frequent comparison with a standard measure, the law for the length of the rod in terms of the standard and temperature, or by direct comparisons with a steel tape of known length.

The latter method is preferable because a steel tape is not affected by humidity and gradual changes from seasoning as is the case with wood. The law of the length of the steel tape once determined may be used almost indefinitely, while the rods are continually undergoing changes. The coefficient of expansion for pine is about 0.000004 per degree Centigrade, and the maximum changes from humidity may amount to 0.3 mm. per meter in the course of a whole summer.

The necessary thermometers and steel scale have already been described, and it will be sufficient here to show only the method of correcting this error.

The general formula for the length of a metallic scale is $1^m = 1 \pm k + \alpha\ t$ in which k is the graduation error, α the coefficient of expansion, and t the temperature in degrees above the normal.

The corrections for one meter of the tape are computed, for every degree of temperature, and these corrections are then tabulated for use.

The temperature and the measured length of one meter on each rod are recorded at the beginning and end of each stretch, and the average is taken to represent the mean rod length for the stretch. The tape correction for the mean temperature is then applied to this rod length, and the difference of elevation is accordingly corrected. When the rods are very nearly alike, as they should be, and always alternate in regular order, this method will prove very satisfactory and not at all complicated.

The past practice of applying a constant rod correction for a whole season's work is not scientific or admissible. The correction should be determined in the field by direct tape comparisons and be applied by stretches.

c. The Error of Estimation.—The smallest, or unit, graduation generally used on precise level rods is the centimeter, though a smaller

unit is very necessary. Rod readings should be as accurate as the telescope pointings, which would necessitate the possibility of estimating with an accuracy of $\pm$ 0.1 mm. at a distance of 100 m. This could be attainable only with graduations as small as 2 mm. All this is on the assumption that the individual rod readings should be of the same accuracy, or excellence, as is indicated by the closure of a stretch of levels, which is not generally true because the error of estimation is a compensating error and may be large on individual shots and finally become zero for a long stretch. This is actually verified in practice.

However, it frequently happens that a stretch is only 50 m. long, and unless a single pair of sights can be taken with an accuracy of $\pm$ 0.1 mm. or less, such a stretch could not be closed with the desired exactness except by many repetitions of readings.

To show the relative value and accuracy of rod readings under various conditions it will be necessary to analyze the effect produced on the appearance of the rod by the distance at which the rod is placed from the instrument, and the position of the thread on the unit graduation.

The apparent magnitude of the unit rod graduation (generally 10 mm.) is expressed by the formula $a = \frac{0.25\,t}{l}\,m$, in which t is the unit rod graduation in millimeters, m is the magnifying power of the telescope and l the length of shot in meters. The constant represents the distance, in meters, at which an object would appear full size to the normal, naked eye.

Table No. 5 gives the apparent magnitude of unit rod graduations for different lengths of shot and magnifying power of 50 diameters, as obtained from the foregoing formula.

TABLE No. 5.

Length of shot.	5	10	20	30	40	50	60	70	80	90	100	meters.
a for t = 10 mm.....	25.00	12.50	6.25	4.17	3.12	2.50	2.08	1.79	1.56	1.28	1.25	millimeters.
a for t = 5 mm.....	12.50	6.25	3.12	2.08	1.56	1.25	1.04	0.90	0.78	0.70	0.63	millimeters.
a for t = 2 mm.....	5.00	2.50	1.25	0.83	0.62	0.50	0.42	0.36	0.31	0.28	0.25	millimeters.

The apparent thickness of good reticule threads will be 0.1 mm., or less, and this would enable the observer to estimate at least one-fourth of the 2-mm. graduation at 100 m., which, combined with the

error of pointing, would bring the total error of a single reading just within the assigned limits of high-grade work.

To estimate only to the nearest millimeter on a 10-mm. graduation would be quite out of the question for a shot which is shorter than 30 m., though this would already exceed the allowable limit. For the 5-mm. graduation the desired accuracy could still be attained on a 20-m. shot, and for a 2-mm. graduation it could be expected for a 10-m. shot. When the respective shots are smaller than those just mentioned, the apparent magnitude of the various unit graduations becomes too large to permit of accurate estimations. Hence, the 2-mm. graduation is the best suited to permit of readings within the required accuracy for all shots between 10 and 100 m.

As to the comparative accuracy of estimating tenths of a division, the following considerations are offered: The 0.5 or middle point of a space can be ascertained with the greatest accuracy, while the one-third or two-thirds points are more difficult to estimate. The quarter points are quite indefinite. This circumstance is a very important factor in the science of estimating, which is often lost sight of. All estimations should be based on the half, third and quarter points, the tenths are practically guesses, if otherwise determined.

According to some experiments made by Kummer,* the error of estimating the individual tenths, calling the error on the 0.5 point unity, is as follows:

Tenths..	0	1	2	3	4	5	6	7	8	9
Error...	1.28	1.37	1.47	1.42	1.17	1.00	1.07	1.23	1.35	1.47

This again points to the desirability of estimating on the smallest possible unit in order that the error of estimation may become insignificant.

Another circumstance affecting the accuracy of estimation is found in the ever-changing illumination, and the definition of the telescope, which latter depends largely on the quality of the illumination.

The difficulty of estimating on 10-mm. graduations with an accuracy commensurate with high-grade leveling is thus clearly shown, and the advisability of adopting a 2-mm. graduation, as suggested by the design of leveling rod proposed by the writer, is made apparent.

* Zeitschr. für Verm., 1897, p. 261.

7. Miscellaneous Errors.

a. Errors of Reading and Recording.—Misreadings may frequently occur, especially by inexperienced observers, and it sometimes happens that the recorder notes a reading erroneously. However, with experienced men errors of this class are very rare, though the possibility of introducing them into the work should be reduced to a minimum.

Various precautions are used to prevent such errors, even when made, from creeping into the work. Some of these are quite effective, and the system adopted by the writer may be regarded as almost absolute protection against errors of both kinds. At least, no stretch ever failed to close, as a result of such errors.

To prevent misreadings, the system of reading on three horizontal threads was introduced many years ago, and is still a valuable check, though it frequently happens that the erroneous reading cannot be identified except when the back and fore sights are known to have been made nearly equal. Also, on short sights, where the decimeter is the same for the three threads, it may happen that all three readings are read wrong by a decimeter. Hence, this method does not afford sufficient assurance of the prevention of misreadings, and a further precaution is necessary.

The French system, formerly used, was for the observer to take the first set of readings and then have them repeated independently by the recorder, and if both sets checked, the work was accepted, otherwise the process was repeated. However, this is slow and unsatisfactory, because the two persons would generally estimate differently on the rod, and thus introduce an element of doubt and divided responsibility.

As will appear later, the method of observing, adopted and used by the writer, necessitates two pairs of readings to eliminate instrumental errors, and incidentally this affords an almost infallible check, as the two sets of sights are independently and separately taken. When, in addition to all this, the observer trains himself not to remember any figures or readings, which is the only true scientific manner of observing, the same errors are not repeated and there remains little opportunity for an error passing undetected by the recorder.

It is also a bad practice, generally followed, to have the recorder repeat back the readings called off by the observer. It was found that this is both annoying and time-consuming and tends to create confusion and introduce errors rather than to prevent them. Absolute quiet

should reign about the instrument when observations are being taken. No talking should be done except by the observer, and he should confine himself to only such utterances as are to be recorded in the notes. When a set of readings is completed the recorder signifies their correctness before the instrument is carried forward, otherwise any existing disparity must first be reconciled.

This system checks the correctness of both the recording and reading, and the order of entering the readings (as will be seen later) is such as to afford a very ready comparison of those readings which ought to correspond.

b. Heaving or Settling of the Instrument or Rods.—This has often been cited as the cause of constant errors, but this explanation was usually offered in want of something better.

There is no doubt that disturbances of this kind take place occasionally when working over soft, springy, or frozen ground, but, when this is the case, the bubble never fails to show the danger, and the observer must then be on his guard.

The turning point pins, Fig. 4, when well driven, are scarcely susceptible to any heaving or settling of sufficient magnitude to be readable with an instrument, even at a very short range. When the instrument is affected it is hardly possible that the three legs of the tripod will move alike, and then the bubble will certainly indicate the disturbance long before it has attained a measurable magnitude.

This subject, therefore, disappears into insignificance, especially when the nature of the constant errors, frequently met, is more fully explained.

c. Air Vibrations and Refraction.—The former are caused by a complex refraction of the line of sight in passing through the air while it is undergoing a mixing process or agitation produced by the heat effect of the sun. Air vibrations become a maximum at or shortly after noon while the earth absorbs heat, and are most rapid and of greatest amplitude for clear, quiet days with high humidity. They cease when the sun has declined to the point where the earth no longer takes on heat and again appear in the coolness of the late afternoon, when the earth begins to radiate the heat absorbed during the day. These vibrations are diminished during the time when the sun is shut off by passing clouds and are entirely checked when the cloudiness is of sufficient duration.

TABLE No. 6.—EXPERIMENTS TO SHOW

JULY 2D, 1899. BACK SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Time.	Bearing to the sun.
		dcm.	dcm.	dcm.	dcm.	dcm.		
1.....	5	14.434	15.198	15.964	....	.....	4.15 A. M.	Before sunrise.
		14.438	15.202	15.968	.....	.....	.....	
		14.436	15.200	15.966	15.201	1.530	.....	
1.....	5	14.540	15.290	16.056	.....	.....	10.00	S. 80° E.
		14.544	15.304	16.060	.....	.....	.....	
		14.542	15.297	16.058	15.299	1.516	....	
1.....	5	14.536	15.294	16.054	.....	.....	10.45	S. 45° E.
		14.536	15.296	16.052	.....	.....	.....	
		14.536	15.295	16.053	15.295	1.517	.....	
1.....	5	14.532	15.294	16.056	.....	.....	11.00	S. 30° E.
		14.538	15.298	16.056	.....	.....	.....	
		14.535	15.296	16.056	15.296	1.521	.....	
1.....	5	14.538	15.296	16.056	.....	.....	12.00 M.	S.
		14.536	15.296	16.056	.....	.....	.....	
		14.537	15.296	16.056	15.296	1.519	.....	
1.....	5	14.536	15.296	16.054	.....	.....	1.30 P. M.	S. 50° W.
		14.534	15.296	16.056	.....	.....	.....	
		14.535	15.296	16.055	15.295	1.520	.....	
1.....	5	14.536	15.292	16.054	.....	.....	2.30	S. 70° W.
		14.530	15.292	16.050	.....	.....	.....	
		14.533	15.292	16.052	15.292	1.519	.....	
1.....	5	14.540	15.300	16.060	.....	.....	3.00	S. 75° W.
		14.538	15.298	16.056	.....	.....	.....	
		14.539	15.299	16.058	15.299	1.519	.....	
1.....	5	14.540	15.298	16.058	.....	.....	3.35	S. 80° W.
		14.536	15.296	16.058	.....	.....	.....	
		14.538	15.297	16.058	15.298	1.520	.....	
1.....	5	14.538	15.296	16.056	.....	.....	4.15	W.
		14.540	15.298	16.058	.....	.....	.....	
		14.539	15.297	16.057	15.298	1.518	.....	
1.....	5	14.540	15.300	16.060	.....	.....	5.00	W.
		14.538	15.296	16.056	.....	.....	.....	
		14.539	15.298	16.058	15.298	1.519	.....	
1.....	5	14.540	15.298	16.058	.....	.....	5.35	W.
		14.536	15.292	16.052	.....	.....	.....	
		14.558	15.295	16.055	15.296	1.517	.....	

Refraction, in the ordinary sense of the word, is a slowly changing deflection of the line of sight in passing obliquely through the concentric air strata surrounding the earth. This form of refraction changes most rapidly at sunrise and sunset, and is practically constant at noon. Since it changes slowly and with some degree of regularity, and is noticeable only on comparatively long sights (500

THE EFFECT OF HEAT ON LEVEL ERRORS.

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Difference of elevation	Remarks. Weather.
		dcm.	dcm.	dcm.	dcm.	dcm.	mm.	
2.....	1	11.564	12.318	13.068	.....	.....	.....	16.7° C. Reading good,
		11.566	12.322	13.072	.....	.....	.....	air calm. Foggy,
		11.565	12.320	13.070	12.318	1.505	+ 288.3	damp.
2.....	1	11.660	12.426	13.178	.....	.....	.....	Clear. 33.9° C. Read-
		11.666	12.434	13.196	.....	.....	.....	ing fair; light warm
		11.663	12.430	13.187	12.427	1.524	+ 287.3	wind.
2.....	1	11.662	12.420	13.184	.....	.....	.....	35°C. Bub. adj. clear.
		11.660	12.420	13.180	.....	.....	.....	Light warm wind.
		11.661	12.420	13.182	12.421	1.521	+ 287.4	Reading fair.
2.....	1	11.660	12.424	13.186	.....	.....	.....	35° C. Reading poor.
		11.664	12.424	13.186	.....	.....	.....	Clear. Light warm
		11.662	12.424	13.186	12.424	1.524	+ 287.2	wind.
2.....	1	11.670	12.424	13.190	.....	.....	.....	35° C. Reading poor.
		11.662	12.424	13.186	.....	.....	.....	Air slightly un-
		11.666	12.424	13.188	12.426	1.522	+ 287.1	steady. Light warm wind.
2.....	1	11.660	12.420	13.180	.....	.....	.....	33.3°C. Reading good.
		11.660	12.422	13.184	.....	.....	.....	Sky overcast. Light
		11.660	12.421	13.182	12.421	1.522	+ 287.4	warm wind.
2.....	1	11.662	12.420	13.182	.....	.....	.....	32.2°C. Reading good.
		11.656	12.420	13.178	.....	.....	.....	Clear. Light W.
		11.654	12.420	13.180	12.418	1.526	+ 287.4	wind.
2.....	1	11.660	12.418	13.180	.....	.....	.....	30° C. Reading good.
		11.658	12.420	13.180	.....	.....	.....	Cloudy; strong cool
		11.659	12.419	13.180	12.419	1.521	+ 288.0	breeze from W.
2.....	1	11.662	12.422	13.182	.....	.....	.....	30° C. Reading fair.
		11.660	12.420	13.180	.....	.....	.....	Partly cloudy, light
		11.661	12.421	13.181	12.421	1.520	+ 287.7	W. wind.
2.....	1	11.660	12.416	13.178	.....	.....	.....	30.2° C. Reading fair.
		11.662	12.422	13.180	.....	.....	.....	Clear; light W.
		11.661	12.419	13.179	12.420	1.518	+ 287.8	wind.
2.....	1	11.660	12.420	13.180	.....	.....	.....	28.9° C. Reading fine.
		11.658	12.416	13.178	.....	.....	.....	Cloudy; cool air;
		11.659	12.418	13.179	12.419	1.520	+ 287.9	light W. wind; storm approaching.
2.....	1	11.660	12.420	13.180	.....	.....	.....	27.8°C. Reading good.
		11.654	12.412	13.174	.....	.....	.....	Slight rain, cool and
		11.657	12.416	13.177	12.417	1.520	+ 287.9	calm.

m. or more), little or no appreciable effect is produced thereby on level readings, unless a considerable time elapses between taking back and fore sight readings late or early in the day. Constant refraction would introduce no errors, as it would affect back and fore sights equally.

The air vibrations can always be avoided by sufficiently reducing the length of sight, but when good readings cannot be taken at 25 m.

the work should be suspended. Generally, the heat effect is such that good work is impossible long before this condition is reached. This will be shown in the following:

Some very interesting experiments* on this subject have been made by Leonard Sewall Smith, Assoc. M. Am. Soc. C. E.

d. Direction of Line, Time of Day and Season of Year.—The variety of effects produced on the results of levels by these conditions has been intimated but never explained. It is believed that these effects are all due to variations in the position of the sun and the intensity and direction of the sun's heat rays at different times during the day and year.

To prove this theory, the writer conducted a set of experiments which are given in part in Table No. 6. A great many readings, which are merely repetitions of those given, are omitted for the sake of brevity. These additional observations add nothing except to show conclusively that the changes found were substantiated by duplicate readings, and there could be no possible doubt as to the facts. Only the characteristic observations are here given. The experiments were made July 2d, 1899, near New Baltimore, Mich.

The instrument was set up on level ground and was carefully shaded with a large wagon umbrella. The rods were rigidly held in a vertical position and supported by the usual steel pins, driven flush with the ground. The rods and instrument were on a straight line, bearing N. 33° E. Rod 5 was at Point 1, or the south end of the line, and Rod 1 was at Point 2, or the north end of the line; both were 30.4 m. from the instrument, thus eliminating all instrumental errors by equalizing the back and fore sights. The rods were graduated to 2 mm., thus reducing the errors of estimation to about ± 0.05 mm. for the set of two readings of three threads each.

The readings were taken by the usual manner of observing, viz.: With the telescope normal and the level tube direct, the back sight (three threads) was read first, and then, without in any way disturbing the instrument, not even to refocus, the fore sight was read, completing the first set. The level tube was then reversed, meanwhile inverting the telescope, and the second set of readings was taken, commencing with the fore sight. The means of the two sets for each

* Bulletin of the University of Wisconsin, Vol. 1, No. 5, June, 1895, pp. 101–145. See also results of observations made by C. H. Van Orden, M. Am. Soc. C. E., in *Transactions*, Am. Soc. C. E., Vol. xxxix, 1898, p. 387.

thread and for the two sets are written on the third line of each observation. All readings were taken with the bubble in the center, and the instrument was not disturbed in any manner during the whole day, except after the first observation, which is taken with a different height of instrument, and the sights are not so nearly equal as for the subsequent observations. Otherwise the notes are self-explanatory.

Since all instrumental errors, except those due to collar inequality, are eliminated by the observations, the mean readings should always be the same unless influenced by atmospheric and temperature effects. Also, if back and fore sights are equally affected, the difference of elevation between the two points should remain constant. However, the difference of elevation did not remain constant, and back and fore sight readings changed, but the changes were unequal.

This phenomenon was not produced by refraction, because the sight (30.4 m.) was altogether too short to show refractive changes. It was not the result of air vibrations for the same reason, and in fact repeated readings by different observers never disagreed by more than 0.1 mm. No material vibrations could be observed which indicated sudden or rapid changes going on, except when passing clouds shut out the sun. The maximum producible change on the rods, due to the range of temperature gone through by the experiments, could not exceed 0.04 mm., which is too small to warrant consideration. The absolute temperature could not have been the cause, for the readings at 4.15 A. M. and 3 P. M., while they agree, were taken at widely different temperatures, and others taken at only slightly different temperatures near noon, show the widest departures.

The following explanation is given to account for this large error, amounting to 0.9 mm. in a distance of 60.8 m. during the hottest part of July 2d, 1899, and becoming zero at about 8 A. M. and 3 P. M. on that day: The afternoon limit was usually between 4 and 5 P. M., but the cloudiness brought on the normal conditions much earlier on the day in question; in fact, the notes indicate that whenever the sun was completely shut off the readings came back to normal. It, therefore, follows that at certain times during the hottest portion of the day the sun affects instrument pointings differently for different directions of pointing, and that before and after this critical portion of the day such an effect does not exist even though the temperature remains

high. Hence, the direction and intensity of the sun's rays seem to be the entire cause of the error.

It appears from the observations that the back-sight readings, pointing southward and toward the sun, do not show much variation, while the fore-sight readings show a maximum variation of 1 mm. This might be explained by the effect of the reflected heat rays from the ground being partly cut off by the observer while reading the fore sight and not while reading the back sight.

The heat effect from above remained about constant. This would tend to curve the eye end of the telescope downward on the fore sight and thus incline the line of sight upward, while the back sight would receive heat from above and below and remain practically unaffected.

For different ground and other umbrella covers, the effect may be very different or even reversed. Hence the importance of more experiments in this direction, and the necessity of confining level work to the early and late hours of the day during warm weather.

The foregoing experiments show that a direct line run shortly after sunrise might differ in result from a reverse line run between 10 and 11 A. M. by practically 0.9 mm. per 60 meters, or 15 mm. per kilometer, the error being entirely in the reverse line. Again, a direct line run between 10 and 11 A. M. might close well with a reverse line run between 1 and 3 P. M., yet both be materially in error.

While errors from this source cannot be eliminated in any possible manner, their presence may be detected by always closing a stretch during the same half of the day. If, then, the closure is within the allowable limits, it is safe to assume that the temperature effect just described did not exist or was extremely small. When the closure is wide, due to this source of error, then the most erroneous line is the one run nearest to noon. An experimental line southward, on a clear warm day, between 2.30 and 4.15 P. M., without any umbrella, differed from the result obtained by duplicate lines on a cloudy day by 100.6 mm. The length of the line was about 1 km., and the one run without the umbrella was too high.

This coincides in sign with the foregoing experiments, though the closure is very large and might be attributed to a decimeter error had not a foot-rod been used on the line in question,

From this it is seen clearly that the umbrella is absolutely indispensable, and a large umbrella of close white duck lined with two

thicknesses of heavy green or black cloth, leaving air spaces between the several coverings, is best suited to the work.

When high-grade work is necessary during the warm summer months, conditions permitting, it would be advisable to run the levels during the night. This will be found far more satisfactory, and failures to close will scarcely ever occur. Such a programme is feasible only when the line passes over good even roads and with proper equipment for illuminating the rods and the bubble.

The best months in the year for precise level operations in the Northern and Central States are April, May, September, October and November, and sometimes March and December. It is best not to work during June, July and August, though, with extraordinary exertions, good results may be obtained. No good work can be done during the cold winter months.

In closing, the following conclusion may be drawn respecting duplicate lines of levels:

1. Lines run simultaneously in the same or opposite directions may close well, yet both be in error, depending on the time of day during which they were run.

2. Lines run in the same or opposite directions during symmetrical parts of the day, with respect to noon, may close well and both be erroneous, depending on the heat conditions.

3. Lines run in the same or opposite directions at different times of the same half of the day may close wide, and the most erroneous line will be the one run nearest to noon.

Instrument Adjustments and Constants.

1. Introductory.

In taking up this subject it is necessary to mention in advance some of the controlling features of accurate level observations in general.

The instrument must be handled with great care, and should always be securely clamped while being carried. The observer should always carry the level tube and umbrella, leaving the umbrellaman to transport the instrument, thus enabling the observer to remain quiet and preserve a steady hand for manipulating the level. The instrument must always be shaded, and a cloudy day is very desirable when determining instrument constants. No work should be done during the heat of the day.

A base line, 100 m. long, should be accurately measured on even ground, marking the 0, 10, 20, 40, 50, 60, 80 and 100-m. points by oak stakes securely driven, with a spike in each on which to hold the rod.

The instrument should be planted firmly and must not be disturbed or shaken. Visitors should be kept at a safe distance.

The telescope should be sharply focused within the reading limits of the reticule, and the temperature of the instrument should not be affected by handling.

2. Instrument Adjustments.

Different instruments may require slightly different adjustments, but, in general, those necessary for an instrument of such a type as the Buff and Berger Precise Level No. 2768, Plate I, apply to nearly all high-grade instruments. These adjustments are as follows:

a. The Adjustment of the Small Tubular Levels.—For coarse setting of the instrument, this adjustment is conducted as for the plate levels of a transit, and would be the same for a watchglass level such as used on the Kern instrument, Fig. 1, Plate II. The levels are brought to the center by means of the leveling screws, and the instrument is then revolved 180° about its vertical axis. If the bubbles remain in the center they are in adjustment, otherwise adjust for half the error and repeat the process.

b. To Make the Horizontal Threads Horizontal.—These threads should be horizontal for both the normal and inverted positions of the telescope. Adjust the stop device until the mean horizontal thread gives the same reading on a rod when the previously leveled instrument is swung slightly in azimuth, so that the rod passes through the entire field of the telescope. This is done for the telescope normal and inverted, and should be repeated until no appreciable error remains.

c. To Make the Thread Adjustment.—Sight on a rod placed about 50 m. away. Read three threads with telescope normal and level tube direct, then invert telescope and read the three threads again, having the bubble in the center each time. The difference between the means for the two sets is the thread error, which is corrected by moving the reticule by one-half the amount of the error. The test is then repeated. This error may readily be reduced to 0.2 mm. in 50 m. and should not be allowed to exceed 1 mm. for the same distance.

After making this adjustment it is well to test the mounting

of the objective lens, which is done by watching the image of an object while the telescope is being revolved. If the image and the reticule appear to move together in a circle, then the objective is eccentrically mounted. Another test is to read a rod with bubble in center, then unscrew the objective lens sufficient to move it through 180° and repeat the reading. Allowing for the play between the threads, the two readings should coincide.

d. To Make the Level Tube Parallel to the Telescope Axis.—This is done by two adjustments, the lateral and the vertical. The instrument is carefully leveled and clamped, the bubble is brought to the center, and then the level tube is tilted laterally and the bubble allowed to come to rest. If this position of rest is not in the center, then the proper end of the level tube must be adjusted laterally until the bubble remains in the center for all positions of tilt permissible between the forked lateral supports. This adjustment should be carefully made and be frequently tested, as errors due to such lack of adjustment cannot be corrected.

The vertical adjustment of the level tube is performed by clamping the instrument in a level position, bringing the bubble to the center, and then reversing the level tube on the telescope collars. If the bubble comes to rest in the center, after reversal, the level tube is in adjustment, otherwise adjust half the error on the vertical adjustment of the level tube and take up the other half on the leveling screws. Repeat the operation until the error does not exceed one second in pointing, or one 2-mm. division of a 2-second level. Strictly speaking, the level tube is made parallel to the elements of contact between the level tube and the telescope collars, while the telescope axis is determined by the relative sizes of the collars, which are generally unequal. This subject is taken up later.

e. The Parallax of the Level Tube Mirror.—This is removed by adjusting the inclination of the mirror to such an angle that the bubble is seen in the same position whether viewed in the mirror by the observer with his eye at the eyepiece of the telescope, or as seen directly by a second observer. This adjustment should be carefully made for the length of bubble to be used, and generally requires no further attention.

f. The Verticality of the Rods.—This must be tested three or four times daily by suspending a plumb-bob between points provided on

the rods for this purpose. The rod is supported against a house or tree, away from the wind, and the rod level is centered when the plumb-bob shows the rod to be vertical.

3. Determination of Magnifying Power.

There are several methods of accomplishing this, and the two simplest will be here given. Both are based on the same principle.

a. First Method.—The rod is held at a convenient distance, L, from the instrument, and the apparent size, T, of the image produced by a decimeter space (0.1 m.), is taken in a pair of dividers held at a certain distance l from the eye. This is accomplished with considerable accuracy by viewing the image with one eye and the dividers with the other. The apparent size of the image is then measured on a scale, and the distance from the eye to the dividers must be measured by an assistant, or, if the dividers are held opposite the objective, the length of the telescope may be used. Thus, for magnifying power $m = \frac{TL}{0.1\ l}$ the Buff and Berger Level No. 2768 gave, for $T = 0.105$ m., $L = 20$ m., and $l = 0.42$ m., $m = \frac{0.105 \times 20}{0.42 \times 0.1} = 50$ diameters.

b. Second Method.—The following method is somewhat simpler, though not so well suited to telescopes of high magnifying power. The instrument is sighted on a rod set as close as the focusing slide will permit. The thread interval is then read through the telescope in the ordinary manner and also in its apparent size, as seen projected against the rod, by viewing the image through the telescope with one eye and the rod with the other. The ratio between the two intervals thus obtained is the magnifying power. Thus, for the same instrument placed 10 m. from the rod, the interval read through the telescope was 0.47 dcm. and the apparent interval projected on the rod was 23.50 dcm., hence $m = \frac{23.50}{0.47} = 50$ diameters.

4. Determination of One Division of the Level Tube.

A rod is held at some even distance from the instrument, and the bubble is brought to rest near one end of the tube, but within limits of the graduation. The three threads are read, likewise both ends of the bubble. The bubble is then brought to rest near the other end of

the tube and the readings repeated. The two sets furnish the observations for one determination. Five to ten determinations will give a very good mean value.

The graduations of the level tube are numbered from the center toward the ends, and the rod readings are expressed in decimeters, likewise the distance from the instrument to the rod. Calling E_1 and O_1 the eye end and object end readings, respectively, when the bubble was near the eye end of the telescope, and R_1 the corresponding rod reading; likewise E_2 and O_2 the bubble readings when the bubble was near the object end for which the rod reading R_2 was taken; also, calling l the distance from rod to instrument, and v the value, in seconds of arc, of one division of the level tube; then,

$$v = \frac{R_2 - R_1}{l\left(\frac{E_1 - O_1}{2} - \frac{E_2 - O_2}{2}\right)\sin 1''}$$

for a single determination, and,

$$v = \frac{\Sigma\,(R_2 - R_1)}{l\,\Sigma\left(\frac{E_1 - O_1}{2} - \frac{E_2 - O_2}{2}\right)\sin 1''}$$

for any number of determinations.

Table No. 7 gives an example in which five determinations were made for level tube Adolf Pessler, Buff and Berger Level No. 2768, with 2-mm. graduations.

The readings were taken on 10-mm. rod graduations.

5. Determination of the Distance Constants.

According to the principle of the stadia, previously cited in describing the parts of a telescope, the length of shot from the center of the instrument to the rod is $l = k\,i + c + f$, in which k is the stadia constant, f is the focal length of the objective and c is the distance from the center of the instrument to the optic center of the objective lens. Hence, if the length l is to be measured, the constants k, c and f must first be determined for the instrument used.

The constants c and f may be found by direct measurement, observing that the telescope should be focused on a distant object before measuring f, which is then the distance from the reticule (or capstan adjusting screws) to the center of the objective.

From the foregoing equation it is seen that $k = \frac{1}{i}\,[l - (c + f)]$ from which k is determined by experiment.

TABLE No. 7.

Grossepointe, Mich., June 19th, 1899.
Temperature, 88° Fahr.; 31.1° Cent.

Point.	Rod No.	Thread Readings. 1	2	3	Mean.	Thread distance.	Level. E	O	E − O	v.
		dcm.	dcm.	dcm.	dcm.	dcm.	div.	div.	div.	sec.
Telescope normal, level tube direct.										
a	1	15.084	16.264	17.500	16.266	2.466	21.8	8.8	+13.0	
		15.104	16.340	17.572	16.339	2.468	8.3	22.3	−14.0	
			R_2-	$R_1=$	+0.073	$\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}=+13.5$				2.230
a	1	15.014	16.250	17.484	16.249	2.470	24.8	6.2	+18.6	
		15.100	16.338	17.568	16.335	2.468	8.9	22.1	−13.2	
			R_2-	$R_1=$	+0.086	$\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}=+15.9$				2.228
a	1	15.050	16.290	17.524	16.288	2.474	18.5	12.8	+5.7	
		15.120	16.358	17.590	16.356	2.470	6.0	25.3	−19.3	
			R_2-	$R_1=$	+0.068	$\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}=+12.5$				2.240
a	1	15.100	16.338	17.570	16.336	2.470	26.9	4.8	+22.1	
		15.200	16.436	17.668	16.435	2.468	8.9	22.8	−13.9	
			R_2-	$R_1=$	+0.099	$\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}=+18.0$				2.267
a	1	15.116	16.350	17.584	16.350	2.468	24.7	7.0	+17.7	
		15.220	16.454	17.688	16.454	2.468	5.1	26.4	−21.3	
			R_2-	$R_1=$	+0.104	2.4686 $\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}=+19.5$				2.198
						true dist. = 499.72 dcm.				
						$\Sigma(R_2-R_1)=+0.430$ dcm.			$\Sigma\left(\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}\right)=+79.4$	2.234 mean.

Hence $v=\frac{0.430}{499.72\times 79.4\times 0.000004858}=2.234''$, or since $1''=0.004858$ mm. per meter, $v=0.0108$ mm. per meter, which is sometimes a more convenient form.

The value as determined by Adolph Pessler, with the aid of a level-trier, was 1.97" (temperature not given). The agreement is as close as might be expected between a laboratory determination and one made in the open air at probably a much different temperature.

For the foregoing determination the probable error of the mean value 2.234" is ± 0.007".

To do this, the instrument is set up on the measured base line, previously mentioned, at a distance $c+f$ behind the zero mark of the base, so that the measured lengths, counting from the zero of the base, are all $l-(c+f)$. If then a rod be set at some one of the base points and the two extreme threads are read, the thread interval thus obtained gives one value for k.

Generally five such sets of readings are taken for each of several points of the base, from which a mean value of k is found. Table No. 8 gives such a determination in detail.

TABLE No. 8.—DETERMINATION OF THE STADIA CONSTANT k FOR BUFF AND BERGER LEVEL, No. 2768. CLAYTON, N. Y., MAY 5TH, 1899.

$l-(c+f)$	Rod No.	Thread Readings			Temp.	Thread Intervals.			k	Remarks.
		1	2	3		Upper.	Lower.	Total.		
m.		dcm.	dcm.	dcm.	deg. C.	dcm.	dcm.	dcm.	ratio.	
15	5	12.624	13.000	13.380	16.7	.....	.....	.....	.....	Tel. normal.
.....		12.610	12.990	13.360	.....	.....	.....	.....	.....	
.....		12.600	12.980	13.350	.....	.....	.....	.....	.....	
.....		12.652	13.026	13.402	.....	.....	.....	.....	.....	
.....		12.650	13.020	13.400	.....	.....	.....	.....	.....	
	m.	12.627	13.003	13.378	.....	0.376	0.375	0.751	199.74	
15	5	12.650	13.024	13.402	.....	.....	.....	.....	.....	Tel. inverted.
.....		12.624	13.000	13.380	.....	.....	.....	.....	.....	
.....		12.600	12.978	13.350	.....	.....	.....	.....	.....	
.....		12.650	13.022	13.400	.....	.....	.....	.....	.....	
.....		12.580	12.950	13.324	.....	.....	.....	.....	.....	
	m.	12.621	12.995	13.371	.....	0.376	0.374	0.750	200.00	
50	5	11.400	12.650	13.900	.....	.....	.....	.....	.....	Tel. normal.
.....		11.350	12.600	13.846	.....	.....	.....	.....	.....	
.....		11.380	12.620	13.878	.....	.....	.....	.....	.....	
.....		11.450	12.702	13.950	.....	.....	.....	.....	.....	
.....		11.400	12.650	13.900	.....	.....	.....	.....	.....	
	m.	11.396	12.644	13.895	.....	1.248	1.251	2.499	200.08	
50	5	11.400	12.648	13.900	.....	.....	.....	.....	.....	Tel. inverted.
.....		11.412	12.664	13.912	.....	.....	.....	.....	.....	
.....		11.340	12.592	13.840	.....	.....	.....	.....	.....	
.....		11.424	12.680	13.922	.....	.....	.....	.....	.....	
.....		11.500	12.752	14.000	.....	.....	.....	.....	.....	
	m.	11.415	12.667	13.915	.....	1.248	1.252	2.500	200.00	
100	5	17.300	19.804	22.300	.....	.....	.....	.....	.....	Tel. normal.
.....		17.220	19.724	22.222	.....	.....	.....	.....	.....	
.....		17.400	19.902	22.400	.....	.....	.....	.....	.....	
.....		17.304	19.806	22.304	.....	.....	.....	.....	.....	
.....		17.404	19.906	22.402	.....	.....	.....	.....	.....	
	m.	17.326	19.828	22.326	.....	2.502	2.498	5.000	200.00	
100	5	17.422	19.926	22.426	16.5	.....	.....	.....	.....	Tel. inverted.
.....		17.590	20.088	22.590	.....	.....	.....	.....	.....	
.....		17.520	20.024	22.524	.....	.....	.....	.....	.....	
.....		17.450	19.946	22.446	.....	.....	.....	.....	.....	
.....		17.504	20.004	22.506	.....	.....	.....	.....	.....	
	m.	17.497	19.999	22.498	.....	2.499	2.502	5.001	199.96	

Mean..199.96

Note: The instrument was set $c+f=0.60$ m. behind the zero of the base. The smallest rod graduation was 1 cm.; magnifying power of telescope 50 diameters. Thread 1 is uppermost in the field of the telescope, whether the latter is normal or inverted, but the upper half-interval is always the same and becomes the lower half-interval for telescope inverted.

The value found is practically 200, or, for distances in meters and rod readings in decimeters, the constant becomes 20, and $c+f=$ 0.60 m.

When k is not an even hundred, the solution of the foregoing formula for distance becomes somewhat laborious, and a tabulation of values of $k\,i$, in meters, for assumed values of i, in decimeters, from 1

TABLE No. 9.—Determination of the Residual Error Due

Clayton, N. Y., May 5th, 1899. Back Sights.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
								Instrument at the 50-m. point of the
0	1	11.570	12.800	14.036	.....	.....	.....	Tel. N., Level D.
		11.558	12.798	14.026	.....	.....	.....	Tel. I., Level R.
	M.	11.564	12.799	14.031	12.798	2.467	16.0	
0	1	11.976	13.206	14.442	.....	.....	.....	
		11.970	13.200	14.430	.....	.....	.....	Note:
	M.	11.973	13.203	14.436	13.204	2.463	.....	Value of the level was 5″ per 2-mm. grad.
0	1	12.428	13.672	14.900	.....	.....	.....	The smallest graduation on the rods = 10 mm. The magnifying power of the telescope = 50 diameters. Length of bubble = 80 mm. Mirror adjusted for parallax. $c + f = 60$ m.
		12.420	13.656	14.894	.....	.....	.....	
	M.	12.424	13.664	14.897	13.662	2.473	.....	
0	1	12.276	13.506	14.746	.....	.....	.....	
		12.270	13.500	14.738	.....	.....	.....	
	M.	12.273	13.503	14.742	13.506	2.469	.....	
0	1	12.258	13.494	14.728	.....	.....	.....	
		12.252	13.492	14.722	.....	.....	.....	
	M.	12.255	13.493	14.725	13.491	2.470	15.8	
				Means	13.3322	2.4684		
								Instrument at the 15-m. point of the
0	1	13.338	13.698	14.058	.....	.....	15.8	
		13.340	13.698	14.060	.....	.....	.....	
	M.	13.339	13.698	14.059	13.699	0.720	.....	
0	1	13.350	13.708	14.064	.....	.....	.....	
		13.348	13.704	14.064	.....	.....	.....	
	M.	13.349	13.706	14.064	13.706	0.715	.....	
0	1	13.364	13.724	14.086	.....	.....	.....	
		13.362	13.724	14.082	.....	.....	.....	
	M.	13.363	13.724	14.084	13.724	0.721	.....	
0	1	12.776	13.140	13.496	.....	.....	.....	
		12.774	13.134	13.494	.....	.....	.....	
	M.	12.775	13.137	13.495	13.136	0.720	.....	
0	1	12.806	13.164	13.524	.....	.....	15.7	
		12.800	13.156	13.516	.....	.....	.....	
	M.	12.803	13.160	13.520	13.161	0.717	.....	
				Means	13.4852	0.7186		

to 100, is very convenient for taking out any distances, even for a whole stretch. To obtain l for a single shot, the constant $c + f$ must be added to the tabulated value of $k\,i$, while, for a stretch of n instrument settings, the length becomes $L = k\,I + 2\,n\,(c + f)$, in which I is the sum of the thread intervals for all back and fore sights of the stretch.

TO COLLAR INEQUALITY, ETC., BUFF AND BERGER LEVEL, No. 2768.

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Diff. of elev.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	mm.	
100-m. base line, Clayton, N. Y.								
100	5	18.714	19.960	21.196	.....	.....	.....	Tel. N., Level D.
		18.710	19.950	21.190	.....	.....	.....	Tel. I., Level R.
	M.	18.712	19.955	21.193	19.953	2.481	−715.5	
100	5	19.122	20.366	21.600	.....	.....	.....	
		19.120	20.360	21.600	.....	.....	.....	
	M.	19.121	20.363	21.600	20.361	2.479	−715.7	
100	5	19.588	20.820	22.052	.....	.....	.....	
		19.580	20.810	22.048	.....	.....	.....	
	M.	19.584	20.815	22.050	20.816	2.466	−715.4	B. S. = 49.968 m. F. S. = 50.100 m.
100	5	19.432	20.670	21.900	.....	.....	.....	Ex. F. S. + 0.132 m.
		19.420	20.660	21.898	.....	.....	.....	
	M.	19.426	20.665	21.899	20.663	2.473	−715.7	
100	5	19.412	20.650	21.888	.....	.....	.....	
		19.410	20.648	21.886	.....	.....	.....	
	M.	19.411	20.649	21.887	20.649	2.476	−715.8	
				Means	20.4884	2.4750	−715.62	
100-m. base line, Clayton, N. Y.								
100	5	18.786	20.898	23.002	.....	.....	.....	
		18.796	20.898	23.010	.....	.....	.....	
	M.	18.791	20.898	23.006	20.898	4.215	−719.9	F. S. = 85.004 m. B. S. = 14.972 m.
100	5	18.800	20.906	23.010	.....	.....	.....	Ex. F. S. + 70.032 m.
		18.800	20.902	23.010	.....	.....	.....	
	M.	18.800	20.904	23.010	20.905	4.210	−719.9	Residual Excess F. S. = + 69.90 m.
100	5	18.804	20.914	23.024	.....	.....	.....	
		18.810	20.920	23.036	.....	.....	.....	
	M.	18.807	20.917	23.030	20.918	4.223	−719.4	True D. = −715.62mm. Err. D. = −719.98mm.
100	5	18.230	20.350	22.460	.....	.....	.....	Resid. = + 4.36mm.
		18.220	20.328	22.446	.....	.....	.....	for 69.9m. Excess F.S.
	M.	18.225	20.339	22.453	20.339	4.228	−720.3	or = + 0.0624 mm.
100	5	18.268	20.380	22.490	.....	.....	.....	per meter Excess of
		18.236	20.350	22.464	.....	.....	.....	F. S.
	M.	18.252	20.365	22.477	20.365	4.225	−720.4	
				Means	20.6850	4.2202	−719.98	

6. Determination of the Residual Error due to Collar Inequality, etc.

The method adopted by the writer is designed to determine the residual inclination in the line of sight caused by inequality in the telescope collars, etc., independent of the various other level tube and telescope errors, by a direct inductive method. The text-book method of determining collar inequality, by level tube and telescope

reversals, has already been explained and found erroneous, because not all of the governing conditions are materialized in practice.

Observations are taken on the measured base previously mentioned, using the method of reading adopted for regular precise level work.

The instrument, which has been carefully adjusted, is set up at the 50-m. point of the base, and five or ten sets of sights are taken on rods held at the zero and 100-m. points, as follows: With telescope normal and level tube direct, the back sight is read on the rod held on the zero point of the base, the instrument is then swung on the rod at the 100-m. point and the fore-sight reading taken. The telescope is then inverted and the level tube is reversed and the readings are repeated, commencing with the fore sight. This constitutes one set of sights. All readings are taken with the bubble in the center.

The instrument is then set at some other point near one end of the base so that the back and the fore sights are very unequal. The 15-m. point was generally used. The same number of sets of sights is then taken as before on the rods at the zero and 100-m. points of the base.

It will be observed that this system of reading eliminates all errors of the level tube, the reticule, eyepiece movements, and mounting of the objective lens, and because the back and fore sights were exactly equal in the first series of sets, the remaining errors due to parallax and to inequality in the collars are also eliminated in these sets. However, the sets of the second series involve both these last-named errors for a 70-m. excess of fore sights. The parallax error should be eliminated by adjustment of the mirror, thus leaving the collar inequality as the only remaining source of error in the second series. Hence, the residual inclination is found by comparing the differences of elevation between the zero and 100-m. points as found by the two series of observations.

Temperature and atmospheric influences must be carefully avoided in these observations by choosing a proper time of day, or cloudy weather.

When the horizontal distance between the cradle supports of the telescope differs from the distance between the contact points of the collars with the level tube, then the only wear on the collars which affects the above residual inclination is that which takes place on the collars under the level tube. This wear is very slow and may be

almost entirely prevented by applying the precaution of lifting off the level tube whenever the telescope is being reversed and when the instrument is being carried.

Past practice has been to construct the cradle and level tube with such dimensions as to bring the four contact points on each collar exactly in a plane. The wear resulting from the cradle supports will then affect the level tube bearings, and the whole matter of collar inequality reaches a chaotic stage beyond all possibility of solution. Hence, this condition should be carefully avoided, and it might be suggested that a pair of collars be provided for the cradle supports independent of those for the level tube.

An example of a complete determination of the residual error due to collar inequality is given in Table No. 9. The result found is + 0.0624 mm. per meter excess of fore sights, and the rule for applying this error is, therefore: On direct lines, the difference of elevation between two successive bench-marks is algebraically increased for a positive excess of fore sights, and on reverse lines the difference of elevation is algebraically increased for a negative excess of fore sights, provided, that the difference of elevation is given the same sign as for the direct line.

7. Eyepiece Movements.

When the eyepiece is afflicted with irregular movements for different focal distances, the thread adjustment becomes very unreliable, and, in fact, an instrument having such a defect is not suitable for accurate work.

The most dangerous error is produced by lateral looseness of the eyepiece slide, which causes the eyepiece to drop slightly after each inversion of the telescope, and thus vitiates all readings. Regular linear motion is necessary, even though the direction is inclined to the axis of the telescope. Such regular inclination is entirely eliminated by readings in the normal and inverted positions of the telescope, while the irregular looseness affects each sight and each position differently. Hence the importance of proper design and careful construction of this detail of the telescope, to which attention has already been invited.

To determine the presence and character of these eyepiece movements, the following tests should be made:

First, perform several inversions of the telescope, each time reading a rod placed 60 to 70 m. from the instrument. If these readings show no changes or marked variations in the thread error, then there is no lateral looseness of the eyepiece slide. However, it frequently happens that such looseness develops as a result of wear, or when working during extremely warm weather. It is well to have the eyepiece move somewhat hard, and a thick lubricant, such as tallow, is preferable in warm weather, while a more fluid oil may be used in cold weather.

Second, to determine the obliquity of motion of the eyepiece slide, the instrument should be set up 5 or 10 m. behind the zero mark of the measured base line and readings taken on such points, the elevations of which have been determined previously by shots of equal back and fore sights. Such a set of readings, which may be repeated several times for level tube direct and reversed, but without inverting the telescope, will give a profile of the line of sight, which should be a straight line. The inclination of this line of sight for the different points sighted should, of course, be constant, and by comparing this mean inclination with the error due to collar inequality, the resultant of the two is the inclination due to obliquity of eyepiece motion.

TABLE No. 10.—Determination of Obliquity of Eyepiece Motion.

Point.		Instrument set at point of base.	Length of		Difference of Elevation.		Error.	Excess of F. S.	Obliquity of line of sight.
To.	From.		B. S.	F. S.	True, for equal sights.	Erroneous for unequal sights.			
		m.	m.	m.	mm.	mm.	mm.	m.	mm. per meter.
20	0	60	60	40	+ 288.1	+ 289.6	−1.5	− 20	−0.0750
40	0	10	10	30	+ 501.6	+ 500.4	+1.2	+20	+0.0600
60	0	20	20	40	+ 737.4	+ 736.3	+1.1	+20	+0.0550
60	0	10	10	50	+ 737.4	+ 735.0	+2.4	+40	+0.0600
80	0	20	20	60	+ 897.9	+ 885.6	+2.3	+40	+0.0575
40	0	60	60	20	+ 501.6	+ 504.4	−2.8	−40	−0.0700
80	0	60	60	20	+ 897.9	+ 900.5	−2.6	−40	−0.0640
80	0	10	10	70	+ 897.9	+ 894.6	+3.3	+60	+0.0550
100	0	20	20	80	+1115.5	+1111.7	+3.8	+60	+0.0633
100	0	10	10	90	+1115.5	+1110.4	+5.1	+80	+0.0638

Mean = 0.0625.

Table No. 10 gives the results of a set of observations, for which each value is the mean of two sets of readings of three threads, each for level direct and level reversed. The error of estimation is prob-

ably large, because the observer (the writer) had not yet attained the necessary skill in estimating on 10-mm. rod graduations.

The instrument, Buff and Berger Level, No. 2768, was here placed at different points of the base line, but it would be better to follow the plan above outlined. The level tube had a curvature of 5 seconds per 2 mm., the smallest rod graduation was 10 mm., and the magnifying power of the telescope was 50 diameters. The ratio of the telemeter threads was 1:100, which is partly the fault of poor estimation on the extreme threads.

The obliquity of the line of sight found for excess of fore sights varying from 20 to 80 m. is practically constant, and hence the eyepiece moves in a straight line.

8. Determination of A of Rods.

This may be done by direct measurement, with a finely divided steel scale, or it may be accomplished with the aid of the leveling instrument.

The latter method consists of repeated readings taken on the rod held on a specially prepared spike head and then a second set of readings taken on a steel scale held on the same mark. The mean difference of the two sets of readings is the A of the rod.

Field Methods of Precise Leveling.

1. Organization and Subsistence of the Field Party.

a. Organization.—A field party, while it may be differently constituted to comply with conditions and circumstances peculiar to certain localities, is generally composed of an observer, who is chief of the party, a recorder, two rodmen and an umbrellaman. Sometimes an axeman is required when lines must be cleared of brush. A cook and camp outfit may be necessary when the party cannot subsist on the farming community.

A proper make-up of energetic, active, well-educated and practical men is the prime necessity when high-grade work is expected. The very best are none too good for precise leveling, and it is economy to employ only efficient men at high compensation, even for the better grades of railroad levels. This applies equally to the instrumental outfit with which a party is equipped.

On all astronomical and precise geodetic work the high degree of accuracy is attained by the most delicate instruments handled in a most skillful manner to obtain a large number of observations of each unknown quantity, from which the best mean values may be deduced.

According to most recent standards of precise leveling the accumulated error of say 9 000 individual rod readings, covering 400 km. of levels, must not exceed 40 mm. and often falls below 10 mm. This must be accomplished in the open air, over any ground, and at nearly all seasons of the year. A certain daily rate of progress is expected, and the atmospheric conditions are frequently prohibitive of good work, thus forbidding the repetition of observations and the rejection of work which, though not perfect, must be accepted because it falls within certain arbitrary limits of requirement. There is no opportunity to select a best value or a mean of many observations. There is but one result and that must be correct within certain specified limits.

From this statement of the facts it becomes apparent that the ability and skill required of a precise levelman is of a very high order, somewhat at variance with the commonly accepted idea.

The observer and chief of a precise level party must be a specialist, possessing a thorough scientific training and having a faculty for careful and patient research united with a natural adaptation to artistic manipulations. He must always lead and provide for the welfare and comfort of his party, frequently to the extent of supplying medical aid. He must often be satisfied with a meager compensation and with little recognition for the quality of his services, because such service is rarely appreciated except by one who has been "through the mill" himself.

The recorder should be a college graduate with several years' practical experience, especially in the computing department of geodetic and survey work. He must be very quick and correct in recording and reducing observations, and must keep clean and legible notes. Much of the success of a party depends on the ability of the recorder to fill his position well.

The rodmen should be college graduates, as besides their regular duties of rodding, they must be able to check the field note reduction and computation made by the recorder. Without such assistance the labor devolving on the recorder and observer would be excessive, and

the progress of the work would be retarded. The duties of the rodmen are exactly the same, and there should be no distinction, such as head and rear rodman.

There are no special qualifications attaching to the umbrellaman or possible axeman, except that the former must carry the instrument and must be cautious and trustworthy.

The chief of party should be allowed to appoint his own assistants, and he alone is then responsible for the quality and quantity of work performed. The several members of a party should be congenial, and pull together. The general principles, as well as the details of the work, should be familiar to all, and ample opportunity should be given for the free discussion of any doubtful matters which may arise.

b. Subsistence.—The peculiarities of leveling over long stretches of country necessarily bring about conditions which are radically different from other field work, and a few remarks on this subject may be in order here.

The engineer has often been compared with the tramp, which is more or less justifiable on some kinds of work; but when leveling is the subject under consideration, the slight distinctions, usually claimed in favor of the engineer, vanish into insignificance.

Owing to the rapid progress of a leveling party, it is necessary to move headquarters about every second or third day, and this prevents the use of a regular camping outfit, unless it be in the form of a gypsy caravan with team and wagon. This is generally more expensive than subsisting on the farming community, and is, therefore, not resorted to, except when passing through unsettled country.

Boarding with the farmers, however, is not always a pleasant existence, as those who have experience will testify. Every farmer wishes to profit by the misery of the party, and the usual result is that hotel rates are charged and tramp accommodations are offered. There are exceptions to this rule, but they are very rare and enjoy a long life in the memory of the levelman.

The personal baggage must be confined to a minimum, as there are times when each man must pack his own belongings and work as he goes. At other times a horse and buggy may be secured for this service.

Working hours are generally from sunrise until atmospheric interference prevents further progress, and again in the afternoon when

work becomes possible until sunset. This makes the busiest time for the levelman at the usual breakfast and supper hours, which adds greatly to the discomfiture of both parties concerned.

The foregoing will suffice to enable the reader to draw his own conclusions regarding the pleasantries of the levelman's occupation.

2. Systems of Leveling.

The value of a single line of levels has often been over-estimated, assuming that careful, conscientious work would yield correct results. It is never known with absolute certainty, however, when work is correct unless at least two results are obtained, and even then cases are on record where both results were erroneous, yet agreeing fairly well with each other.

Several systems of duplicate leveling have been used with the distinct purpose to check work and at the same time to eliminate injurious atmospheric influences; also to detect heaving or settling of instruments and other sources of error. The methods commonly used were to run duplicate simultaneous lines, using one instrument and two sets of turning points, or two instruments and one set of turning points; to run duplicate simultaneous lines in opposite directions using two complete party outfits; to run duplicate lines in opposite directions by the same party, dividing the work variously between forenoons and afternoons. The method generally prevailing is to run one way in the forenoon and to duplicate the work in the opposite direction in the afternoon.

The experiments conducted by the writer, however (see Table No. 6), to show the effect of time of day and direction of line on the closure of level lines, answer this question very conclusively. Accordingly, there appears to be but one method of detecting the principal error due to temperature or radiation and that is to run duplicate lines in opposite directions by a single party, closing each stretch in the same half of the day. This, however, is merely a means of detecting the presence of errors from this source; such errors may be eliminated only by dividing the work equally between the two halves of the day for the entire line of levels.

This view is quite different from that ordinarily held by most authorities, but the conclusion above reached supersedes all theoretical reasoning, being, as it is, the result of actual observation and experiment.

On some recent Government work, a system of single lines has been adopted with the expectation of checking or closing the work by leveling in large polygonal lines forming a net. If such work be carried on in a continuously forward direction, the errors are very apt to be cumulative. If work be run direct and reverse on alternate days, the error will be materially reduced, but such a level line will be unsatisfactory and may embrace many serious local undulations and still close well on an entire polygon.

Hence, if high-grade work is required, it is absolutely necessary to run forward and back over each stretch, and, according to the writer's experience, each stretch should be closed in the same half of the day, choosing the length of the stretch to make this requirement possible.

3. Instrument Adjustments and Constants.

The manner of adjusting a level and determining its constants has already been given in full detail, and hence only a few remarks are necessary here.

When a party enters the field to commence leveling operations, the first thing is to measure a base line and mark the ends and intermediate points on bed-rock or on very solid stakes with large spikes for turning points. The instrument constants should then be carefully determined and a permanent record kept of all work done. The same observations should be repeated at the completion of the work, or at the close of the field season, if possible on the same base line.

The adjustments of the level should be tested every day before doing any leveling, and, if found excessive, the instrument must be carefully adjusted. As will be seen presently, the combined error of the cross-threads and level tube is measured by each set of observations, and, whenever this becomes excessive (exceeds 1 mm. in 50 m.), it is well to readjust the cross-threads and level tube. The lateral adjustment of the level tube and the plate levels do not require much attention, though the eyepiece movements should be constantly watched, as, in extreme temperatures, irregularities often occur which would prove serious if not detected at the time.

When working with a very delicate level, great care should be exercised not to expose the instrument to sudden changes of extreme temperatures, as is very apt to occur in cold weather. The level should be maintained as nearly as possible at the atmospheric temperature,

and should not be left standing in the sun or be placed in a warm room after being used in the cold.

Attention should be called to the fact that it is useless to attempt the adjustment of an instrument before it has stood long enough to take on an even temperature in the air where it is to be used. This may require 15 minutes or more after being removed from the case, and can be best determined by observing the constancy in the length of the bubble, which is a determining factor. A single reversal of the level tube is not considered sufficient to decide whether or not the level tube requires adjustment, but at least two, and generally three or four, are necessary to determine the real state of affairs. Hasty action will often cause much useless labor and delay. A level which was in adjustment when placed in the case will scarcely ever be so when again set on the tripod, but in a few minutes it frequently returns to perfect adjustment.

The rods should be tested regularly, morning, noon and evening, and, when not plumb, the rod levels should be adjusted.

It is not necessary, according to the method of observing advocated in the following, to measure and record errors of adjustment, as these errors are not sufficiently constant to justify their use in applying corrections.

4. Atmospheric Conditions Permitting Acceptable Work.

The opinions regarding this important factor are so varied that, to the inexperienced, it may seem impossible to follow any definite rule.

The majority of writers take the stand that work may be profitably carried on when the air is sufficiently quiet to enable the observer to obtain a good, steady image, when the rod is placed at about 30 m. from the instrument, independent of the time of day. This is entirely inadmissible in warm weather, as is clearly shown by the experiments given in Table No. 6.

A steady image is, however, only one of the many governing conditions for obtaining good work. It may be accepted as satisfactory evidence of normal conditions when the sun is completely hidden by dense clouds so that not merely the instrument, but the entire distance sighted over is in the shade.

On clear or partially cloudy days, great care and good judgment are required to obtain acceptable readings during midday hours, which

in the warm summer months may comprise from 8 A. M. to 5 P. M., leaving only the early morning and late evening hours for work. In the cooler weather of spring and fall, leveling may be done at nearly all hours, though excessive dampness of the ground, combined with warm air, may frequently cause trouble.

The following method of observing is intended to furnish the means of detecting any abnormal atmospheric conditions or rapid changes, and the observer, if he is very careful, will soon learn to judge the reliability of his readings by the behavior of the bubble and the coincidence of his pairs of observations.

However, the error shown to exist in very warm weather (see Table No. 6) cannot be detected, except by the closure of a line, and the best and most careful work may be done under seemingly good conditions without yielding satisfactory results.

To test the atmospheric conditions, the following method may prove serviceable when working in very warm weather. At a convenient place near each camp, drive three stakes in approximate direction of the level line and about 50 m. apart (set by the level). Large spikes should be driven in the first and third stakes. The instrument is set up over the center stake, and the difference of elevation between the first and last stakes is carefully determined very early or very late in the day or during cloudy weather. Whenever the atmospheric condition is uncertain it may be tested by setting the instrument over the middle stake and taking a set of readings on the first and third stakes. If the resulting difference of elevation does not check the one previously found under known conditions, within the allowable error of estimation, then no acceptable work can be executed.

Working during the warm summer months, June, July and August, has many disadvantages, and the recommendation previously made, respecting the advisability of night leveling will be well worth consideration if within the limits of possibility.

5. Method of Observing.

The method of observing, advocated by the writer, is such as to eliminate all instrumental errors in the observations themselves, and obviate, as far as possible, all mathematical manipulations in the form of corrections. It is held that careful observations, taken only under suitable atmospheric conditions, will lead to better results than can

ever be expected by applying more or less speculative corrections to work of less exact execution.

The instrument should always be completely shaded from the direct rays of the sun, and should be protected from rain and annoying winds. This can, with rare exceptions, be accomplished with a large instrument umbrella, as previously described. So-called wind protectors are not considered practical when using a good firm tripod, such as supplied with Buff and Berger Level No. 2768, though they are valuable when using less rigid tripods. One of the best forms of protectors was designed by E. E. Haskell, M. Am. Soc. C. E., and consists of three 10-ft. poles forming a tripod, which latter is covered with canvas to form a portable tent. Openings for taking readings are provided.

All rod readings are taken on rods, held perfectly plumb by means of spirit levels, and three threads are read for each sight. The bubble is always in the center when readings are taken, and the observer sees the rod image with one eye simultaneously with the bubble image in the mirror with the other eye.

The rods are placed equidistant from the instrument by the rodmen counting their paces. The rodmen become alternately head and rear rodman, thus having exactly the same duties and responsibility.

One rod is held on a starting bench-mark and the other on the first turning point; the readings are taken and the instrument and rear rod are then carried forward to new positions. The rear rodman counts his paces from the first turning point to the new instrument position and then proceeds ahead, repeating his count to the second turning point. This process is continued until a new bench-mark is reached. The instrumentman thus fixes the length of shot he wishes to take, and both rods are always in position when readings are taken.

The readings, constituting one pair of back and fore sights, are taken as follows: With the telescope normal and the level tube direct, the first sight of the back sight is taken with the bubble in the center.* Then, without disturbing the telescope, not even touching the focusing screw or level tube, the instrument is quickly swung on to the front rod and the first sight taken on the fore sight as soon as the

* Professor J. B. Johnson is probably entitled to the credit of having introduced the method of reading with the bubble in the center, as applied to precise leveling in the United States.

bubble can be centered by the micrometer screw. Should the lengths of the shots be materially unequal, the front rod is set correctly and the sights are repeated. The level tube is then removed and the telescope is inverted, carefully replacing the level tube in a reversed position. The second sight of the fore sight is then taken with the bubble in the center; the instrument is quickly swung on to the rear rod and the second sight of the back sight is taken.

Before taking up the instrument, the recorder inspects the two sets of readings to ascertain any possible inconsistencies, and, if any are discovered, the readings are repeated; otherwise, the instrument is carried forward, and the recorder takes out the mean readings and thread distances while the instrument is being reset.

In closing on a bench-mark, the instrument is so placed, on the last pair of sights, as to make the sum of the thread distances of the back sights equal to the sum of the thread distances of the fore sights for the entire stretch between bench-marks. The difference of elevation and total length of stretch is then figured and the stretch run in the opposite direction during the same half of the day. The closure of the loop thus run must be less than the specified limit given in the following.

By this method of observing, the errors of pointing and of estimation are materially reduced and become almost insignificant, as compared with those incident to other methods, while the errors of the telescope and level tube are eliminated, to all intent and purpose. The only correction to be applied to the work is for rod length and temperature as above described.

Should the sums of sights not be entirely equalized, then the "residual correction" due to collar inequality, etc., must be applied for the resulting excess of fore sights. Further than this, no corrections of any kind are necessary or applicable to the field notes.

When a bench-mark is so placed that the level rod cannot be held directly on the mark as on a turning point, then a steel tape is most advantageously used at short range. Such a tape has been described in connection with level rods, and the value "A" for the rods used must be subtracted from all tape readings when the tape is stretched upward from the mark.

On direct lines, the back sights are considered positive and the fore sights are negative, while on reverse lines these signs are advisedly

reversed, thus giving the same sign to the difference of elevation whether determined from a direct or reverse line.

A line is considered "direct" when going from the initial or starting point toward the terminal point. The opposite direction is considered "reverse."

6. System of Notes.

The method of observing was treated somewhat briefly, with the expectation of illustrating the details of the method here advocated by a complete example of notes, thus presenting the subject in a more understandable manner.

The following books constitute the records of a precise level party: A notebook of bench-mark descriptions; as many notebooks as may be necessary for level notes; and a summary-book containing a compilation of final results and computation of elevations. In addition to these notes the level line and all fairly permanent bench-marks should be platted on some large-scale map of the route, whenever such a map is obtainable.

The level fieldbooks should be about 5 x 7½ ins., with printed headings, vertical column ruling, and horizontal lines ruled $\frac{3}{16}$ in. apart. The summary-book should be about 9 x 12 ins., of paper ruled crosswise with about seven squares to the inch. The headings can be printed, or stamped with a large rubber stamp, and the columns ruled by hand.

Each fieldbook should contain on the front page a brief statement of programme of observing, definitions of instrument positions, names of observer and recorder, instrument and rods used, units of measure adopted, etc. The notes should then follow in chronological order in strict accordance with the system adopted. The last double page of each book should contain a summary of results with page index of work recorded in such book.

The summary-book should contain a similar introduction, with definitions of terms used therein, authority and elevation of starting bench-mark and a brief history of the work.

It is scarcely necessary to give an example of bench-mark descriptions, though it must be emphasized that too much attention cannot be given to the accurate and complete description of every permanent bench, as appears from the purpose for which such a description should serve. A very good test of the adequacy of such

descriptions is for the observer to place the bench and draft the first description. The recorder should then find the bench with the aid of this description, and, if the latter is found deficient, it must be improved.

A complete example of a field level-book is given in the following, and applies to Buff and Berger Level, No. 2768 (see Plate I).

Designation of Instrument Positions.

"1. The telescope points direct when its eye end is over the micrometer bearing of the wye-cradle. This position is always maintained when leveling.

"2. The telescope is normal when the focusing screw at the eye end is on the upper right-hand side as seen from the position of the observer.

"3. The level tube is in its direct position when the end marked 'chamber' is at the eye end of the telescope.

Programme of Readings.

"1. For each position of the telescope, the three threads are read consecutively from the upper thread down, and the readings are recorded in this order as 1, 2 and 3, whether the telescope is normal or inverted.

"2. Each shot is the mean of six thread readings taken for the two following positions of telescope and level tube: 1. Telescope normal, level direct; 2. Telescope inverted and level reversed. Thus each shot or pair of sights eliminates errors in the adjustment of the cross-threads and the level tube.

"3. The thread distance corresponding to the mean value of the two instrument pointings, or sights, is obtained by subtracting the mean for thread 1 from the mean for thread 3.

"4. Thread distance, in decimeters, is converted into true distance, in meters, by the formula

$$L = kI + 2n\,(c + f),$$

in which $I = \Sigma_{n}^{n}\, i =$ total thread distance, in decimeters, for any stretch of n instrument settings. The stadia constant, k, is 20 for thread ratio 1:200. The telescope constant $c + f = 0.6$ m.

"5. All rod readings are read and recorded in decimeters.

"6. No bubble readings are recorded, because the bubble is always in the center when rod readings are taken. However, when the bubble is read for experimental purposes, the readings termed 'eye end' (E) are taken for the eye end of the telescope whether the level tube is direct or reversed. For regular level work, the length of the bubble is maintained at 40 divisions = 8 cm.

"7. The allowable limit of error for the closure of each stretch is

$2\sqrt{L}$, in millimeters, in which L is the length of the stretch, in kilometers.

"8. Buff and Berger Precise Level, No. 2768, and Lake Survey rods 1 and 5 were used. For summary of results see p. — of this book.

"D. M., Observer; M. B., Recorder; A. E. and A. H., Rodmen."

Instrument Constants.

The instrument constants were determined whenever the instrument underwent any changes or alterations, and at the beginning and end of each field season. The values used in this book are given in Table No. 11.

TABLE No. 11.

Designation of constants.	Values.
Magnifying power of telescope, in diameters	50
Focal length of objective, in millimeters	402
Diameter of objective, in millimeters	38
Ratio of telemeter threads	1:200
Curvature of level tube, in seconds per 2-mm. divisions	2.23
Residual error, in millimeters per meter excess of F. S. for + difference of elevation	+ 0.0630
Weight of instrument, in kilogrammes	5.3
Weight of tripod, in kilogrammes	7.6
Smallest graduation on rods, in millimeters	10
A of rods 1 and 5, in decimeters	0.486

Level Notes.

Tables Nos. 12 and 13 are the complete field notes of a stretch of levels, direct (D) and reverse (R). During the progress of the work the recorder carries (in lead pencil) a continuous summation of thread distances, to prevent any excessive inequality in sums of sights (say larger than 2 m.). On the last setting the instrument is so placed that the final sums should balance. Only the final sums are retained in the notes, the others are again erased when the notebooks are checked, which is done each night by the rodmen. All notes, as given in the sample, except final sums, are recorded in ink, thus preventing any erasures or obliterations.

An example of a river crossing (Tables Nos. 14 and 15) is inserted to indicate the special features of such a problem, which often presents itself and which frequently involves numerous difficulties. The notes also show in a remarkably striking manner the correctness of the new theory of collar inequality developed herein.

When a river crossing necessitates a very long sight, it is not advisable to introduce two such sights for the purpose of equalizing back and fore sights, even though the ground would permit of so doing. The corresponding long sight over the ground would in all probability be subject to different refractive influences from those existing over the water surface. Hence, the difference in elevation is determined between two temporary bench-marks, A and B, on the respective banks of the river, in such a manner as to eliminate all errors of inclination in the line of sight. This is done as follows (see also Fig. 16):

Drive stakes C and D so that $A\,D = B\,C$ and $A\,C = B\,D$. This is not quite true in the example, but is near enough for the purpose. The direction of the line is from A toward B, and the distances are in meters.

Set up the instrument over C and read back sight on A and fore sight on B, reading as many pairs of sights as conditions may require. Then shift the instrument to D and repeat the readings on A and B. The mean difference of elevation from the two instrument settings eliminates all errors when $A\,D = B\,C$ and $A\,C = B\,D$. Otherwise, a slight correction is applied for the resulting mean excess of fore sight. The same work with sights in opposite directions will constitute the reverse line.

In the example cited, the residual correction for the instrument was + 0.0635 mm. per meter excess of fore sights, and, applying this to the several single lines, gives the differences of elevation from A to B shown in Table No. 16.

The correction for collar inequality, found by the commonly adopted method of level tube and telescope reversals, was + 0.0235 or a little over one-third of the actual existing error, which is properly accounted for by the residual correction above applied. This also shows a very close agreement of the four values found, which is due to careful work and good weather conditions. Wide river crossings should never be made except during cloudy weather or at sunrise and sunset.

(*Left-hand page.*)

TABLE No 12.—EXAMPLE

Direct Line T. B. M. 128-

JULY 5TH, 1899. BACK SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
T. B. M. 128.	1	24.174	26.006	27.838	.....	.....	23.9	3 P. M.
		24.176	26.008	27.840	.....	.....	.....	Cloudy. fresh
		24.175	26.007	27.839	26.007	3.664	.....	wind, reading good.
T. P. 1.....	5	20.336	22.050	23.764	.....	.....	.....	Puffy, cool wind.
		20.336	22.052	23.766	.....	.....	.....	
		20.336	22.051	23.765	22.051	3.429	.....	
2	1	14.606	16.290	17.970	.....	.....	.....	Cloudy.
		14.602	16.284	17.964	.....	.....	.....	
		14.604	16.287	17.967	16.286	3.363	.....	
3	5	17.648	19.244	20.842	.....	.....	.....	Sun shining;
		17.652	19.248	20.844	.....	.....	.....	brisk puffy
		17.650	19 246	20.843	19.246	3.193	.....	wind.
4	1	8.524	10.104	11.688	.....	.....	.....	Cloudy.
		8.512	10.096	11.678	.....	.....	.....	
		8.518	10.100	11.683	10.100	3.165	.....	
5	5	21.024	22.646	24.270	.....	.....	.....	Cloudy; wind
		21.024	22.646	24.268	.....	.....	.....	abating.
		21.024	22.646	24.269	22.646	3.245	.....	
								B. S. + 156.357
6	1	19.260	20.922	22.580	.....	.....	.....	F. S. — 112.384
		19.258	20.916	22.576	.....	.....	.....	D = + 43.973
		19.259	20.919	22.578	20.919	3.319	.....	Res. Cor. 0.000
7	5	17.520	19.104	20.690	.....	.....	.....	Light wind.
		17.516	19.098	20.684	.....	.....	.....	
		17.518	19.101	20.687	19.102	3.169	.....	
					156.357	26.547		

* Taken from precise level line along

OF PRECISE LEVEL NOTES.* (*Right-hand page.*)

T. B. M. 129.

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
T. P. 1.....	5	14.262	16.094	17.922	.....	.....	.....	
		14.264	16.096	17.924	.....	.....	.....	
		14.263	16.095	17.923	16.094	3.660	.....	
2	1	8.924	10.684	12.450	.....	.....	.....	
		8.924	10.686	12.450	.....	.....	.....	
		8.924	10.685	12.450	10.686	3.526	.....	
3	5	16.398	18.002	19.600	.....	.....	.....	Sun out on first reading of F. S.
		16.390	17.994	19.598	.....	.....	.....	
		16.394	17.998	19.599	17.997	3.205	.....	
4	1	13.934	15.558	17.186	.....	.....	.....	
		13.938	15.562	17.190	.....	.....	.....	
		13.936	15.560	17.188	15.561	3.252	.....	
5	5	7.374	8.990	10.600	.....	.....	.....	Thread dist. F. S. = 26.550
		7.364	8.974	10.586	.....	.....	.....	B. S. = 26.547
		7.369	8.982	10.593	8.981	3.224	.....	I = 53.097
								Ex.F.S. =+0.06m.
6	1	3.716	5.298	6.880	.....	.....	.....	
		3.716	5.296	6.880	.....	.....	.....	
		3.716	5.297	6.880	5.298	3.164	.....	
7	5	10.962	12.634	14.300	.....	.....	.....	
		10.960	12.633	14.298	.....	.....	.....	
		10.961	12.632	14.299	12.631	3.338	.....	
T. B. M. 129	1	23.548	25.140	26.730	.....	.....	22.8	4.30 P. M.
		23.542	25.134	26.722	.....	.....	.....	Delayed 60 min.
		23.545	25.137	26.726	25.136	3.181	.....	by rain.
					112.384	26.550		

Lake St. Clair, Mich., by D. Molitor, 1899.

(*Left-hand page.*)

TABLE No. 13.—REVERSE LINE

JULY 5TH, 1899. BACK SIGHTS

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	5.30 P. M.
T. B. M. 129.	1	23.514	25.108	26.694	.....	.....	22.2	Cloudy, calm.
		23.522	25.112	26.704	.....	.....	.....	Light rain.
		23.518	25.110	26.699	25.109	3.181	.....	Reading good.
T. P. 1.....	5	10.864	12.536	14.204	.....	.....	.....	
		10.864	12.536	14.202	.....	.....	.....	
		10.864	12.536	14.203	12.534	3.339	.....	
2.....	1	3.754	5.338	6.916	.....	.....	.....	
		3.756	5.336	6.918	.....	.....	.....	
		3.755	5.337	6.917	5.336	3.162	.....	
3.....	5	7.214	8.828	10.442	.....	.....	.....	
		7.220	8.834	10.450	.....	.....	.....	
		7.217	8.831	10.446	8.831	3.229	.....	F. S. = + 158.269
								B. S. = − 114.296
4.....	1	14.218	15.846	17.470	.....	.....	.....	+ 43.973
		14.216	15.844	17.472	.....	.....	.....	Res. Cor. + 0.0002
		14.217	15.845	17.471	15.844	3.254	.....	
								R = + 43.9732
5.....	5	16.596	18.194	19.794	.....	.....	.....	
		16.590	18.192	19.790	.....	.....	.....	Closing error
		16.592	18.193	19.792	18.193	3.199	.....	= 0.02 mm.
6.....	1	8.988	10.746	12.504	.....	.....	.....	
		8.982	10.738	13.500	.....	.....	.....	
		8.985	10.742	13.502	10.743	3.517	.....	
7.....	5	15.878	17.706	19.540	.....	.....	.....	
		15.876	17.702	19.534	.....	.....	.....	
		15.877	17.704	19.537	17.706	3.660	.....	
					114.296	26.541		

* Taken from precise level line along

T. B. M. 128-T. B. M. 129.* (*Right-hand page.*)

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
T. P. 1.....	5	17.488	19.072	20.654	.....	.....	.....	
		17.496	19.080	20.662	.....	.....	.....	
		17.492	19.076	20.658	19.075	3.166	.....	
2.....	1	19.164	20.826	22.486	.....	.....	.....	
		19.164	20.828	22.486	.....	.....	.....	
		19.164	20.828	22.486	20.826	3.322	.....	
3.....	5	21.064	22.688	24.304	.....	.....	.....	
		21.064	22.690	24.310	.....	.....	.....	
		21.064	22.689	24.307	22.687	3.243	.....	
4.....	1	8.364	9.950	11.534	.....	.....	.....	
		8.370	9.954	11.536	.....	.....	.....	Thread dist.
		8.367	9.952	11.535	9.951	3.168	.....	F. S. = 26.528
								B. S. = 26.541
5.....	5	17.938	19.522	21.106	.....	.....	.....	I = 53.069
		17.940	19.522	21.110	.....	.....	.....	Ex. F. S. = 0.26 m.
		17.939	19.522	21.108	19.523	3.169	.....	
6.....	1	14.804	16.486	18.170	.....	.....	.....	
		14.800	16.480	18.164	.....	.....	.....	
		14.802	16.483	18.167	16.484	3.365	.....	
7.....	5	20.388	22.100	23.814	.....	.....	.....	
		20.380	22.094	23.808	.....	.....	.....	
		20.384	22.097	23.811	22.097	3.427	.....	
T. B. M. 128.	1	25.796	27.626	29.462	.....	.....	21.1	
		25.790	27.622	29.460	.....	.....	.....	
		25.793	27.624	29.461	27.626	3.668	.....	6.30 P. M.
								Cloudy, calm.
					158.269	26.528		

Lake St. Clair, Mich., by D. Molitor, 1899.

(*Left-hand page.*) TABLE No. 14.—EXAMPLE OF PRECISE

Grass River Crossing.

AUG. 22D, 1898. BACK SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
		Instrument at *C*, 1st set.						
T. B. M. A.	5	11.41	12.52	13.64	.....	.....	23.9	Tel. N., Bub. D.
		11.42	12.53	13.65	.....	.....	.....	" I., " D.
		11.42	12.53	13.65	.....	.....	.....	" I., " R.
		11.42	12.53	13.64	.....	.....	.....	" N., " R.
	Means.	11.418	12.528	13.645	12.530	2.227	.....	
		Instrument at *C*, 2d set.						
T. B. M. A.	5	11.415	12.535	13.64	.....	.....	.....	Same positions of Instr. used on all sets.
		11.42	12.535	13.65	.....	.....	.....	
		11.43	12.54	13.65	.....	.....	.....	
		11.415	12.53	13.645	.....	.....	.....	
	Means.	11.420	12.535	13.646	12.534	2.226	.....	
				Means..	12.532	2.226	.....	B. S. = + 12.532
								F. S. = — 11.294
		Instrument at *D*, 1st set.						D_1 = + 1.238
T. B. M. A.	5	8.57	19.11	29.68	.....	.....	.....	
		8.63	19.18	29.75	.....	.....	.....	
		8.62	19.19	29.75	.....	.....	.....	
		8.56	19.13	29.69	.....	.....	.....	
	Means.	8.595	19.152	29.718	19.155	21.123	.....	
		Instrument at *D*, 2d set.						
T. B. M. A.	5	8.57	19.14	29.69	.....	.....	.....	
		8.61	19.15	29.72	.....	.....	.....	
		8.62	19.17	29.73	.....	.....	.....	
		8.58	19.14	29.69	.....	.....	.....	
	Means.	8.595	19.150	29.708	19.151	21.113	.....	
				Means..	19.153	21.118	.....	B. S. = + 19.153
				Total..	.....	23.344	.....	F. S. = — 17.672
								D_2 = + 1.481
				Res. C or. for		Mean of + 2.08 m.	D_1 and ex. F.	D_2 = + 1.3595
								S. = + 0.0013
								D = + 1.3608

* Taken from precise level line along St.

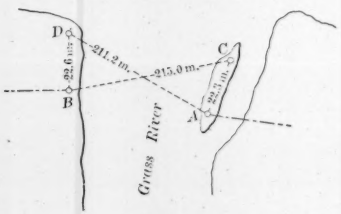


FIG. 16.

LEVEL NOTES FOR A RIVER CROSSING.* *(Right-hand page.)*

Direct Line.

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
T. B. M. B.	1	0.51	11.27	22.01	.....	.....	.....	Weather conditions very favorable; cloudy, calm.
		0.58	11.33	22.09	.....	.....	.....	
		0.58	11.34	22.10	.....	.....	.....	
		0.49	11.28	22.00	.....	.....	.....	4.00 P. M.
	Means.	0.540	11.305	22.050	11.298	21.510	.....	
T. B. M. B.	1	0.51	11.28	22.00	.....	.....	.....	
		0.58	11.31	22.09	.....	.....	.....	
		0.58	11.30	22.08	.....	.....	.....	Ratio of telemeter threads 1:100. Value of level tube 5″ per 2 mm. Instrument otherwise as above described.
		0.50	11.26	22.00	.....	.....	.....	
	Means.	0.542	11.288	22.042	11.291	21.500	.....	
				Means..	11.294	21.505	.....	
T. B. M. B.	1	16.54	17.67	18.80	.....	.....	.....	
		16.545	17.675	18.803	.....	.....	.....	
		16.545	17.675	18.802	.....	.....	.....	
		16.54	17.67	18.80	.....	.....	.....	
	Means.	16.542	17.672	18.801	17.672	2.259	.....	
T. B. M. B.	1	16.54	17.67	18.80	.....	.....	23.5	5 P. M.
		16.555	17.675	18.80	.....	.....	.....	
		16.55	17.68	18.802	.....	.....	.....	
		16.54	17.67	18.80	.....	.....	.....	
	Means.	16.546	17.674	18.800	17.673	2.254	.....	Thread Dist.
				Means..	17.672	2.256	.....	F. S. = 23.761
				Total ..	.....	23.761	.....	B. S. = 23.344
							Mean...	T. D. = 23.552
							Mean...	ex. F. S. = + 0.208 = + 2.08 m.

Lawrence River, by David Molitor, in 1898.

(*Left-hand page.*) TABLE

Grass River Crossing.*

AUG. 22D, 1898. BACK SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
		Instrument at *D*, 1st set.						
T. B. M. B.	1	16.34	17.47	18.598	.....	.....	23.5	Tel. N., Bub. D.
		16.345	17.475	18.60	.....	.....	.....	" I., " D.
		16.345	17.475	18.60	.....	.....	.....	" I., " R.
		16.335	17.47	18.598	.....	.....	.....	" N., " R.
	Means.	16.341	17.472	18.599	17.471	2.258	.....	
		Instrument at *D*, 2d set.						
T. B. M. B.	1	16.335	17.465	18.598	.....	.....	.....	Same positions of instr. used on all sets.
		16.34	17.475	18.60	.....	.....	.....	
		16.345	17.475	18.60	.....	.....	.....	
		16.34	17.465	18.598	.....	.....	.....	
	Means.	16.340	17.470	18.599	17.470	2.259	.....	
				Means..	17.470	2.258	.....	F. S. = + 18.983
								B. S. = — 17.470
								R_1 = + 1.463
		Instrument at *C*, 1st set.						
T. B. M. B.	1	0.75	11.50	22.23	.....	.....	.....	
		0.81	11.54	22.30	.....	.....	.....	
		0.81	11.55	22.31	.....	.....	.....	
		0.71	11.49	22.19	.....	.....	.....	
	Means.	0.770	11.520	22.258	11.516	21.488	.....	
		Instrument at *C*, 2d set.						
T. B. M. B.	1	0.75	11.51	22.25	.....	.....	.....	
		0.82	11.56	22.31	.....	.....	.....	
		0.81	11.55	22.30	.....	.....	.....	
		0.73	11.49	22.21	.....	.....	.....	
	Means.	0.778	11.528	22.268	11.524	21.490	.....	
				Means..	11.520	21.489	.....	F. S. = + 12.766
				Total...	.....	23.747	.....	B. S. = — 11.520
								R_2 = + 1.246
						Mean of R_1 and		R_3 = + 1.3545
				Res.	Cor.	for — 2.01	m. ex.	F. S. = + 0.0013
								R = + 1.3558

* Taken from precise level line along

No. 15. (*Right-hand page.*)

Reverse Line.

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
T. B. M. A.	5	8.31	18.91	29.47	.....	.....	.....	Weather conditions very favorable; cloudy, calm.
		8.41	18.95	29.52	.....	.....	.....	
		8.40	18.95	29.53	.....	.....	.....	
		8.35	18.91	29.47	.....	.....	.....	
	Means.	8.368	18.930	29.498	18.932	21.130	.....	5 P. M.
T. B. M. A.	5	8.33	18.92	29.48	.....	.....	.....	
		8.42	18.97	29.55	.....	.....	.....	
		8.40	18.95	29.52	.....	.....	.....	
		8.32	18.90	29.45	.....	.....	.....	Ratio of telemeter threads 1:100. Value of level tube 5″ per 2 mm. Instrument otherwise as above described.
	Means.	8.368	18.935	29.500	18.934	21.132	... .	
				Means..	18.933	21.131	.....	
T. B. M. A.	5	11.655	12.77	13.87	.....	.....	23.0	6 P. M.
		11.66	12.77	13.875	.....	.....	.....	
		11.67	12.775	13.88	....	.....	.....	
		11.655	12.77	13.87	.....	.....	.....	
	Means.	11.660	12.771	13.874	12.768	2.214	.....	
T. B. M. A.	5	11.66	12.77	13.87	.....	.....	.....	
		11.655	12.77	13.875	.....	.....	.....	
		11.655	12.77	13.875	.....	.....	.....	
		11.655	12.765	13.865	.....	.....	.....	
	Means.	11.656	12.769	13.871	12.765	2.215	... .	
				Means..	12.766	2.214	.....	Thread Dist. F. S. = 23.345
				Total...	.....	23.345	.....	B. S. = 23.747
							Mean...	T. D. = 23.546
							Mean...	ex. F. S. = — 0.201 = — 2.01 m.

St. Lawrence River, by David Molitor, in 1898.

TABLE No. 16.

Line.	Excess of F. S.	Residual correction.	Difference of Elevation.	
			Observed.	Corrected.
1st direct D_1....	+ 215.0 − 22.3 = + 192.7 m.	192.7 × 0.0635 = + 12.24 mm.	+ 123.8 mm.	+ 136.04 mm.
2d direct D_2.....	− 211.2 + 22.6 = − 188.6 m.	−188.6 × 0.0635 = − 11.88 mm.	+ 148.1 mm.	+ 136.22 mm.
1st reverse R_1...	+ 211.2 − 22.6 = + 188.6 m.	188.6 (− 0.0635) = − 11.88 mm.	+ 146.3 mm.	+ 134.42 mm.
2d reverse R_2....	− 215.0 + 22.3 = − 192.7 m.	−192.7 (− 0.0635) = + 12.24 mm.	+ 124.6 mm.	+ 136.84 mm.
		Means.....	+ 135.70 mm.	+ 135.88 mm.

Summary of Results and Index.

Table No. 17 gives an example of a summary of results intended to occupy the last double page of each notebook. This tabulation gives a very comprehensive statement of the work contained in the book, and furnishes directly the data required in the compilation of the final results and computation of elevations.

Results and Elevations.

Table No. 18 serves to illustrate the system adopted for the compilation of results and computation of elevations in the large summary notebook (7 x 9 ins.).

Column 1 gives the number or name of the bench-mark, the elevation of which is found. P. B. M. designates a permanent bench and T. B. M. a temporary bench.

Column 2 gives the bench-mark from which the bench-mark in Column 1 was determined.

Column 3 gives the length of each stretch or distance, in meters, between consecutive bench-marks given in Columns 1 and 2.

Column 4 gives the total length of level line, in kilometers, from the initial point to the bench-mark in Column 1.

Column 5 gives the direction in which the stretch was run. *D* designates direct line or leveled in a direction away from the initial or known point, and *R* designates reverse line, or leveled toward the initial point. *M* represents the arithmetical mean of the accepted results.

Column 6 gives the differences of elevation, in millimeters, between two successive bench-marks, as found by the direct and reverse lines, also the arithmetical mean of the two values. The + sign indicates that the bench found is above the bench from which it was determined on the direct line, calling back sights positive and fore sights negative. These signs are reversed on reverse lines, thus giving the same sign to the difference of elevation as found on the direct line. The mean of accepted values is used in computing elevations above datum.

Column 7 gives the residual error from the mean difference of elevation for each stretch.

Columns 8 and 9 give the algebraic summations of these residuals, and show the total discrepancy of each line from the mean, for the entire line from the initial point.

Column 10 gives the probable error in the mean difference of elevation for each stretch as found from the formula $\pm r = 0.674 \sqrt{\frac{\Sigma V^2}{m(m-1)}}$ where $V =$ the residual from the mean for each observation, and $m =$ the number of observations. For a single pair of observations this formula reduces to $\pm r = 0.674\ V$ (see Table No. 25 in the Appendix).

Column 11 gives the probable error r_o in the elevation of the point, as found from the initial point, and is $\pm r_o = \sqrt{\Sigma r^2}$. This formula is applicable only when all stretches are of equal length, though it is generally used without regard to the actual weights resulting from unequal lengths.

Column 12 gives the mean atmospheric temperature while leveling each stretch.

Column 13 should give the correction to rod length to reduce the latter to standard length and temperature. The law was not yet determined for the rods used.

Columns 14 and 15 give the elevations in meters and feet, respectively, above mean sea level. The history and authority for the determination of the initial bench-mark should follow here.

7. Rules to be Observed on Precise Level Work.

The following axiomatic principles are the result of careful study and observation, and may serve as a guide in answering many of the questions confronting the beginner:

1. Little or no reliance should be placed on an instrument, though

(*Left-hand page.*)

TABLE No. 17.—EXAMPLE OF

Summary

B. M. To	B. M. From	No. of Settings. D.	No. of Settings. R.	Mean thread distance. dcm.	Mean length of stretch. m.	Excess F. S. D. m.	Excess F. S. R. m.	Observed Diff. of Elev. D. mm.	Observed Diff. of Elev. R. mm.
T. 126	T. 125	8	9	43.926	888.7	+0.32	+0.44	−1190.3	−1187.4†
126	125	—	8	43.898	887.6	.....	+0.14	.....	−1188.8
127	126	9	9	52.784	1066.5	+0.18	+0.46	−2547.2	−2547.1
128	127	6	7	37.346	754.7	+0.02	+0.20	− 548.3	− 547.3
129	128	8	8	53.083	1071.3	−0.06	−0.26	+4397.3	+4397.3
130	129	11	11	70.256	1418.3	+0.02	+1.90	−2258.7	−2258.9
P. 34	130	7	7	32.356	655.5	+0.24	−0.02	+1661.8	+1663.8

†rejected.

*NOTE.—The final correction applied to all differences of elevation is + 0.063 mm. excess of F. S. on reverse lines, taking account of the sign of the differ-

The temperature is recorded in degrees Centigrade.

The allowable error of closure for each stretch is $e = 3\sqrt{L}$, where L is the

T = temporary bench-mark; P = permanent bench-mark; P. = afternoon;

The above example is taken from the notes of the precise level line along

it be of the most superior quality. If good results are sought they may be achieved only by constant vigilance and alertness, combined with most painstaking exertions on the part of the observer, as well as of every member of the party.

2. A line of levels is no better than the poorest portion thereof. Hence the importance of maintaining the same high standards of accuracy on every single rod reading. A slip in one place can never be made good even by the most careful work done thereafter.

3. The shortest line between two terminals is always the best line, provided no unnecessary climbs are encountered thereby. However, it is not advisable to pursue a roundabout route merely to avoid a slight rise or depression. In making a steep descent or ascent it is most advisable to adopt the shortest route perpendicular to the contours, utilizing full rod lengths with minimum lengths of shots. An extra setting costs less delay than to attempt to place rods for proper height.

4. Fore and back sights should always be equal, independent of the nature of the ground leveled over. That is, they should be made so nearly equal as not to require refocusing between reading fore and back sights. Any disturbance of the focusing mechanism would

INDEX PAGE FOR FIELD NOTEBOOKS.* (*Right-hand page.*)

of Results.

CORRECTED DIFF. OF ELEV.		CLOSING ERROR.		Direction of line.	No. of run.	TIME OF DAY.				M. TEMP.		PAGE.	
						D.		R.					
D. mm.	R. mm.	Actual. mm.	Allow. mm.			From	To	From	To	D.	R.	D.	R.
−1190.28	−1187.43†	2.85	2.83	N. 75° E.	1	5.15 P.	6.30 P.	4.00 P.	5.15 P.	30.0	32.2	32	31
.....	−1188.81	1.47	2.83	N. 75° E.	2	.....	.....	6.00 A.	7.15 A.	....	21.1	....	35
−2547.19	−2547.13	0.06	3.08	N. 80° E.	1	6.15 A.	7.45 A.	4.45 A.	6.15 A.	21.1	17.8	30	29
− 548.30	− 547.31	0.99	2.59	N. 70° E.	1	7.00 P.	7.45 P.	4.30 P.	5.30 P.	21.1	18.9	33	34
+4397.30	+4397.32	0.02	3.08	N. 45° E.	1	3.00 P.	5.00 P.	5.30 P.	6.30 P.	23.3	21.7	36	37
−2258.70	−2259.02	0.32	3.56	E.	1	7.00 P.	7.15 A.	5.00 A.	8.00 A.	20.0	20.0	38	40
+1661.82	+1663.80 †rejected	1.98	2.42	E.	1	5.00 P.	6.15 P.	6.15 P.	7.15 P.	23.3	21.1	42	43

per meter excess of F. S. for a positive difference of elevation, or − 0.063 mm. per meter
ence of elevation as given for the direct line.

length of the single stretch in kilometers and e is in millimeters.
A = forenoon; D = direct line; and R = reverse line.
Lake St. Clair, Mich., by David Molitor, 1899.

destroy the relative accuracy of readings taken before and after such disturbance. On short sights, the permissible inequality is very small and may frequently require resetting the forward rod. On sights of 50 to 100 m. such inequality may range from 2 to 3 m., but this should not be allowed to accumulate. The sums of sights for an entire stretch should be made exactly equal by properly setting the instrument on the last station.

5. The limiting length of sight for any given instrument and any atmospheric conditions is one for which repeated back and fore sight readings, taken on two fixed points equidistant from the instrument, shall give equal values for the difference of elevation between these points within the attainable limits of accuracy of rod estimations. Ordinarily, good conditions will permit of taking sights of 50 to 70 m., using a 1 to 3-second (per 2-mm. division) level. Very good conditions will permit of reading at 90 m. and exceptionally at 100 m. Fair progress can be made when only 30-m. sights are possible, provided the weather is not too warm, and this limit is solely the result of atmospheric vibrations. Sights exceeding 100 m. should not be taken unless extraordinary precautions are used, such as indicated for a river crossing.

6. The length of the bubble should be retained practically constant and should be a little over half as long as the graduation.

7. Extreme quiet should prevail about the instrument when work is in progress. The observer calls his readings in quick succession and the recorder should not repeat them back but should test their correctness by careful inspection. This is contrary to general practice, but, in this particular case, it will be found highly commendable.

8. The observer should make it a practice not to remember any readings after he has called them off, as otherwise his memory will frequently deceive him by causing him to call something that he has in mind in place of the real readings which he sees. As in all scientific work, results should be fact and not fiction, and the observer's sole aim should be directed toward obtaining impartial and truthful results, whether these be pleasing or otherwise. The observer must feel absolutely satisfied with every reading taken, and when this is not the case, readings must be repeated until such satisfaction is realized, even though the notes necessitate changes or alterations. A failure to attain perfectly trustworthy readings is always a good cause for the rejection of such work when reasons therefor are given in the remarks.

9. The strict requirement, frequently imposed, regarding the forbiddance to erase, alter or obliterate figures in notes, is not considered a safeguard against fraudulent perpetrations on the part of an observer. Should he have such intention he can easily accomplish his purpose and still comply with all requirements of this kind. If the employer is not willing to accept the final results of his employee, then the latter is not a suitable person to entrust with precise leveling operations.

8. Advantages of the Method Advocated.

The most reliable results in precise leveling are undoubtedly to be obtained by careful and systematic observations designed to prevent and eliminate errors directly in the observations, avoiding, as far as possible, all more or less speculative mathematical manipulations intended to correct and distribute errors inherent in ordinary work and imperfect methods. The labor spent in computing and applying corrections as practiced in the past, when applied to direct elimination of errors in the field, will yield the highest attainable accuracy in the results, without necessarily increasing the cost of the work. This is the fundamental idea in the method described and illustrated.

TABLE No. 18.—EXAMPLE OF SUMMARY NOTEBOOK.*

B. M. (1)	Determined from. (2)	Distance. Partial. (3)	Distance. Total. (4)	Direction. (5)	Difference of elevation. (6)	Resid. V (7)	ΣV Direct line. (8)	ΣV Reverse line. (9)	Prob. Error. $r \pm$ (10)	Prob. Error. $r_o \pm$ (11)	Mean Temp. (12)	Rod Cor. (13)	Elevation. Meters. (14)	Elevation. Feet. (15)
		m.	klm.		mm.	mm.	mm.	mm.	mm.	mm.	deg. C.	mm.		
T. B. M. 126.....	T. B. M. 125	887.6	38.654	*D.*	—1190.28	+0.74	—3.41	+3.41	0.50	3.1	30.0	.....	179.18169	587.8717
				R.	—1187.43†	.....	.....	.....	....	....	32.2	.....		
				R2.	—1188.81	—0.74	.....	.....	....	....	21.1	.....		
				M.	—1189.54	.....	.....	.....	....	....	....	.....		
T. B. M. 127.....	T. B. M. 126	1066.5	39.720	*D.*	—2547.19	+0.03	—3.38	+3.38	0.02	3.1	21.1	.....	176.63453	579.5148
				R.	—2547.13	—0.03	.....	.....	....	....	17.8	.....		
				M.	—2547.16	.....	.....	.....	....	....	....	.....		
T. B. M. 128.....	T. B. M. 127	754.7	40.475	*D.*	— 548.30	+0.50	—2.88	+2.88	0.34	3.2	21.1	.....	176.08673	577.7175
				R.	— 547.31	—0.50	.....	.....	....	....	18.9	.....		
				M.	— 547.80	.....	.....	.....	....	....	....	.....		
T. B. M. 129.....	T. B. M. 128	1071.3	41.546	*D.*	+4397.30	+0.01	—2.87	+2.87	0.01	3.2	23.3	.....	180.48404	592.1445
				R.	+4397.32	—0.01	.....	.....	....	....	21.7	.....		
				M.	+4397.31	.....	.....	.....	....	....	....	.....		
T. B. M. 130.....	T. B. M. 129	1418.3	42.964	*D.*	—2258.70	—0.16	—3.03	+3.03	0.11	3.2	20.0	.....	178.22518	584.7335
				R.	—2259.02	+0.16	.....	.....	....	....	20.0	.....		
				M.	—2258.86	.....	.....	.....	....	....	....	.....		
P. B. M. 34.....	T. B. M. 130	655.5	43.620	*D.*	+1661.82	+0.99	—2.04	+2.04	0.67	3.2	23.3	.....	179.88799	590.1889
				R.	+1663.80	—0.99	.....	.....	....	....	21.1	.....		
				M.	+1662.81	.....	.....	.....	....	....	....	.....		

† Rejected.

* Taken from Precise Level Line along Lake St. Clair, Mich., by David Molitor, in 1899.

TABLE No. 19.—Errors in Lines Run Direct

Direct Line.

Back sight.			Fore sight.			Diff. Elevat.	Remarks.
Point.	M. Read.	Th. dist.	Point.	M. Read.	Th. dist.		
	dcm.	dcm.		dcm.	dcm.	mm.	
T. B. M. 128	26.007	3.664	T. P. 1	16.094	3.660	+ 991.3	Cloudy; fresh wind; reading good.
T. P. 1	22.051	3.429	2	10.686	3.526	+1136.5	Cool, puffy wind.
2	16.286	3.363	3	17.997	3.205	— 171.1	Sun out on first set of F. S.
3	19.246	3.193	4	15.561	3.252	+ 368.5	Sun shining; brisk wind.
4	10.100	3.165	5	8.981	3.224	+ 111.9	Cloudy.
5	22.646	3.245	6	5.298	3.164	+1734.8	Cloudy; wind abating.
6	20.919	3.319	7	12.631	3.338	+ 828.8	Delayed 1 hr. by rain.
7	19.102	3.169	T. B. M. 129	25.136	3.181	— 603.4	Cloudy; light wind.
Totals..	156.357	26.547		112.384	26.550	+4397.3	
T. B. M. 129	21.806	3.448	T. P. 1	16.216	3.494	+ 559.0	Cloudy; light wind; reading very good.
T. P. 1	16.640	3.409	2	17.555	3.567	— 91.5	" " "
2	16.838	3.323	3	15.458	3.317	+ 138.0	" " "
3	13.355	3.213	4	17.356	3.014	— 400.1	" " "
4	16.038	3.038	5	20.713	3.068	— 467.5	" " "
5	9.652	3.120	6	20.521	3.207	—1086.9	Cloudy; calm; reading good.
6	15.042	3.239	7	21.216	3.205	— 617.4	" " "
7	15.419	3.149	8	16.151	3.043	— 73.2	Cloudy; light wind.
8	11.213	3.164	9	14.839	3.155	— 362.6	" " "
9	14.487	3.251	10	12.716	3.228	+ 177.1	Partly cloudy; light wind.
10	14.578	2.761	T. B. M. 130.	14.914	2.818	— 33.6	Sun shining.
Totals..	165.068	35.115		187.655	35.116	—2258.7	

Note.—Value of level tube, 2.23" per 2-mm. graduation; magnifying power of telescope

AND REVERSE, ON PERMANENT TURNING POINTS.

REVERSE LINE.

Back sight.			Fore sight.			Diff. Elevat.	Remarks.	Error D—R.
Point.	M. Read.	Th. dist.	Point.	M. Read.	Th. dist.			
	dcm.	dcm.		dcm.	dcm.	mm.		mm.
T. P. 7	17.706	3.660	T. B. M. 128	27.626	3.668	+ 992.0	Cloudy; calm; light rain.	—0.7
6	10.743	3.517	T. P. 7	22.097	3.427	+1135.4	" " "	+1.1
5	18.193	3.199	6	16.484	3.365	— 170.9	" " "	—0.2
4	15.844	3.254	5	19.523	3.109	+ 367.9	" " "	+0.6
3	8.831	3.229	4	9.951	3.168	+ 112.0	" " "	—0.1
2	5.336	3.162	3	22.687	3.243	+1735.1	" " "	—0.3
1	12.534	3.339	2	20.826	3.322	+ 829.2	" " "	—0.4
T. B. M. 129	25.109	3.181	1	19.075	3.166	— 603.4	" " "	0.0
Totals..	114.296	26.541		158.269	26.528	+4397.3	Prob. error ±0.517 mm.	0.0
T.P. 10	16.062	3.480	T. B. M. 129	21.655	3.472	+ 559.3	Partly cloudy; reading good.	—0.3
9	17.441	3.359	T.P. 10	16.524	3.409	— 91.7	Partly cloudy; reading good.	+0.2
8	15.083	3.318	9	16.462	3.330	+ 137.9	Partly cloudy; reading good.	+0.1
7	17.036	3.017	8	13.089	3.211	— 399.7	Partly cloudy; reading good.	—0.4
6	20.277	3.071	7	15.602	3.050	— 467.5	Clear; light wind; reading good.	0.0
5	20.245	3.207	6	9.373	3.122	—1087.2	Clear; fresh wind; reading good.	+0.3
4	20.936	3.202	5	14.761	3.236	— 617.5	Clear; fresh wind; reading good.	+0.1
3	15.889	3.053	4	15.157	3.150	— 73.2	Clear; light wind; reading good.	0.0
2	14.683	3.159	3	11.057	3.165	— 362.6	Clear; light wind; reading good.	0.0
1	12.506	3.217	2	14.277	3.252	+ 177.1	Partly cloudy; reading good.	0.0
T. B. M. 130	14.942	2.800	1	14.604	2.791	— 33.8	Clear; reading good.	+0.2
Totals..	185.100	35.093		162.511	35.188	—2258.9	Prob. error ± 0.32.	+0.2

50 diameters; smallest graduation on rods, 1 cm.; Lake St. Clair Levels, 1899.

Furthermore, the readings are so taken and recorded that, by careful inspection, the recorder can always see whether there has been an error in reading or recording. Should any disparity exist, or any irregularity from atmospheric influences be noticeable, the readings can be repeated until a rational set is obtained. This is the secret to the success of this method.

The only disturbing element which cannot be detected in the readings, even though it may be acting, is the warping of the telescope caused by direct and radiated heat in warm weather. This error may, however, be detected by closing each stretch in the same half of the day, and it may be almost entirely eliminated by closing an equal number of stretches forenoons and afternoons.

The office reduction consists merely of compiling results and computing elevations, and no corrections of any kind are necessary save those already applied in the field reduction and checked in the field.

While the ultimate attainable accuracy in precise leveling depends directly on the number of levelings or duplications of work, it is not deemed necessary, from a utilitarian standpoint, to run more than two lines, one direct and the other reverse, thus forming a loop. To go beyond this would result in merely a slight increase in accuracy at a very considerable increase in cost over that for a double line. Thus, three single lines would yield a theoretical accuracy of 1.4* times that obtained from two lines, at 50% increase in cost, and four lines would give an accuracy of 1.7* times that from two lines, at 100% increase in cost.

Hence, conceding the necessity for running duplicate lines, it does not appear practicable to seek greater accuracy by duplicating lines more than twice, and, whatever process of leveling is used, it should be designed with a view of producing the highest accuracy in the results for a single pair of lines. This claim attaches to the method above described.

Accuracy of Precise Level Work.

1. The Actual Error of Closure.

The various sources of error have been treated at considerable length in a previous part of this paper. The residual or resultant error of a loop of levels formed by a pair of lines (direct and reverse) will now be discussed.

*Note (added by author after reading discussions).—Should be 1.2 and 1.4 times, respectively, as given in Mr. Hayford's discussion.

A very prevalent idea among men of experience in leveling is that the resultant error of closure of a stretch of levels is essentially accidental, being made up of errors of estimation and pointing, and atmospheric disturbances, all of which are considered of a compensating nature. Heaving or settling of rods or instrument have been cited as explanations for errors of a constant nature. Cumulative errors have also been attributed to the personal equation of the observer.

According to the writer's experience, however, the important cumulative error is that produced by the warping effect of the sun's direct and reflected heat rays on the telescope (see experiments in Table No. 6). All other errors are generally of a compensating nature when good instruments are used. Errors due to heaving or settling, when they reach a measurable magnitude, are the result of carelessness, improper instrumental equipment or very unsuitable ground.

To illustrate more conclusively the views here expressed, the levelings in Table No. 19 are given, in which two stretches were leveled over fixed turning points by the method described. The individual thread readings are omitted for the sake of brevity, and only the mean pointings are given. The settings of the reverse lines are given in the order taken on the direct lines, thus offering a more ready comparison.

The weather conditions on the direct line 128-129 were good, though somewhat changeable, while on the reverse line the conditions were uniformly good. This fact, as explained by the remarks, clearly shows the effect of atmospheric changes on the level error. While the error was entirely compensated in the eight instrument settings, yet the difference of elevation between T. P. 1 and T. P. 2 found on the direct line is 1.1 mm. larger than that found on the reverse line.

On the stretch 129-130, the weather conditions were quite uniform, and the partial errors, as also the residual closing error, are all very small, showing that the small final closure is not an accidental balance of large errors, but the result of accurate pointings and close rod estimations, without atmospheric disturbances. This stretch, or any portion thereof, closes within the assigned limits of permissible error for work of the highest grade.

Other data relating to these two lines are given in the example of index page for field notebooks, Table No. 17.

The errors in the two lines 128–129 and 129–130 are entirely of a compensating nature. However, to give an idea of the effect of temperature and atmospheric changes producing cumulative errors, a few other readings are given in Table No. 20, which were taken on the last T. P. 10 and T. B. M. 130, when the sun was higher, but good readings still possible.

TABLE No. 20.

Time, July 6th, 1899.	Difference of elevation, T. P. 10 to T. B. M. 130.	Temp.
	mm.	deg. C.
7.15 A. M.	−33.6*	18.3
7.25 A. M.	−32.7	19.4
7.30 A. M.	−32.2	20.5
8.00 A. M.	−32.2	21.1
4.45 P. M.	−33.8*	26.7

* Accepted.

Such wide differences from the values known to be good cannot escape detection, and, when temperature conditions are such as to produce a marked inclination in the line of sight, the error will always show as a cumulative effect, provided a stretch is closed during the same half of the day.

Many other examples of this kind might be cited to locate the causes of cumulative errors in temperature effects rather than to indulge in a speculative mathematical elaboration intended to show effect of rod supports or personal errors which may or may not be founded on facts.

L. L. Wheeler, M. Am. Soc. C. E., United States Assistant Engineer, in discussing the cumulative errors of precise level work done under the Mississippi River Commission,* attributes these errors almost wholly to personal errors of the observers, and even derives the personal equations of the various observers engaged on this work. These cumulative errors more probably represent the susceptibility to temperature effects of the various instruments used. This presumption seems the more justifiable because a large portion of the work was done during very warm weather and generally in a north and south direction.

* Annual report of Miss. Riv. Com., 1883, p. 141 *et seq.*

TABLE No. 21.—ACCURACY OF PRINCIPAL PRECISE LEVEL WORK IN THE UNITED STATES.

NAME OF LINE.	OBSERVER.	Time of work.	Direction of line.	Length of line.	Prob. error per kilometer.
		WORK BY THE UNITED STATES COAST AND GEODETIC SURVEY.		km.	mm.
St. Louis, Mo., to New Haven, Mo.	A. Braid	Oct. to Dec. '82.	W.	116.2	±0.83
Mobile, Ala., to Okolona, Miss	J. B. Weir and J. E. McGrath	1884-1886.	N.	423.5	±0.83
Villa Ridge, Ky., to Odin, Ill.	J. B. Weir	Apr.-Jun. '85.	N.	176.0	±0.91
New Haven to Jefferson City, Mo.	G. Bradford and I. Winston	Apr.-Jun. '88.	W.	84.4	±0.98
Greenfield, Tenn., to Villa Ridge, Ky.	I. Winston and P. A. Welker	Oct. '88-Jan. '89.	N.	128.1	±1.02
Okolona, Miss., to Greenfield, Tenn.	I. Winston and F. A. Young	Oct. '89-Mar. '90.	N.	265.4	±0.93
Jefferson City, Mo., to Holliday, Kan.	I. Winston and F. A. Young	Apr.-Oct. '91.	N. W.	286.7	±1.47
Corinth, Miss., to Memphis, Tenn.	I. Winston	Nov. '90-Jan. 91.	W.	151.4	±1.05
Old Point Comfort—Richmond, Va.	I. Winston	Dec. '91-Feb. '92.	N. W.	140.5	±1.80
Holliday—Salina, Kan.	I. Winston	Jul.-Oct. '95.	W.	275.9	±0.85
		WORK BY THE MISSISSIPPI RIVER COMMISSION.			
Keokuk, Iowa—Grafton, Ill.	J. B. Johnson and O. W. Ferguson	May-Aug. '81.	S. E.	242	±0.74
Grafton—Cairo, Ill.	J. A. Paige and O. W. Ferguson	Aug. '80-Mar. '81.	S. E.	346	±1.18
Carrollton, La.—Biloxi, Miss.	J. B. Johnson and O. W. Ferguson	Early 1882.	N. E.	140	±0.67
Keokuk, Iowa—Fulton, Ill.	J. B. Johnson and O. W. Ferguson	Sept.-Nov. '82.	N. E.	267	±0.67
Fulton, Ill.—Chicago, Ill.	J. B. Johnson and O. W. Ferguson	May-Aug. '83.	N. E.	261	±0.84
St. Paul, Minn.—Savannah, Ill.	O. W. Ferguson and A. L. Johnson	May-Oct. '91.	S. E.	484.7	±0.62
*Duluth—St. Paul, Minn.	J. A. Paige	May-Sept. '91.	S. S. W.	251.2	±0.99
*New Orleans—Head of Passes	J. A. Paige and E. J. Thomas	Jan.-Mar. '93.	N. W.	164.3	±0.69
*St. Paul—Aitkin	W. S. Williams	June-Nov. '98.	N. W.	317.6	±0.54
*New Orleans—Port Eads.	W. S. Williams and E. J. Thomas	Jan.-Mar. '98.	S. E.	183.5	±0.68
*Baton Rouge—New Orleans	W. S. Williams and E. J. Thomas	Dec. '97-Jan. '98	S. E.	143.7	±0.68
*Fort Adams — Baton Rouge	W. S. Williams and E. L. Harman	Jan.-Mar. 1900.	S. E.	129.3	±0.63
*New Orleans—Biloxi	W. S. Williams and E. L. Harman	" "	E. N.E.	135.3	±0.61
*Brainard—Lake Itasca, Minn.	W. S. Williams and E. L. Harman	Aug.-Nov. 1900.	N. W.	203	±0.59
		WORK BY MISSOURI RIVER COMMISSION.			
St. Joseph, Mo.—Mouth Missouri River	O. W. Ferguson and A. L. Johnson	Mar.-Nov. '92.	S. S. E.	622.5	±0.58
		WORK BY ENGINEER DEPARTMENT, U. S. A.			
Corinth, Miss.—Decatur, Ala.	O. W. Ferguson and F. B. Williamson	Apr.-Jul. '95.	S. S. E.	173	±0.62
Birmingham, Ala.—Meridian, Miss.	O. W. Ferguson		S. W.		±0.55
Grossepointe—New Baltimore, Mich.	David Molitor	June, 1899.	N.	43.6 41.0	±0.48 ±0.33
		WORK BY U. S. BOARD OF ENGINEERS ON DEEP WATERWAYS.			
Gibraltar — Grossepointe, and N. Baltimore—Pt. Huron, Mich.	O. W. Ferguson	Sept. 21-Dec. 15, '98.	N.	117.6	±0.48
St. Regis—Oak Point, N. Y.	David Molitor	Aug. 10-Nov. 1, '98.	S. W.	109.2	±0.61
Oak Point—Tibbetts Pt., N.Y., St. Lawrence River	" "	Nov. 1, '98-Jun. '99.	S. W.	88	±0.48

*This information was kindly furnished by J. A. Ockerson, M. Am. Soc. C. E., Member, Miss. Riv. Com.

A very fair idea of the accuracy generally attained on precise level work in the United States, and which has not been excelled by other countries, is given in Table No. 21.

2. The Probable Error.

Having briefly discussed the actual error of closure found between a pair of direct and reverse level lines, it is intended here to illustrate the extent to which the theory of error is applicable to the subject of leveling.

The formulas for mean and probable errors are applied to precise level work more particularly to give a standard of comparison between different lines and observers. They are also used to furnish a criterion as to the accuracy of the ultimate result of a line of levels.

In reality, however, these formulas serve the purpose intended in a very imperfect manner, being, as they are, only applicable to repeated observations of the same function, taken under similar conditions, and assuming the errors of observation to be strictly of a compensating nature. It is useless to add here that the errors of leveling, and the prevalent conditions producing them, do not morally justify the application of the theory of error to leveling. There is only one excuse for such application, and that is for want of something better.

The theory of error is applicable to observations taken under the same conditions, so that the error of a single observation is as likely to be plus as minus. Also, when comparing groups of observations, each group must contain the same number of observations, and all must be taken under similar circumstances.

In leveling, the principal errors are due to variable conditions which in no way obey natural laws of a constant character. Also, the number of observations forming a group (rod readings in a stretch) differs very widely, thus giving each group a certain weight in the determination of the residual error.

Hence, the essential requirements necessary to render the theory of error applicable are entirely wanting in level observations.

In a general way, however, the errors of leveling, resulting from the above-described system, are more inclined to be compensating than cumulative, and a computation of probable error, while not truly reliable, would nevertheless constitute a fair measure of the average accuracy of a piece of work.

In comparing the accuracy of different lines of levels, it is, of course, necessary to consider the number of instrument settings per kilometer of line as well as the lengths of stretches, because each stretch constitutes a group of observations for which only the algebraic sum of the errors is known and each of the quantities in the group is observed but twice.

It is necessary, therefore, that the stretches be all of the same length and leveled over with a constant length of shot, all of which is impracticable and almost impossible.

Generally, the length of a stretch is determined by the amount of labor or number of settings necessary in covering a certain line, and would imply about 10 to 12 instrument settings, covering from 100 to 1 200 m. The length of shot is chosen to fit the ground and atmospheric conditions for reading, and may vary between 6 and 100 m. It would be practically impossible to assign weights to groups of observations in which such widely different factors constantly enter.

As an illustration, the probable error is computed for the stretch T. B. M. 128 to T. B. M. 129, Table No. 19, in which the errors on the individual turning points were determined (see Table No. 22).

This stretch of 1071.6 m. length, leveled direct and reverse over seven permanent turning points, with eight instrument settings each, closed with a final error of 0.00 mm., and in the computation of Table No. 18, with a closure of $\pm$ 0.01 mm. as a result of the correction for excess of fore sights. In the computation of Table No. 22 the probable error is found to be $\pm$ 0.517 mm., while for the final closure of the stretch it is only $\pm$ 0.003 mm.

A similar computation of the stretch T. B. M. 129 to T. B. M. 130 (length 1418.3 m.) results in a probable error of $\pm$ 0.32 mm. (Table No. 19), while the actual error of closure was $\pm$ 0.16 mm. (Table No. 18) with a probable error of $\pm$ 0.11 mm. This stretch is undoubtedly better than the other, though it has a more detrimental effect on the apparent accuracy of the work.

The inevitable conclusion to be drawn from the foregoing is, as previously stated, that the theory of error is not justly entitled to a place of honor in discussing level errors, though it may be used, as in the past, to give an approximate measure of comparison between different lines.

Assuming, then, that a computation of the probable error of a line

of levels is desirable, the formulas previously given will be repeated here, and may be applied with proper reserve.

For a stretch over which m level lines have been run, the probable error in the mean of the m values found for the difference of elevation will be $r = \pm 0.674 \sqrt{\frac{\Sigma v^2}{m(m-1)}}$ in which v is the residual error from the mean for each value. When only two lines (direct or reverse) have been run, then $m = 2$, and the two residuals become equal; hence for this case $r = \pm 0.674 \sqrt{\frac{2 v^2}{2}} = \pm 0.674 v$. The values of r for any given values of v may be taken from Table No. 25, in the Appendix.

The probable error per kilometer for any stretch, of length L, in kilometers, is $\pm \frac{r}{\sqrt{L}}$, in millimeters.

TABLE No. 22.—PROBABLE ERROR OF A STRETCH OF LEVELS.

B. M.	Determined from	Distance. Partial.	Distance. Total.	Direction.	Difference of elevation.	Resid. v.	Σv. Direct line.	Σv. Reverse line.	Prob. Error. $\pm r$.	Prob. Error. $\pm r_o$.
		m.	m.		mm.	mm.	mm.	mm.	mm.	mm.
T. P. 1	T. B. M. 128	147.7	147.7	D.	+ 991.3	+0.35	+0.35	−0.35	0.236	0.236
		.....	.....	R.	+ 992.0	.....	.....	.....	.....	.....
		.....	.....	M.	+ 991.65	.....	.....	.....	.....	.....
2	T. P. 1	140.3	.....	D.	+1136.5	−0.55	−0.20	+0.20	0.371	0.439
		.....	.....	R.	+1135.4	.....	.....	.....	.....	.....
		.....	.....	M.	+1135.95	.....	.....	.....	.....	.....
3	2	132.6	.....	D.	− 171.1	+0.10	−0.10	+0.10	0.067	0.445
		.....	.....	R.	− 170.9	.....	.....	.....	.....	.....
		.....	.....	M.	− 171.00	.....	.....	.....	.....	.....
4	3	130.1	.....	D.	+ 368.5	−0.30	−0.40	+0.40	0.202	0.488
		.....	.....	R.	+ 367.9	.....	.....	.....	.....	.....
		.....	.....	M.	+ 368.20	.....	.....	.....	.....	.....
5	4	129.0	.....	D.	+ 111.9	+0.05	−0.35	+0.35	0.034	0.490
		.....	.....	R.	+ 112.0	.....	.....	.....	.....	.....
		.....	.....	M.	+ 111.95	.....	.....	.....	.....	.....
6	5	129.4	.....	D.	+1734.8	+0.15	−0.20	+0.20	0.101	0.500
		.....	.....	R.	+1735.1	.....	.....	.....	.....	.....
		.....	.....	M.	+1734.95	.....	.....	.....	.....	.....
7	6	134.3	.....	D.	+ 828.8	+0.20	0.00	0.00	0.134	0.517
		.....	.....	R.	+ 829.2	.....	.....	.....	.....	.....
		.....	.....	M.	+ 829.00	.....	.....	.....	.....	.....
T. B. M. 129	7	128.2	.....	D.	− 603.4	0.00	0.00	0.00	0.000	0.517
				R.	− 603.4	.....	.....	.....	.....	.....
		1071.6	.....	M.	− 603.40	.....	.....	.....	.....	.....

NOTE.—For method of computation see explanatory remarks accompanying Table No. 18.

The probable error r_o of a line of levels of length ΣL is $r_o = \pm \sqrt{\Sigma r^2}$, and the probable error per kilometer is as before $\pm \frac{r_o}{\sqrt{\Sigma L}}$.

These formulas are readily solved with any ordinary table of squares and square roots.

3. The Allowable Error.

In view of the many perplexities attending the results and accuracy of precise level work, the question naturally arises as to what shall constitute acceptable work. This question is not easily answered.

As a general rule, that work is acceptable which is as good as, or better than, has ever been done before. Any work falling within this requirement is certainly beyond adverse criticism. However, certain local and temporary conditions may preclude the possibility of achieving such results, and then it becomes necessary to give an explanation of the reason.

In order that work may be carried on with a uniform degree of accuracy, a certain limit of allowable error is usually stipulated, and whenever a pair of lines does not close within this limit one or more additional lines must be run until a pair (direct and reverse) is obtained which fulfills the requirement. When weather remarks, etc., are carefully noted, the observer will have little difficulty in deciding which one of a pair of lines is most likely to be in error and a third line will usually give an acceptable pair.

According to the law of the continuity of error in a series of observations, the allowable error of closure is made a function of the $\sqrt{L}$, thus, $e = c\sqrt{L}$, assigning various values to c, according to the accuracy desired. In this expression, L is generally taken as the length of the loop line, in kilometers, and e is in millimeters.

The values assigned to the constant c range from 2 to 10, the former being the smallest limit at present attainable, without unnecessarily increasing the cost of the work, when instruments of the highest grade are used. With very good common levels work may be done within the limit $10\sqrt{L}$, though such work cannot be classed with precise levels at this time. The largest limit at present permitted on precise levels is $3\sqrt{L}$.

Some work under the Engineer Department, U. S. Army, has been done within the limit $e = 3\sqrt{L}$, in which L is the length of the single

stretch. This limit is very nearly the same as $2\sqrt{L}$ when L is the length of the loop line, which latter is perhaps a better form, and offers a more ready means of comparison.

It follows from the foregoing, that the resulting accuracy of a line of levels, as also the cost and progress of the work, depends almost entirely on the limit assigned, since all work not within such limit must be rejected.

The formula assumes that the error of leveling is purely a function of the length of the line leveled over. It makes no allowance for the fineness of the instrument used, the nature of the ground leveled over, or the length of shot, all of which are important factors which largely govern the magnitude of the closing error.

While it is quite impossible to derive a formula for allowable error which shall include all the variables in their true mathematical relation, the writer ventures to propose a new formula which is intended to give due weight to the principal factors which go to make up the error of closure between a pair of lines.

This formula is $e = c\sqrt{L\sqrt{l}}$ in which L is the length of the loop line, in kilometers; l is the average length of shot, in meters; e is the allowable error, in millimeters; and c is a constant representing the quality of the instrument, and differs for different instruments. Calling v the curvature of the level tube, in seconds per 2-mm. graduation; m the magnifying power of the telescope, in diameters; and t the smallest graduation on the rod, in millimeters; then $c = \sqrt{\frac{v\,t}{m}}$.

As this formula does not make any allowance for atmospheric variations it may be generally accepted only for excellent weather conditions, and errors which exceed this limit are usually the result of poor observing or injurious temperature effects.

As the principal source of error, however, is caused by temperature and atmospheric effects, as previously explained, the foregoing value will be found to be extremely small and difficult to maintain, even on the most careful leveling. It is, therefore, necessary to add a numerical constant k, to cover these errors to the extent desired. The final formula then becomes

$$e = k\sqrt{\frac{v\,t}{m}}\quad\sqrt{L\sqrt{l}} = k\,c\sqrt{L\sqrt{l}}$$

To facilitate the use of this formula, as also those in common use, a diagram, Fig. 17, is given in the Appendix, from which values of e may be read off for any ordinary values of L and l. The diagram is self-explanatory and will be found far more convenient than a tabulation of allowable errors.

4. The Adjustment of Errors.

Whenever a stretch of levels is run in two directions between successive bench-marks, thus forming a closed polygon, the resulting error of closure must be distributed in some manner prior to using the difference of elevation thus obtained in the computation of elevations above datum.

Since there are generally but two measurements (direct and reverse) of the difference of elevation between successive benches, the most probable value is represented by their arithmetic mean. This is quite as true when three or four accepted measurements exist, though this is rarely the case, since a single pair, closing within the stipulated limit, is considered sufficient. The foregoing examples of level notes are reduced in this manner.

When a line of levels closes on itself, thus forming a polygon, the resulting error of the polygon must be distributed over the entire periphery (either in proportion to the length, or, perhaps better, in proportion to the square root of the length, from the known starting point).* As there is always more or less guesswork attending such distribution, it matters little which method is adopted.

For large polygons, the spheroidal correction should first be applied to each side before attempting any final distribution of the closing error, as this error may be materially affected thereby.

It sometimes happens that a succession of polygons, or a polygonal system, requires adjustment. Such an adjustment may be made by the method of least squares, as given in some text-books on the "Adjustment of Errors" (Wright and others), or it may be accomplished by a repetition of the method just mentioned for a single polygon, which latter is less laborious and quite within the knowable exactness. All methods applicable to this problem must be regarded as guesswork, be they methodical or otherwise. A good common-sense

* NOTE (added by author after reading discussions).—This should read—so that the difference in elevation of each side receives a correction proportional to the length, or, perhaps better, to the square root of the length of such side.

adjustment is entitled to as much confidence as any other. It is, therefore, deemed unnecessary to add more on this subject.

Rapidity of Work.

The rapidity with which precise level work may be prosecuted is governed by the quality of the work, the training and experience of the party, the grade of the instrumental equipment, the nature of the ground traversed and the weather.

It is, therefore, difficult to give exact information on this subject, though some few facts will enable the reader to make a fair estimate for various conditions under consideration.

With a well-equipped and experienced party, following the method previously described, the average rate of progress over comparatively level country will be about 4.5 km. of single line per day worked. This is for a limiting error of closure $e = 3\sqrt{L}$, in which L is the length of the loop, in kilometers.

The weather conditions generally limit the length of sight to from 30 to 85 m., with an average of about 50 m. This would necessitate from 17 to 6 instrument settings per kilometer of single line. Occasionally, short sights, ranging from 6 to 10 m., are necessitated by undulating ground, in which case the rate of progress becomes very slow.

When the previously described method of observing is followed, the average time required to walk between stations, set up the instrument, and take the readings, is about $9\frac{1}{2}$ minutes. For exceptionally good runs, with clear, steady air and good footing on the road, this time can easily be reduced to 7 minutes, enabling the work to proceed at the rate of about 1 mile an hour. However, 1 km. per hour is very good progress, and, for average conditions of weather and walking, not much over 600 m. per hour will be accomplished. This latter rate represents a good average for a whole field season.

The number of Sundays and days on which weather conditions prevent field work being done can be estimated as between 20 and 25% of the days spent in the field. However, most of this time is well spent in working up the notes and preparing the summary of results, etc.

The average working day will rarely exceed 8 hours, and will generally be about $7\frac{1}{2}$ hours, because the midday hours do not permit

of good work being done, except during cloudy weather. This apparently lost time can always be advantageously devoted to the checking of notes.

Table No. 23 will suffice to give a good idea of the work performed in a field season. The St. Lawrence River levels were run by the writer, from August 10th to December 1st, 1898, and from May 1st to June 1st, 1899, for the U. S. Board of Engineers on Deep Waterways.*

The Lake St. Clair levels were run by the writer in June, 1899, for the Engineer Department, U. S. Army. The former line was generally very undulating, and over poor roads for much of the distance; while the Lake St. Clair line was over good level roads.

TABLE No. 23.—Data on Rapidity of Work.

Item.	St. Lawrence River levels.	Lake St. Clair levels.
Total number of days in the field	185	27
Number of Sundays	20	5
Number of days lost by rain, etc	9.5	0
Number of days actually worked	105.5	22
Number of hours actually worked	746	159
Number of kilometers of single line run	433.55	98.05
Number of kilometers single line per day in field	3.21	3.63
Number of kilometers single line per day worked	4.11	4.46
Number of kilometers single line per hour worked	0.58	0.62
Average time consumed per setting	10.10 min.	9.81 min.
Number of instrument settings, entire work	4 430	972
Average number of instrument settings per kilometer	10.6	9.9
Average length of stretch, in meters	815.7	1 014.4
Average length of shot, in meters	49.0	50.4
Actual closure at end of line, residual from mean	± 1.07 mm.	±2.04 mm.
Residuals passed through zero, times	14	0
Maximum value of residuals, from the mean	±14.72 mm.	±5.44 mm.

Cost of Work.

The reliable data bearing on this subject are very meager, and must always be used with reserve.

Nearly everyone, intrusted with work of this class, will take pride in producing it as cheaply as possible, yet all high-grade work demands proper pecuniary recognition, and as the cost of leveling is almost entirely a question of salaries, it is readily seen that a wide range is possible, even for the same quality and rate of progress of work.

The factors of weather, season of year, nature of ground covered,

* Final Report of U. S. Board of Engineers on Deep Waterways.

etc., all enter, as for rate of progress, though the governing item is liberality in the wages paid.

The average salaries which should be paid to members of a party doing high-class precise level work are about as follows:

Chief of party or observer,	$200	per month	and subsistence	in field.
Recorder	100	"	"	"
Rodmen	60	"	"	"
Umbrellaman	30	"	"	"

This schedule is intended only for the highest grade of precise levels and for experts in that line. Subsistence in the field, when a party subsists on the country, will range from 70 cents to $1.00 per day per man.

Table No. 24 gives the cost of work done by the writer during 1898 and 1899, and may be accepted as a very reasonable expenditure when the quality of the work is considered. The traveling expenses of the party, to and from the field, are excluded, while salaries, subsistence, reduction of notes and traveling expenses incidental to the work are all included.

TABLE No. 24.

Item.	Line of Levels Along	
	St. Lawrence River.	Lake St. Clair.
Establishment of permanent bench-marks, each	$4.51	.....
Cost of leveling 1 km., single line	5.43	$4.61*
Cost of leveling 1 km., direct and reverse	10.86	9.22*
Cost of compiling and reducing notes per single kilometer	0.83	0.84

A good liberal value for estimating total cost of precise level work (exclusive of transportation to and from the field) is about $15 per kilometer, or $24 per mile, of double line.

* Note (added by author after reading discussions).—Including establishment of bench-marks.

APPENDIX.

A diagram, Fig. 17, for taking off the allowable error on precise level work, and Table No. 25, for the computation of probable error, are appended. These will serve a good purpose in the field as also in the final compilation of results.

The writer takes this opportunity of expressing his sincere thanks to the U. S. Board of Engineers on Deep Waterways; C. W. Raymond, M. Am. Soc. C. E., Col., Corps of Engineers, U. S. Army; Alfred Noble and George Y. Wisner, Members, Am. Soc. C. E. and members of that Board, and to Colonel G. J. Lydecker, Corps of Engineers, U. S. Army, in charge of the Detroit Office of U. S. River and Harbor Improvements, for granting their kind permission to use some of the data pertaining to the St. Lawrence River and Lake St. Clair Levels.

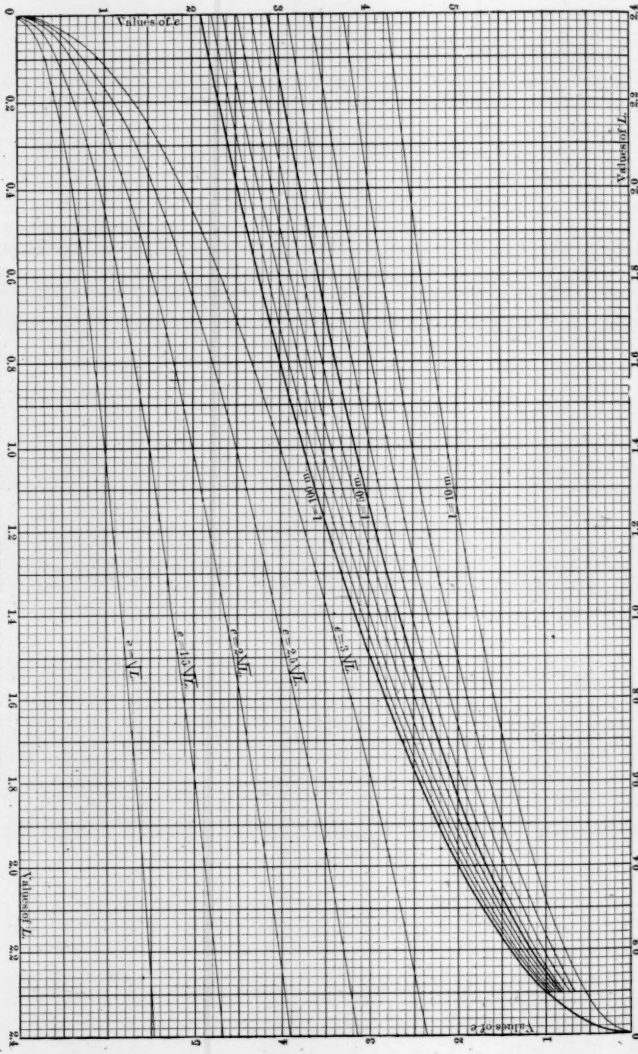
The writer is also indebted to Professor H. S. Pritchett, former Superintendent, U. S. Coast and Geodetic Survey, for photographs of instruments shown in Plates I and II.

TABLE No. 25.—PROBABLE ERRORS FOR TWO OBSERVATIONS.

Equation: $\pm r = 0.674\sqrt{\frac{\Sigma v^2}{m(m-1)}} = 0.674\ v$ for two observations;

r = probable error, v = residual from mean, m = number of observations.

v.	$\pm r$.	v.	$\pm r$.	v.	$\pm r$.	v.	$\pm r$.	v.	$\pm r$.
0.01	0.007	0.11	0.074	0.21	0.142	0.31	0.209	0.41	0.276
0.02	0.013	0.12	0.081	0.22	0.148	0.32	0.216	0.42	0.283
0.03	0.020	0.13	0.088	0.23	0.155	0.33	0.222	0.43	0.290
0.04	0.027	0.14	0.094	0.24	0.162	0.34	0.229	0.44	0.296
0.05	0.034	0.15	0.101	0.25	0.168	0.35	0.236	0.45	0.303
0.06	0.040	0.16	0.108	0.26	0.175	0.36	0.243	0.46	0.310
0.07	0.047	0.17	0.115	0.27	0.182	0.37	0.249	0.47	0.317
0.08	0.054	0.18	0.121	0.28	0.189	0.38	0.256	0.48	0.323
0.09	0.061	0.19	0.128	0.29	0.195	0.39	0.263	0.49	0.330
0.10	0.067	0.20	0.134	0.30	0.202	0.40	0.270	0.50	0.337
0.51	0.344	0.61	0.411	0.71	0.478	0.81	0.546	0.91	0.613
0.52	0.350	0.62	0.418	0.72	0.485	0.82	0.553	0.92	0.620
0.53	0.357	0.63	0.425	0.73	0.492	0.83	0.559	0.93	0.627
0.54	0.364	0.64	0.431	0.74	0.499	0.84	0.566	0.94	0.634
0.55	0.371	0.65	0.438	0.75	0.505	0.85	0.573	0.95	0.640
0.56	0.377	0.66	0.445	0.76	0.512	0.86	0.580	0.96	0.647
0.57	0.384	0.67	0.452	0.77	0.519	0.87	0.586	0.97	0.654
0.58	0.391	0.68	0.458	0.78	0.526	0.88	0.593	0.98	0.660
0.59	0.398	0.69	0.465	0.79	0.532	0.89	0.600	0.99	0.667
0.60	0.404	0.70	0.472	0.80	0.539	0.90	0.607	1.00	0.674



FORMULA: $e = C\sqrt{L\sqrt{l}}$ in which $C = \sqrt{\frac{vt}{m}}$

Above curves are drawn for $C = 1$.

L = Length of loopline in kilometers, l = average length of shot in meters, v = seconds, curvature of level per division of 2.0 millimeters
t = Smallest rod graduation in millimeters, m = magnifying power of telescope in diameters, e = error of closure of loop in millimeters.

DIAGRAM OF MAXIMUM ALLOWABLE ERRORS IN PRECISE LEVEL WORK.

FIG. 17.

DISCUSSION.

Mr. Hering. RUDOLPH HERING, M. Am. Soc. C. E.—It might be said, in connection with Mr. Wilson's discussion, that the European practice, which has very thoroughly covered this field of precise leveling, is inclined toward obtaining the greatest attainable precision and accuracy, but within practicable limits; which seems to correspond with Mr. Molitor's views. In important cases, it is certainly better to avoid errors in leveling as you go along, rather than to rely simply on the indications of your closure results, which may be due more or less to accident, from the possible fact that the closure error may be smaller than some intermediate errors. The speaker believes it to be proper, and in the line of progress, always to secure the greatest practicable precision warranted by the specific case.

Mr. Marshall. HORACE M. MARSHALL, M. Am. Soc. C. E. (by letter).—The treatment of the subject in this paper is exhaustive, but would lead one to believe that the work is very difficult; in fact, it is stated distinctly that "very few men have made a success of precise leveling." In the first place, that statement applies probably only to this country, where the method is of recent application, and in the next, only a few men in this country have tried it.

Under the United States Engineer Office at Vicksburg, Miss., no less than nine men have been engaged, from time to time, on precise level work, and there has been but one instance of failure. That was due to the man getting "rattled" by becoming imbued with the idea that the work was especially "scientific." It is exactly this feature which needs to be combated in order that the method may become more generally adopted on private and corporate work.

When any one talks about running levels to hundredths of a millimeter with a self-reading rod, they excite derision, though they may be happy in the knowledge that no one can prove the results incorrect. It smacks too much of theory and too little of experience. There is absolutely nothing in precise leveling save an accurate instrument (with a sensitive bubble) and a thorough programme for making the readings, to get good results. By thorough programme is meant one which will balance or detect errors; and it is not necessary to balance out the same errors twice. Equal sights balance instrumental constants; eccentricities cannot be balanced. To repeat the sight with instrument inverted and reversed is simply to balance out the constants twice, so far as the instrument is concerned, and to delay the progress.

The record given in Table No. 21 shows that the progress, where the double readings were presumably taken by the author, was about 25 miles per month. The last line run under the Vicksburg Engineer Office was at the rate of 59.3 miles per month in the field, for a distance

Mr. Marshall. of 222 miles. The best day's run was almost exactly 10 miles of single line, and the line checked within the limit

$$5 \text{ mm.} \sqrt{\text{distance in kilometers}},$$

the distance being counted only in one direction. This limit does not very greatly exceed the 3 mm. $\sqrt{2 \text{ distance in kilometers}}$ mentioned in the paper. A recomputation of the notes in the office usually changes the final result less than 1 mm.

For practical purposes, the source of error matters little so long as the programme remains unchanged by later discoveries. Whether it be the settling of the instrument or the change due to alteration of the temperature cannot be detected by any peculiarity in the movement of the bubble, nor is there any good to come from defining refraction differently from the meaning accepted in physics.

The condition of the atmosphere being unstable, of course, change in refraction is continually going on, and, by the way, this change, which is most detrimental to good work, is usually most rapid near sunrise and sunset, the very time the author selects for best results.

With a stable condition of the atmosphere the absolute temperature makes no difference, except that cold weather is not conducive to comfort, and it is a fact that some of the best work done under the United States Engineer Office at Vicksburg, Miss., was done during the summer time. An attempt was made to run at night, but proved unsuccessful because of the difficulty of properly illuminating the rods and instruments. Almost any nightwork is distasteful to the men, and is seldom as rapid as day work.

The programme proposed by the author varies very little from accepted practice, and the reversal of the telescope and bubbles is hardly worth while. In fact, any increase in the time required to complete a set of observations permits an increase in error due to change in atmospheric conditions, and adds weakness, unless the gain in some other feature more than counterbalances. Of course, it is equivalent to repeating the observation, but it delays the work and is likely to trip the recorder. To move the instrument before the notes of observation have been checked is distinctly a bad feature.

The refinement of subdividing the rod is probably most generally detrimental, for it increases the blurring effect of any boiling of the air. Possibly these fine divisions on the rod forced the choice for best running to the time of day when the air is steady, even if refraction is changing most rapidly.

To prescribe a particular direction for running, according to the time of day, having reference to the light, pre-supposes a constant direction of the line, when in fact the line is seldom straight. The convention in regard to signs for difference of elevation by lines in opposite directions seems hardly well taken. If it is up in one direction, it is down in the other, and the signs should show actualities.

Mr. Marshall.

Table No. 21 purports to show the accuracy of the "Principal Precise Level Work in the United States," and gives results on 2 048 km. by the Coast Survey and 4463.5 km. by various United States Engineer Offices, but it seems a little curious that it leaves out that office which has done more precise level work than any other and about seven-ninths as much as all other engineer offices combined.

The same thing has occurred before, when an assistant engineer published a list in the annual report of the Chief of Engineers of the United States Army. The Engineer Officer in Charge made the *amende honorable*, however, when his attention was called to it.

Perhaps the Vicksburg office, formerly under Major J. H. Willard, and now under Major Thomas L. Casey, does not do "principal" precise level work, but it is, nevertheless, the only Engineer District which is fully covered by a system where all the polygons are closed. There are seven polygons, the perimeters of which vary from 20 to 294 km., with errors of closure from 0.7 to 101.6 mm. Taking into account the closures on lines run by the United States Coast Survey, there are six other polygons with errors from 4.1 to 146.8 mm.

The size of the polygons, the actual error of closure and the error in the closure, in millimeters per kilometer, are shown in Table No. 26.

TABLE No. 26.

Polygons, Nos.	Perimeters, in kilometers.	Closure error, in millimeters.	Error, in millimeters per kilometer.	Remarks.
1.....	20	+ 0.7	0.035	
2.....	32	— 1.5	0.047	
3.....	165	— 14.8	0.079	
4.....	259	— 27.7	0.108	
5.....	528	— 46.0	0.089	
6.....	629	+101.6	0.161	
7.....	694	+ 98.9	0.142	
8.....	69	+ 4.1	0.060	Closed on Coast Survey.
9.....	303	— 93.5	0.308	" " "
10.....	343	— 26.6	0.077	" " "
11.....	471	+146.8	0.311	" " "
12.....	536	+ 45.2	0.084	" " "
13.....	680	— 84.1	0.124	" " "

It will be noticed that the greatest actual error of closure per kilometer is less than the least probable error per kilometer in the work cited in Table No. 21.

Of the allowable error, or the limit of error permitted, a good deal might be said, but it is, after all, simply an approximate test of the accuracy of the work, and only serves as a warning to show that the conditions will not permit good results under the usual programme when it is exceeded. The proper form would be an equation of the second degree, having the distance in one term to the first power and in another term to the second power. Probably the present form was

Mr. Marshall. worked out empirically from practice where the use of the level has got beyond the experimental stage.

The probable value of two results is the mean, and the error probably does not exceed half the difference between the two results; that is, if the difference is about the usual size for that character of work. To compute the probable error by least squares from two observations is foolishness; and as the observations on different lines, and even on the same line, are not made under similar conditions, *i. e.*, they cannot be considered as repetitions of the same observation, the probable error by least squares cannot properly be used even for comparison.

As said, the paper goes to the full limit in the matter of precise levels, but it is to be hoped that members of the Society will understand that the work is easy in practice. Almost perfect lines of levels can be run nearly as cheaply as ordinary levels are now run, and, if the mistake of cutting down the limit too small be not made, the work can be done more rapidly. All lines of levels on permanent works would be better if run by the precise level; lines on all water courses should certainly be.

The American instrument makers turn out a more finished instrument than foreign makers usually do, but the Kern level is of a good, substantial make, which has stood the test of time for wear and good work.

Mr. Kastl. ALEXANDER E. KASTL, M. Am. Soc. C. E. (by letter).—The writer, having been engaged on precise spirit leveling, under the Corps of Engineers, U. S. Army, both as recorder and observer, has read with much interest this valuable paper.

Referring to Table No. 21, the writer wishes to call attention to a line of precise levels, not given in the table, run in connection with the Red River Survey, in 1889, 1891 and 1892.* The line runs from Delta, La. (opposite Vicksburg, Miss.), on the Mississippi River, to Shreveport, La., on the Red River; thence along the Red, Atchafalaya and Mississippi Rivers to Smithland, La., on the Mississippi River. The length of this line is 654 km., or 406 miles. In 1880 and 1881, the U. S. Coast and Geodetic Survey had run a line of precise levels along the Mississippi River from Greenville, Miss., to New Orleans, La , which includes the Mississippi River from Delta, La., to Smithland, La. The length of the Mississippi River line from Delta to Smithland is 248.4 km., or 154 miles. The Red River Survey levels started from the U. S. Coast and Geodetic Survey Bench-mark No. 215 at Delta, with its elevation of †24.084 m. above Cairo Datum, and closed on the U. S. Coast and Geodetic Survey Bench-mark No. XLV at Smithland, with an elevation of 21.005 m. above Caro Datum. The elevation of Bench-mark No. XLV, by the U. S.

* Annual Report of the Chief of Engineers, U. S. Army, Report of Captain J. H. Willard, Corps of Engineers, for 1893, page 1944 *et seq.*

Coast and Geodetic Survey is *20.983 m.; that is, the error of closure of the entire polygon or loop, Smithland, Delta, Shreveport and back to Smithland, is 0.022 m., being 22 mm., or less than $\frac{1}{8}$ in. The total length of the polygon is 902.4 km., or 560 miles. Mr. Kastl.

The writer does not know the probable error per kilometer

$$\left(= \pm \sqrt{\frac{\Sigma r^2}{\Sigma L}}\right)$$

for the entire line, as on the Red River Survey it was not deemed of sufficient importance to take the time to make the computations. The writer ran the last stretch of the Red River precise levels from Grand Bend, La., on the Red River, to Smithland, La., on the Mississippi River, a distance of 126.2 km., or 79 miles. For his own information he computed the values of r and r° for his line of levels and found the probable error for the 126.2 km. to be $\pm$ 12.3 mm., or $\pm$ 1.1 mm. per kilometer. The writer's method of computing the probable error was the same as that given by the author on pages 108 and 109.

In running the Red River precise levels care was taken to keep the instrument closely adjusted and to keep the back sights and fore sights equal. The instrument was shaded with an umbrella, and no work was done from about 11 A. M. to 2 P. M., unless the day was cloudy. The length of sights was usually not over 100 m., except at river crossings, where the method of reciprocal levelings was used. The rod readings were taken with the level bubble in the center, telescope normal and level direct. Each stretch of levels was run in duplicate in opposite directions by the same observer, and during the same forenoon or afternoon. The level and collimation errors, although small, were observed and recorded at least at the beginning and close of each day's work and sometimes of each half-day's work. The correction for inequality of the pivots and the value of one division of the level for a distance of 1 m. were determined at the beginning and close of the season's work. However, as the instrument was kept closely adjusted, and the back and fore sights kept equal, the corrections for the inequality of the pivots and the level and collimation errors were rarely used. Is is a good plan to keep a record of the adjustments. The allowable limit of error for each stretch was 5 mm. $\sqrt{}$ distance in kilometers, the distance being the length of the stretch from one bench-mark to the next, and not the length of the loop leveled forward and back. The instruments used were precise levels and rods, manufactured by Kern of Aarau, Switzerland, and had been used on the U. S. Lake Survey and on the Mississippi River Survey.

It would be very interesting if the author would give comparisons between the lines of precise levels along the St. Clair and Detroit

*Report of the Mississippi River Commission for 1884; or, Annual Report of the Chief of Engineers for 1885, pages 2676 and 2678.

Mr. Kastl. Rivers as run under the U. S. Lake Survey * in 1877, and as run under the U. S. Board of Engineers on Deep Waterways in 1898 and 1899. The U. S. Lake Survey precise levels were about the first of the kind which were run in the United States, and a comparison of results between the first and latest methods of doing this class of work would be worthy of record.

Mr. Wilson. HERBERT M. WILSON, M. Am. Soc. C. E. (by letter).—This exhaustive paper on precise spirit leveling covers thoroughly the thecretic conditions involved in the practice of this class of surveying. It is especially pleasing to the writer and should be to this Society, because it substantiates, theoretically, the principal conclusions brought out by the discussion of the paper entitled, "Spirit Leveling of the United States Geological Survey."† Had Mr. Molitor examined this paper, he would not have stated that he is unable to give examples of speed or cost. Many such are to be readily obtained from a study of the reports of the United States Engineers, the United States Coast and Geodetic Survey and the United States Geological Survey, and a few are published in the paper above referred to.

The writer is glad to see that in this paper the title "precise spirit leveling" is used, thus bringing out clearly the fact that it is spirit leveling by which the most precise work is done, and not that variety of leveling wherein numerous instrument constants and corrections for "errors" are applied, to the detriment of the results.

Mr. Molitor points out very clearly that "the most perfect is not necessarily the most complicated instrument. On the contrary, simplicity is a prime necessity in the usefulness of a level." Elsewhere, he repeats, in order to point out clearly that in his opinion the best results are to be obtained by the simplest methods of running and of instrument and by the avoidance of those methods, practiced in the past, which require the application of corrections and constants to the instrument and its results before the latter are available for use. Yet, these admirable conclusions have been hidden by him under a mass of theoretic discussion of errors and instrument constants and their correction, which are so voluminously stated as to befog the conclusions to be drawn. In point of fact, scarcely any of the constant corrections are to be applied to the methods preferred by Mr. Molitor, and few of the errors are not shown by him to be eliminated by the methods of observation practiced.

Level.—The writer partially agrees with Mr. Molitor that the level made by Messrs. Buff and Berger for Professor Mendenhall is one of the best precise leveling instruments ever made. Yet he speaks slightingly of the quality of the level made by the same firm for the Geological Survey, when, in point of fact, the differences between the two are

* Professional Papers, Corps of Engineers, U. S. Army, 1882, No. 24, p. 599 *et seq.*

† *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 339.

almost wholly in minor points of construction. They differ prominently in but one feature—the omission of the attached bubble from the Mendenhall instrument and the use of the striding level alone on that instrument. The writer is inclined to agree with Mr. Molitor and others that the striding level is theoretically, and possibly practically, slightly preferable to a well-made attached level. The writer will show, farther on, however, that it is doubtful if any better results in precise leveling have ever been obtained than those secured by the Buff and Berger level (Fig. 2, Plate III), and it is not proven yet that the slightly more complicated, improved Buff and Berger is superior. A comparison made from Table No. 1 shows that in most of the essentials these two levels are superior to the other instruments described. After a conference on the subject of a new level for the Geological Survey, the maker of these two instruments was inclined to agree with the writer that it was doubtful whether the changes made in the Mendenhall level had not affected some of its simplest and most admirable qualities as a purely precise spirit level. Mr. Molitor makes a slight mistake in stating that on the Geological Survey instrument 6- and 8-second bubbles are used. In point of fact, 4- and 8-second bubbles are used, though the latter only under unusual conditions; the 4-second bubble being that which is generally used. The new Coast Survey instrument should be of superior make because of its reputed great simplicity. Mr. Wilson.

Rods.—A very shallow coating of paraffine is preferable to any other treatment, as it will keep out dampness, etc., more effectually than painting, and yet does not affect a sufficient amount of the cross-section of the rod to have an appreciable influence on its expansion. On this point, however, there is much to be learned, and rods either treated as above, or oiled and kept freshly coated with shellac, or painted, as suggested by Mr. Molitor, are probably equally satisfactory.

The conclusions to be drawn from a series of careful experimental levels run and re-run last year by the Geological Survey between Elmira and Corning, New York, with both target and self-reading rods of most approved patterns, indicated that there was little difference in the quality of the results obtained by either. In the matter of speed and relative cost there was a decided advantage shown in favor of the target rod, and these are points which engineers thoroughly appreciate. Self-reading rods are slower than target rods, because on even moderate grades the length of the sights must be materially reduced, since only a relatively small portion of the rod can be seen between the extreme of the three wires. Moreover, since it is desirable in such work not to observe nearer the ground than 1½ to 2 ft., because of refraction, it is apparent that only a very short rod is available, and the lengths of sights and rate of speed are correspondingly

Mr. Wilson. reduced. Finally, the use of self-reading rods retards the work because of the time required to read and record all these wires.

Turning Points and Bench-Marks.— The use of pins for turning points, in preference to plates, has, as stated by Mr. Molitor, received the approval of nearly all precise levelers. The writer, however, disapproves of the hole sunk in the foot of the turning pin into which a projection on the foot of the rod is placed, because of the danger of dirt getting into such a cavity and being carelessly left there, thus vitiating the results. He also disagrees with Mr. Molitor's recommendation for the concealment of bench-marks. A very wide experience with these, in every part of the United States, and in the setting of thousands during the past few years by levelers for the United States Geological Survey indicates that publicity is better, and this is in accordance with the best European practice. The threat of fine, etc., printed on the Geological Survey bench-marks has its effect in deterring many from disturbing them, though in point of fact, as stated by Mr. Molitor, the conviction of a transgressor would be a very difficult matter. However, bench-mark tablets of large size attractively labeled and placed in prominent positions in public buildings or grounds are decidedly more immune from injury than those which are smaller and concealed—this, chiefly for the reason that people living in the vicinity are proud of and become interested in the public and official mark showing the height of their locality, and would interrupt any vandal who would attempt to injure the same. Moreover, its very publicity makes it difficult for any one to disturb it without detection.

Simplicity of Precise Leveling.—It is feared that the imposing array of mathematical formulas and the statements as to the very high grade of personal attainments necessary in precise levelers will deter civil engineers who have good spirit leveling to do from attempting work of this character. To counteract this effect the writer insists that precise spirit leveling is not at all a complicated operation. It differs little from ordinary spirit leveling except in the quality of instruments and rods used and in the precautions taken in their use. At little expense, over that incurred in the more crude methods of spirit leveling ordinarily used on engineering work, leveling of the highest order is possible. Nor are scientific attainments of a very high order requisite in the leveler. Such qualifications as are to be found in well-educated instrumentmen of a few years' experience with transit or level will generally suffice. Although for the best results only the best instruments should for safety be used, almost equally good results are being obtained daily over distances of less than 100 miles, such as engineers will ordinarily have to operate in, with nearly any spirit leveling instrument of good make. It is not necessary to point out to engineers that the ultimate test of the quality of such work is not in theoretic

discussions of minute errors. These are of value in pointing out the methods to be used and the cure for the errors; but the test of the result is in the error shown in starting from a given point, running 50, or 100, or 1 000 miles, and closing back on that point. No better results of this kind have ever been obtained than those procured under the direction of Mr. William H. Brown, Chief Engineer of the Pennsylvania Railroad, in running precise levels with a first-class Heller and Brightly instrument, or by the United States Geological Survey with an instru- Mr. Wilson.

FIG. 18.

ment of which Mr. Molitor says: "It is not really a precise level, but merely a good common level." Yet with these instruments the very best attainable closures have been invariably procured.

Unfortunately, the writer is not at this time able to quote the closure errors exactly, because the leveling parties are still at work in the field, or because his present duties render them inaccessible; however, in a line 1 043 miles in length, from the sea coast in North Carolina through Asheville and Knoxville to Atlanta, and back to the sea coast

Mr. Wilson. at Brunswick, Ga., including the errors of tidal observations at two points on the coast, the closure error was 1.057 ft., a result within permissible limits, when it is remembered that the total length of the duplicate line is twice 1 043, or 2 086, miles, and includes errors of two tide gauges. Equally good results have been had in closing a circuit of 780 miles, half of which consists of the United States Engineers' line from Albany to Oswego and water levels on Lakes Ontario and Erie to Dunkirk, and the other half of Geological Survey levels from Dunkirk *via* Binghamton to Albany, the closure error being 0.645 ft.

This year, the Geological Survey "common levels" have closed, within 0.246 ft., a line run from the old United States Engineers' bench-mark at Utica north to Mr. Molitor's own line for the Deep Waterways Commission at Fort Covington. At Pittsburg the Geological Survey levels checked on a bench-mark of the Pennsylvania Railroad "common levels" and with a line brought by the Geological Survey from the Coast Survey bench-mark at Grafton, W. Va., to Pittsburg. The extreme range of closure was 0.368 ft. in a distance of 1606 miles. The elevation brought by the Pennsylvania Railroad direct from Sandy Hook is 738.491 ft. That brought by the Coast Survey from Sandy Hook to Grafton, and by the Geological Survey thence to Pittsburg, is 738.468 ft. The discrepancy between the elevation brought by the Geological Survey from Lake Erie, dependent on the old United States Engineers' levels from Albany, and the Pennsylvania Railroad, in 1 227 miles, was 0.215 ft., accepting the present height of Albany, or about 0.4 if Albany be corrected. At Harrisburg the closure error between the Pennsylvania Railroad, the Geological Survey and the Coast Survey was 0.079 ft. in 376 miles.

The line just closed by the Geological Survey on a Coast Survey bench-mark at Poughkeepsie, from a Geological Survey bench-mark at Binghamton, shows an error of 0.607 ft. This includes a circuit from Poughkeepsie by the Coast Survey to Albany, thence a mean of United States Engineers' and Geological Survey levels *via* Dunkirk to Binghamton. This closure goes to show that the revised Grist-mill bench-mark at Albany, characterized in the writer's paper of 1898,* is still in error by about half a foot, since the application of the closure error just obtained to results had through Harrisburg and through Pittsburg improves the closure at those and at other points made by lines brought from Albany and from other directions. Thus applying it, it makes a line to Harrisburg from Elmira and Albany close with the Pennsylvania Railroad 0.044 ft. low.

The writer desires to emphasize Mr. Molitor's statements about heat effects on the instrument, as he fully agrees with him that practically all the errors which cannot be eliminated in the instrument and in the running are those due to changes of temperature. Therefore,

* *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 339.

to do careful work of this kind, every effort must be made to minimize heat effects, and one of the essentials is that the instrument shall be at all times shaded from the rays of the sun. Even in ordinary leveling the bubble can be so much more quickly brought to center when the instrument is shaded, and the results will be so much more trustworthy, that the cost of an umbrellaman will practically be saved in a short time. Mr. Wilson.

The members of this Society will be interested to know that, as a result of the writer's paper on leveling,* he has the personal statement of the Superintendent of the United States Coast Survey, Professor H. S. Pritchett, that the Coast Survey is indebted to that article and its discussion for its clear proof of the necessity of a revision in their methods of precise leveling. That Bureau discarded in 1899 the instruments and methods theretofore used, and is now using a new instrument made by it which is similar to but even simpler than the Kern and the Buff and Berger levels. The chief point brought out, as a result of the conference of experts which recommended the changes, was that the largest source of error was due to temperature and the expansion of metal in the instruments; and that, therefore, any attempt to introduce instrumental constants, as had been done in the past, was erroneous because of the uncertainty of the coefficient of expansion and its effects on the parts of the instrument. This alone is a striking argument for the use of the simplest instrument and the simplest methods, without the application of corrections for so-called instrument constants.

The members of this Society will be interested to know that now nearly the entire eastern half of the United States is thoroughly gridironed by lines of precise levels. It is scarcely necessary to go more than 100 miles in any direction to get a bench-mark which is connected directly with sea level by precise levels. For the benefit of those who are interested in the use of such results there is published herewith an outline map, Fig. 18, showing the distribution of such level lines. In many places there are adjusted between them other lines of levels derived from numerous reliable sources. The completeness of the precise level net in New York and Pennsylvania attests the progress being made in the co-operative topographic surveys of those states. Descriptions and elevations of all precise bench-marks can be procured by addressing the United States Geological Survey or the United States Coast and Geodetic Survey Offices.

E. G. Fischer,† Esq. (by letter).—The writer, being an instrument maker, was particularly impressed by those portions of this interesting paper which treat of the form and material of the various precise levels and level rods. Having had charge of the construction of the instrument described therein under "*d*," page 10, also of the design Mr. Fischer.

* *Transactions*, Am. Soc. C. E., Vol. xxxix. p. 339.
† Chief of the Instrument Division, Coast and Geodetic Survey.

Mr. Fischer. for a reconstruction of that instrument, and the design and construction of a new one which has been in use during the past season, the writer feels that he may add to the interest in the subject of precise leveling by a description of an instrument and rod designed and constructed since any of those described by the author. A few remarks are added in regard to the instrument selected by the author as "the most perfect precise level yet produced."

It should be said here that the difficulty between the engineer and the "peculiarly obstinate" instrument maker did not prevail in the planning of what may be called the Coast and Geodetic Survey 1900 Precise Level. The engineer designated the method and the principal features required to satisfy it, and the instrument maker made the design which was approved by the engineer.

The method of observation adopted in the Coast and Geodetic Survey obviates the use of a revolving telescope and a reversible stride level. As will be seen in Figs. 1 and 2, Plate IV, the advantages arising from this fact have been utilized by mounting the level directly upon the telescope. By making the support for the telescope cylindrical, it was not only given the strongest and lightest form, but it could be made to serve at the same time as a protection to the level. The whole thus offers perhaps a minimum of surface to wind pressure, and is less readily affected by temperature changes.

Considering the minuteness of the quantities producing what are called cumulative errors, so minute that a most sensitive level is required to indicate them, the aims in designing the new precise level were to select the material with a view to the smallness of its expansion coefficient, to protect the vital parts against sudden and unequal changes of temperature, to reduce to the smallest possible dimension the linear distance between level vial and line of collimation, to ensure stability by reducing the distance between the center of gravity and the plane of support, and to enable the observer to obtain the rod reading, as nearly as possible, simultaneously with the setting of the level.

The material selected was the same as that which had been used in the reconstruction of the older instruments of the Coast and Geodetic Survey in the spring of 1899. The nickel-steel alloys, brought out by Professor Guillaume of the International Bureau of Standard Weights and Measures, had attracted considerable attention by reason of their low expansion coefficients (down to 0.000001 per degree Centigrade). Inquiry established the fact that tubing and castings, almost indispensable in the construction of instruments, could not be obtained, because attempts to produce them had not been successful. The writer, therefore, secured the co-operation of a brass founder, and after a number of trials succeeded in obtaining an alloy with a coefficient of 0.000004, which could readily be cast into suitable shapes. It is composed of 66

Mr. Fischer.

Fig. 19.

U.S.C. & G.S. 1900 PRECISE LEVEL.

LONGITUDINAL SECTION

Fig. 20.

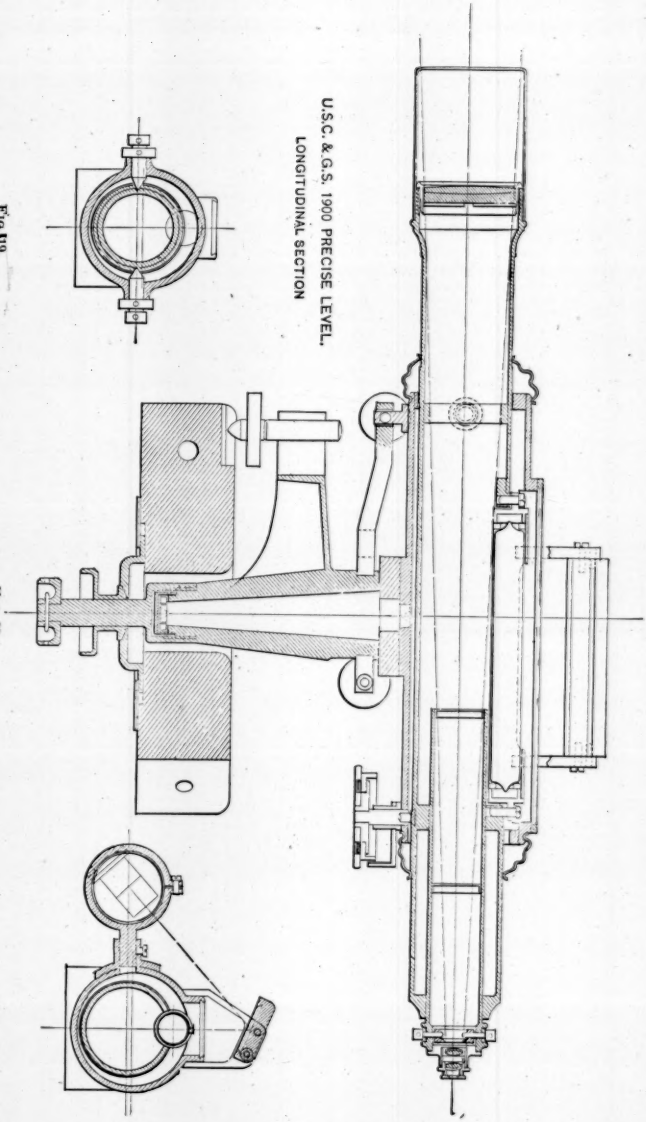


Fig. 21.

Mr. Fischer. parts of a soft grade of cast iron and 34 parts of grain nickel. This is a metal almost proof against corrosion, though not better, as far as strength is concerned, than cast iron. A peculiar property is the readiness with which it takes polish and the smoothness with which it wears against itself even under considerable pressure. The nickel-iron draw tubes of the three reconstructed instruments, though moving in bearings of the same metal, do not show the slightest wear or looseness, though they have been used in running 200, 300 and 600 miles of double lines of leveling, respectively. This alloy, having proven its worth by a considerable improvement in the results obtained during the season of 1899, was also used in the construction of the 1900 levels of the United States Coast and Geodetic Survey, Nos. 7 and 8. The telescopes were cast with objective head and draw-tube bearings complete, with the aid of cores, as were also the tubes surrounding the telescopes, while the draw tubes and level tubes were cast solid and bored out. The screws which pivot the telescope, adjust the position of the level vial and the disc carrying the spider threads, and the leveling screw, are made of nickel-steel with a co-efficient of 0.000001, obtained from the firm* producing Professor Guillaume's nickel-steel alloys. As will be seen from Fig. 20, the telescope tube is cut open to admit of the level tube being placed as near to its axis as the cone of rays formed by the apertures of the objective and reticule will permit. The outer cylinder supports the telescope by means of the pivot screws shown in Fig. 19, the points of which enter at a place reinforced by a ring forming also a diaphragm; and the micrometer screw, the end of which carries a glass-hard steel tip upon which rests the telescope, which is provided with a small plate of glass-hard steel at the point of contact. A white celluloid head, graduated to 100 parts, and an index, serve to obtain quickly the horizon setting when placing the instrument. The pitch of the screw being about 39 threads per millimeter (100 per inch), gives a value of about 2.6 seconds per division.

The telescope has a vertical motion of about 2 mm. above and below the horizontal position. Just back of the micrometer screw is fastened to the outer tube a small eccentric with a lever handle, by means of which the telescope can be lifted off the screw and pressed against a spring above, to prevent jarring and disturbing the level adjustment while carrying the instrument from station to station. This device is not shown in the diagram, but can be seen on one of the photographs. The necessary openings between the telescope and the ends of the outer tube are closed by leather cones which effectually shut out dust and air currents, without in the slightest degree influencing the freedom of the telescope to assume the position determined by pivots and micrometer screw. The level tube holds the vial by six points of metallic

* The Société Anonyme de Commentry-Fourchambault, 16 Place Vendome, Paris.

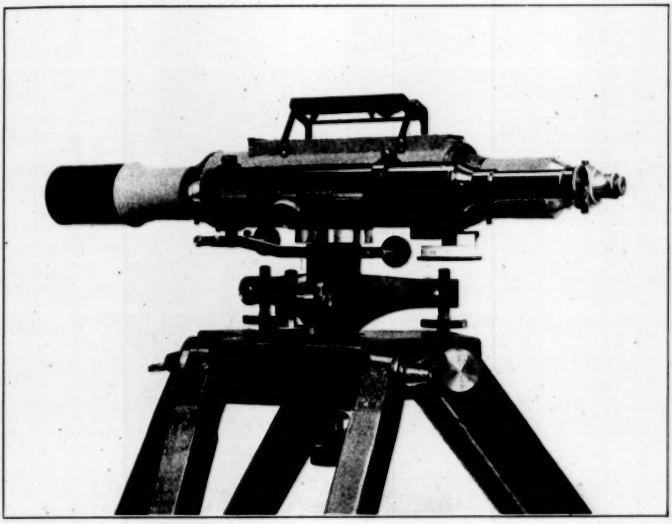


FIG. 1.—THE COAST AND GEODETIC SURVEY LEVEL OF 1900.

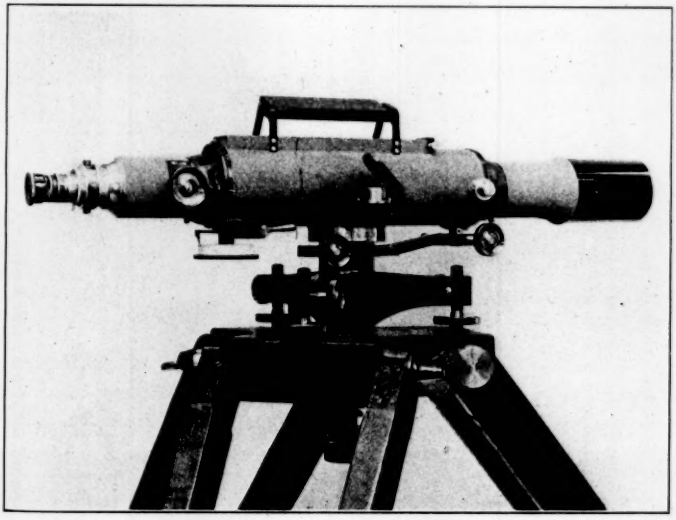


FIG. 2.—THE COAST AND GEODETIC SURVEY LEVEL OF 1900.

contact, three at each end, 120° apart, the two lower ones being formed by the ends of screws piercing the tube, the upper one by a metal tip at the end of a flat spring fastened to the tube itself. The vial is confined longitudinally by cork rings, which, however, leave a clearance of about $\frac{1}{2}$ mm. The level tube is fastened to the telescope at the end nearest to the ocular by means of a square-headed vertical adjusting screw of about 27 threads per centimeter, working against two very strong helical steel springs, and moved by a socket wrench with long lever arms, enabling the observer to make very delicate adjustments with ease. A turn or two of a small milled-head screw permits the glass cover over the oblong opening in the outer tube to be slipped back sufficiently for the admittance of the wrench. At the other end the tube is held by a clamping screw and two opposing screws for adjusting the level vial laterally. Mr. Fischer.

Two slightly enlarged portions of the telescope tube, which are indicated in Fig. 20, but hidden under the leather cones in the photographic views, are turned to equal diameters, and as nearly as possible co-axial with the objective head and draw-tube bearings. By means of these collars, and suitable shop contrivances, the intersection of the vertical spider thread and middle horizontal thread is adjusted to fall in the geometric axis of the telescope, thus avoiding errors due to divergence between the line of sight and the line of motion of the draw tube. The level vial is also placed parallel to the line of sight in the shop, before the instrument is finally put together. The objective has a focal length of 41 cm. and a clear aperture of 4.2 cm. Two Steinheil eyepieces, of 9.5 mm. ($\frac{3}{8}$ in.) and 12.5 mm. ($\frac{1}{2}$ in.) equivalent focus, are provided, having magnifying powers of 43 and 32 diameters, respectively. The system of spider threads consists of one vertical and three horizontal lines. The upper and lower horizontal threads were placed to embrace a space of 30 cm. at a distance of 100 m., as requested by the engineer.

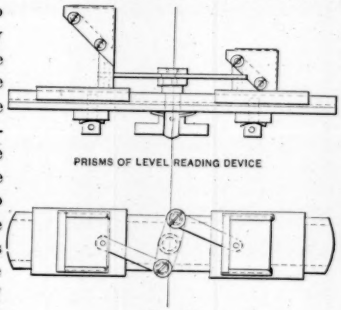


FIG. 22.

The outer tube and the telescope and level tube are covered with a substantial coating of cloth dust, to protect them against sudden changes of temperature.

The level-reading device (Figs. 21 and 22), is a modification of that made by Berthelemy (Fig. 2, Plate II), and was first applied to the three Coast and Geodetic Survey instruments when remodeled in the

Mr. Fischer. early part of 1899 (Figs. 1 and 2, Plate V). The same device was adopted for the new instruments where it could be applied in a more compact form. The difference between the French device and its modification is readily apparent from the illustrations. The large and weighty mechanism has been reduced and placed near the base of the instrument, and, what is considered the most important change, the eye tube has been placed at binocular distance from the eyepiece of the telescope, enabling the observer to control both rod and level bubble as nearly simultaneously as his ability to change mental attention from one image to the other will permit. The distance between the axes of the telescope and level-reading tube can be adjusted to suit the individual observer. The images of the ends of the bubble and the scale divisions in their vicinity are reflected by the mirror above the opening of the outer tube into two prisms and are by them reflected, in a direction parallel to the telescope, into the observer's eye. The prisms have curved surfaces, and, together with a lens mounted between them and the eye end, reduce the distances between the ends of the bubble and the eye to the normal distance of distinct vision. The cap forming the eye end is arranged to hold such a lens as an individual observer may need, to adapt the normal distance to an abnormal eye. The means of keeping the prisms adjusted symmetrically to the ends of the bubble are illustrated in Fig. 22. They are much simpler than those in the French instrument, in which racks and pinions are used.

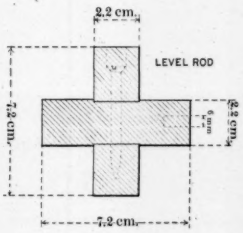


FIG. 23.

The level vials are graduated to 2-mm. spaces, and have values of somewhat less than 2 seconds per space. A small universal level with 45° reflector aids in quickly placing the instrument in operating position.

The centers are of the single-cone type, and are made of the hardest and toughest steel (Sanderson's Tool Steel No. 6), and fit closely into the tripod socket, which is made of a hard and fine-grained cast iron, furnished by the Brown and Sharpe Manufacturing Company. The distance from the line of collimation to the plane of support was reduced from 17.7 cm. in the old instrument to 10.9 cm. in the new, while the length of the center was increased from 7.3 to 10.0 cm. The radial distance of the foot screw is 9.0 cm.

The device for clamping the instrument to the tripod head for carrying from station to station is arranged so that the former cannot rest upon the latter unless the large clamp screw under the tripod head is screwed home, that is, the observer when mounting the instrument upon the tripod for the day's work, cannot forget to secure it so

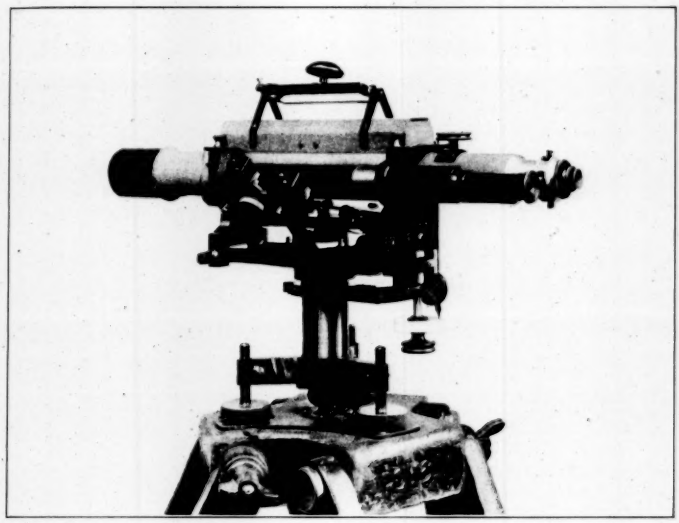


FIG. 1.—THE COAST AND GEODETIC SURVEY LEVEL OF 1899.

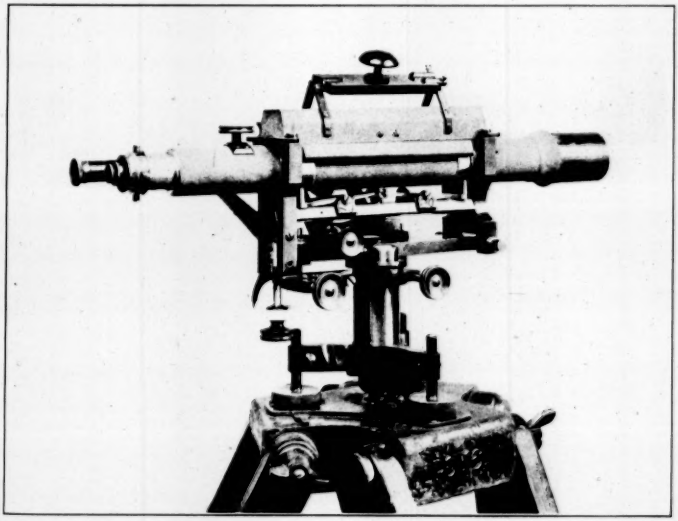


FIG. 2.—THE COAST AND GEODETIC SURVEY LEVEL OF 1899.

that the whole may be picked up and carried without accident. The weight of the instrument is 5.2 and that of the stand 7.2 kgr. Mr. Fischer.

The writer feels seriously the weakness of the described instrument due to the loose texture of the alloy used for the telescope, level, draw and outer tubes, which would cause the instrument to suffer materially in an accident that would but slightly disarrange an instrument constructed of wrought metal; but the love and esteem it has won for itself from the user during the past season aids, to some extent at least, in shielding it against mishaps.

With the aid of the several photographs, Figs. 1, 2 and 3, Plate VI, and Fig. 23, there will be no difficulty in forming a clear conception of the level rod used during the seasons 1899 and 1900 in the Coast and Geodetic Survey. As will be seen from Fig. 23, the cross-section is symmetrical with reference to a central vertical line. A broad strip, carrying the graduation on one edge, is provided with two side ribs, forming with it a cross. The center of the foot, made of the hardest bell metal, is placed in the plane of the graduated face of the rod and is spherical, with a radius of 2.6 cm. For control of the length of the rod, silver-faced plugs of 6 mm. diameter are inserted at the first decimeter above the foot, and at the 11, 21 and 31-dcm. marks above it, fine lines across them forming part of the graduation. Frequent measurements with the same steel tape during the season's work enable the observer to keep informed as to any changes taking place in the field and at what time they take place. The material from which these rods are made was treated with paraffine, but under considerably lower temperature than those mentioned by the author, so that the life of the wood, which is pine of a very light weight, is not appreciably impaired. The treatment was changed after the construction of the first pair of rods referred to in the paper, so that, although the paraffine was made to penetrate to the center of the strips of wood, all but about 19% was driven out again before allowing them to cool.

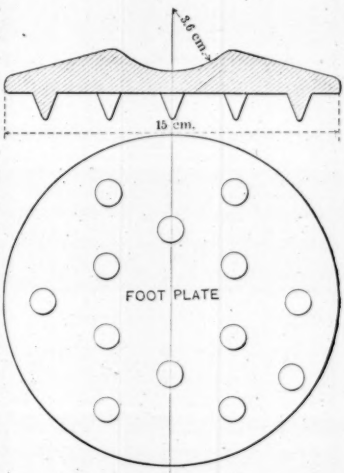


FIG. 24.

The author's statement that the coefficient of expansion of wood is

Mr. Fischer. increased by about 50%, according to determinations of the Coast and Geodetic Survey, must be based upon some mistake, since careful measurements gave a coefficient of 0.0000042, while that of untreated pine is usually given as 0.000004 (see page 51).

The handle, universal level and thermometer are shown in Fig. 3, Plate VI. The latter has a bent bulb which rests in the center of the cross-section and is surrounded by sawdust made of paraffined wood; the view shows the wooden protecting shield raised for reading the temperature. The weight of the rod, obtained from weighing two of them, is 4.7 kgr.

Fig. 24 is a plan and section of the foot-plate. It is made of cast iron and the spherical cavity is turned to a radius of 3.6 cm., which is but little greater than that of the foot of the rod.

While, after all that may be said for or against the form and material of an instrument, the final test lies in the results obtained with it; and while the results obtained with the one selected by the author as the best may be better or worse than those secured with the one here described and others described in his paper, a matter which is left for the engineer and computer to decide, the writer cannot help but ask what particular features in the Buff and Berger level, No. 2768 decided the author to select it from the rest of those he described. Its optical power is superior to that of any of the others; that of the Swiss level, though equal to it in diameters, is yet inferior by reason of its shorter focal length. The telescope is mounted nearer to the top of the vertical axis than in any of the other instruments, and the distance between level and line of collimation is apparently smaller than in any of the others except the Swiss level. These are points in its favor. But, on the other hand, would it not be a better instrument if it were not for the tall mirror and its supports, mounted where the wind must be most effective and forbid its use when another would be comparatively steady? Should not the level vial be protected from currents of air? The writer has observed, in an instrument mounted on a heavy stone pier, fluctuations of the level amounting to 1 to 2 divisions, in precise coincidence with puffs of wind too gentle to have caused the movements of the bubble by any other than temperature effects. Such air currents or puffs of wind would very likely attack also the upper half of the portion of the telescope shielded below by the cradle, and cause temporary warping of the line of collimation in a vertical direction which would not be controlled by the level.

The mounting of the micrometer screw at the end of a long horn extending out from the cradle support, to make contact with a similar one attached to the cradle, seems a large price to pay for the unimportant advantage of having the cradle swiveled in the center of the instrument. It admits of disturbances due to temperature effects and adds considerably to the already large area of surface exposed to wind

FIG. 1.

FIG. 2.

FIG. 3.

THE COAST AND GEODETIC SURVEY LEVEL ROD.

pressure. The use of brass for the telescope and other parts of the instrument, with its high temperature coefficient of 0.000018, and especially the use of steel collars on a brass telescope tube, seems to the writer contrary to good principles in designing an apparatus for such delicate uses as those of a precise level. In the case of astronomical transits, meridian circles and other instruments of the highest order, where the control of the pivot axes by a striding level is of the greatest importance, it has been held for many years that, in order to preserve the true figure and equality of the cylinders forming the pivots, they should be supported in material which will itself yield and wear rather than cause their deformation. This was not regarded in the design of the author's favorite level, in which the steel collars of the telescope rest upon agate points, which must certainly indent them after but little use. Why not have had the collars rest across the whole width of bearings made of softer material than hardened steel? The author's statement that the four contact points of the cradle wyes should not lie in the same circles as those of the striding level would not be necessary if grooves were not worn in the collars. In the writer's opinion, ideal conditions can be reached more nearly by providing line contacts with wyes of some material wearing more quickly than that of the collars; in a very short time, really before the instrument goes to the field, these lines are worn into narrow cylindrical surfaces; if the wyes of the stride level are fitted in the same manner its readings can be relied upon for indicating the direction of the line of sight and inequality of collars, provided, of course, that longitudinal motion is prevented. The writer cannot admit that mounting the level vial in its support by a strip of blotting paper or cork will secure the extreme stability in the make-up of a precise level. For the past twelve years all level vials, except the very smallest, of the instruments belonging to and made for the Coast and Geodetic Survey, have been mounted in the manner above described, that is, in metallic supports, free to follow temperature changes and free from effects due to swelling or shrinking of cork, paper, wood, etc. Mr. Fischer.

JOHN F. HAYFORD, Assoc. M. Am. Soc. C. E. (by letter).—The author's treatment of the subject of precise leveling has been read with interest, because it is evidently a serious attempt to solve a problem to which the writer has devoted much time during the last two years. The writer finds that he must dissent from several of the main points reached in the paper, as well as from many of the minor points. In expressing that dissent fully there is no intention of reflecting on the author's intellectual honesty, his ability, or his sincerity of purpose. The writer must express here his confidence in the high degree of accuracy attained in the two precise level lines run by the author. In the expressions of dissent in this discussion an attempt has been made to avoid making merely a statement of difference of Mr. Hayford.

Mr. Hayford.

opinion, thus leaving the reader to decide between opposing opinions without an adequate basis for his decision, and to state as fully as is possible in a discussion of readable length the points wherein the writer believes that the author's conclusions rest upon an insufficient basis, or wherein the foundation for an opposing opinion is more substantial.

The paper is very largely an advocacy of a certain instrument and a certain method of observation, together with a statement of the reasons for believing in such an instrument and method. It happens that another instrument and another method embody the views of the writer. It has, therefore, seemed that the best form for this discussion is to first describe the writer's favorite instrument and method, and to call especial attention to the most important points in which they differ from those of the author; to set forth in brief the considerations which led to the adoption of this particular instrument and method for use in the Coast and Geodetic Survey; then to call attention specifically to certain points in the paper; and to close, as the author has done, with a statement of the speed and cost of leveling.

Coast and Geodetic Survey Level of 1900.—As this instrument, together with the rods used with it, has already been fully described by Mr. E. G. Fischer, the designer of the instrument, it will suffice here to call attention anew to three main points.

1st. The instrument is irreversible, and as simple as possible. It will be seen later that this principle is coupled with the simplest possible programme of observation.

2d. Great care has been taken to prevent errors due to changes in the relative temperature of the different parts of the instrument.

3d. A device for reading the bubble has been supplied which enables the observer to stand erect at all times and see the bubble and the rod alternately and in quick succession, without moving the eyeballs, and without even refocusing the eye, the only change required being in fact a mere shifting of the attention from one eye to the other.

This instrument, put into use in 1900 in the Coast and Geodetic Survey, is radically different from the instrument used in the same organization previous to 1899, Fig. 1, Plate III. The 1900 instrument was preceded in 1899 by an intermediate type, illustrated on Plate IV. The 1899 type was produced by modifying the instrument, Fig. 1, Plate III, by redesigning the striding level in such a way as to bring the level vial and the line of collimation of the telescope much nearer together; by constructing the telescope barrel, including the collars and the principal metallic parts of the striding level of a nickel-iron alloy, having a low coefficient of expansion, about the same as that of pine; and by providing a device for reading the bubble similar to that described fully by Mr. Fischer in connection with the 1900 level.

Though it was possible to reverse the telescope of the 1899 level

about its axis of figure, and to reverse the striding level, such reversals were not actually made during the course of the regular leveling observations, but only when determining instrumental constants. Hence, when the non-reversing method of observation had, during the working season of 1899, been found thoroughly satisfactory in every respect, it was but a logical step, in designing the new instrument, to make it irreversible, and, in doing so, to insure greater stability in the relative positions of its parts and to make its manipulation easier and quicker. Mr. Hayford.

Aside from the advantages which may be claimed for the 1900 Coast and Geodetic Survey instrument over the Mendenhall instrument, which is the author's favorite—and also over the Kern instruments, which have been used extensively under the U. S. Corps of Engineers—which have already been mentioned and which imply an instrument maker's or office point of view, the following points will be especially appreciated by the observer in the field. The quotation is from an observer* who has probably run more miles of precise leveling than any other man in the United States, and whose work has been subjected to very severe tests by being involved in many circuits and has been uniformly found to be of the highest degree of accuracy. This observer used a Kern level until 1899.

"The 1900 instrument is a great success. In using it there need be no fear of the effects of dirt or dust, the Y's having been eliminated; no fear that the level does not 'set right,' as there is with other instruments; no fear that the wind will tilt the striding level a little between back sights and fore sights and so change the inclination; no fear that the collars may have needed wiping; and no fear that an ill-directed motion of the hand has disturbed the relation between the level vial and telescope, or the verticality of the wires. This new instrument is about 15% quicker to manipulate than the Kern level or other instruments of that type. It is leveled up and made ready to observe quicker than the other instruments, and after observing it requires very little time to put the instrument in condition to be carried forward to the next station. The prisms, reflecting the ends of the bubble through the false telescope, are a great comfort to the observer. This device is superior to the inclined plane mirror often used for that purpose, for it concentrates the vision on the ends of the bubble and shuts out distracting patches of light and images of trees and other objects which sometimes appeared so prominent in the inclined plane mirror that it was hard to read the bubble."

Present Method of Observation, Coast and Geodetic Survey.—The method of observation used in the Coast and Geodetic Survey in 1899 is indicated in the following general directions which were issued to the observers in the field:

"1. Except when specific instructions are given to proceed otherwise, all lines are to be leveled independently in both the forward and the backward direction.

* Mr. O. W. Ferguson, formerly engaged on various level lines under the Corps of Engineers, and now an Assistant in the Coast and Geodetic Survey.

Mr. Hayford. "2. It is desirable that the backward measurement on each section should be made under different atmospheric conditions from those which occurred on the forward measurement. It is especially desirable to make the backward measurement in the afternoon if the forward measurement was made in the forenoon, and *vice versa*. The observer is to secure as much difference of conditions between the forward and backward measurements as is possible without materially delaying the work for that purpose.

"3. On all sections upon which the forward and backward measures differ by more than 4.0 mm. $\sqrt{K}$ (in which K is the distance leveled between adjacent bench-marks, in kilometers), both the forward and backward measures are to be repeated until two such measures fall within the limit.

"4. The programme of observation at each station is to be as follows:

"Set up and level the instrument. Read the three lines of the diaphragm as seen projected against the front (or rear) rod, each reading being taken to the nearest millimeter (estimated) and the bubble being held continuously in the middle of the tube (*i. e.*, both ends reading the same). As soon as possible thereafter read the three lines of the diaphragm as seen projected against the rear (or front) rod, estimating to millimeters as before, and holding the bubble continuously in the middle of the tube. During all observations the telescope is to be erect and the striding level with a particular leg toward the objective—that is, there are to be no reversals of the telescope or striding level.

"5. At each rod station the rod thermometer is to be read and the temperature recorded.

"6. At stations of odd numbers the back sight is to be taken before the fore sight, and at even stations the fore sight is to be taken before the back sight.

"7. The maximum difference in length between a fore sight and the corresponding back sight is to be 10 m. The actual difference is to be made as small on each pair of sights as is feasible by the use of good judgment without any expenditure of time for this particular purpose.

"8. The recorder shall keep a record of the rod intervals subtended by the extreme lines of the diaphragm on each back sight, together with their continuous sum between bench-marks. A similar record shall be kept for the fore sights. The two continuous sums shall be kept as nearly equal as is feasible without the expenditure of extra time for that purpose, by setting the instrument beyond (or short of) the middle point between the back and front rods. The two continuous sums shall not be allowed to differ by more than a quantity corresponding to a distance of 20 m.

"9. The inequality of collars shall be determined at the beginning and end of each season of work.

"The collimation error shall be determined once for each day of work, by a set of three readings in the direct position and two in the reverse position of the telescope.

"The error of adjustment of the striding level shall be determined at least twice on each day of work by a set of three readings in the ordinary position and two in the reverse position.

"10. Notes for future use in studying leveling errors shall be inserted in the record, indicating the time of beginning and ending of the work of each half day; indicating the weather conditions, espe-

cially as to cloudiness and wind; indicating whether each portion of the line is run toward or away from the sun; and such other notes as promise to be of value in studying errors. Mr. Hayford.

"11. The instrument shall be shaded from the direct rays of the sun, both during the observations and the movement from station to station.

"12. The maximum length of sight shall be 150 m., and the maximum is to be attained only under the most favorable circumstances."

When instruments of the 1900 design were issued to two of the parties during the past season it was necessary to make the following changes in these general directions on account of the fact that the new instruments are not reversible:

"The last sentence of Section 4 of the general directions for precise leveling is now unnecessary.

"In place of Section 9 substitute 'Once during each day of observation the error of the level should be determined in the regular course of the leveling, and recorded in a separate opening of the record book as follows: The ordinary observations at an instrument station being completed, transcribe the last fore sight reading as part of the error determination, call up the back rod and have it placed about 10 m. back of the instrument, read the rod, move the instrument to a position about 10 m. behind the front rod, read the front rod and then the back rod. The rod readings must be taken with the bubble in the middle of its tube. The required constant C to be determined, namely, the ratio of the required correction to any rod reading to the corresponding subtended interval, is

$$C = \frac{(\text{sum of near rod readings}) - (\text{sum of distant rod readings})}{(\text{sum of distant rod intervals}) - (\text{sum of near rod intervals})}$$

The level should not be adjusted if C is less than 0.005. If a new adjustment of the level is made, C should at once be re-determined. It is desirable to have the determinations of level error made under the ordinary conditions as to length of sight, character of ground, elevation of line of sight above the ground, etc.'"

It is important to note, although nothing in the foregoing directions bears upon that point, that the usual practice in the Coast and Goedetic Survey of continuing the observations throughout the day was not changed. The only concession made to the idea that observations should be taken only during the early morning hours or late evening hours was, that the period of rest at noon was sometimes made unusually long.

In order to obtain the actual lengths of the rods while in use in the field, the practice has been to have the rods measured accurately at the Office of Standard Weights and Measures, at the beginning and end of the field season, and to supplement these measures by tape measures made in the field once or twice a month during the field season.

The following examples of the record and computation will serve to explain the method still further:

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TABLE No. 27.—SPIRIT LEVELING.

Date, August 29th, 1900. Sun, C. Forward. From B. M. 68 to B. M. *G*. Wind, S. T.

No. of station.	Thread reading back sight.	Mean.	Thread interval.	Sum of intervals.	Rod and temp.	Thread reading fore sight.	Mean.	Thread interval.	Sum of intervals.
Time, 2.00 P. M.									
43	0 674	.....	99	.....	*V*	2 683	.....	99	.....
	0 773	0773.0	99	.....	38	2 782	2782.3	100	.....
	0 872	.....	198	.....	.....	2 882	.....	199	.....
	0 925	.....	106	.....	.....	2 415	.....	103	.....
44	1 031	1030.3	104	.....	*W*	2 518	2518.0	103	.....
	1 135	.....	210	408	35	2 621	.....	206	405
	0 484	.....	98	.....	.....	2 510	.....	96	.....
45	0 582	0582.3	99	.....	*V*	2 606	2606.0	96	.....
	0 681	.....	197	605	35	2 702	.....	192	597
	0 398	.....	97	.....	.....	2 859	.....	96	.....
46	0 495	0495.0	97	.....	*W*	2 955	2954.7	95	.....
	0 592	.....	194	799	34	3 050	.....	191	788
	1 027	.....	26	.....	.....	1 006	.....	29	.....
47	1 053	1053.3	27	.....	*V*	1 035	1034.7	28	.....
	1 080	.....	53	852	34	1 063	.....	57	845
		3933.9	.....	.....	.....	.....	12895.7	.....	
							−7961.8	.....	
								2.25 P.M.	

The explanation of the symbols used after the words "Sun" and "Wind" is shown on the bottom of the computation form, Table No. 28. The unit in the record is the millimeter. The instrument stations (not turning points) are numbered consecutively throughout the day. A rod once placed at a point stays there, both for the fore sight and back sight, the rodman thus being front and rear rodman, alternately. To carry out the requirement of the general directions, that at stations of odd numbers the back sight is to be taken before the fore sight, and at even stations the fore sight is to be taken first, it is only necessary to remember that this is equivalent to the statement that one particular rodman must always show his rod first after each placing of the instrument. The position of the rod is indicated in the record on the fore sight only. The temperature is read by the rear rodman just before he moves forward, and is called out to the recorder when the rodman passes.

The columns headed "Thread interval" show the intervals between the lower and middle threads as seen projected on the rod, and the middle and the upper, and finally the total interval. The columns headed "Sum of Intervals" show the continuous sum of the total intervals, and, as these values are proportional to the sums of the back-sight distances and fore-sight distances, respectively, they enable the observer to keep these two sums nearly equal at all times.

Such portions of the computation as are shown in the tables as forming a part of the record, are kept up by the recorder as the work progresses. The instrument is not moved forward from any station until the recorder announces that the readings at that station check properly. The recorder uses, as a short method of computing the mean of the three thread readings, the fact that the difference of the upper and lower intervals divided by three is the correction to be applied with the proper sign to the middle thread reading to give the mean of the three. Mr. Hayford.

A party which has just returned from the field, after having completed nearly 120 miles of single line per month for over five months, has turned in record books in the form indicated above, in which every value has been checked by applying the same process as used by the recorder. In addition to this the sums of the mean rod readings at the bottom of each page have been checked by adding the second column of each page and dividing by three, so that these sums have been derived three times, and by two independent processes, and two or three different men.

But little explanation is needed in connection with the computation form, Table No. 28. The forward line from B. M. 68 to B. M. *G*, on this form, is that for which the record is given.

The fifth column on the left-hand page is derived from the fourth by using the sufficiently exact relation that 287 mm. subtended on the rod corresponds to 100 m. along the line, regardless of the lengths of the separate sights, the quantity $c + f$ on page 28 being ignored in both the determination of the stadia constant and its use.

The corrections for curvature and refraction shown in the first column of the right-hand page are those due to the differences of corresponding fore sights and back sights, no correction being necessary when the corresponding sights are exactly equal. The correction only becomes appreciable occasionally, and may be applied very quickly by the use of properly prepared tables and a rapid inspection of the record books. It seldom exceeds 0.1 mm. under actual conditions. It is important to note that this is in the main a correction for curvature, a quantity which is not uncertain, the uncertain refraction being upon an average about one-eighth as great as the curvature.

The level correction shown in the second column of the right-hand page is equal to the constant C (defined in the general directions for leveling) times the value in the sixth column of the left-hand page. This correction will very seldom exceed 0.3 mm. under actual conditions, and will be sensibly zero for most sections.

The third column gives the correction due to the excess of length of the rod at zero degrees, this particular rod being 0.28 mm. too long on each meter. The examinations of the rods made at the office show that the error of graduation is, with sufficient accuracy, proportional to the

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TABLE

Left-hand page.

COMPUTATION OF

Line: Somerset, Ky., to Knoxville, Tenn.

B. M's.	Forward or backward.	No. of stations.	Sum of rod intervals.	Distance, meters.	Rod intervals, $\Sigma B - \Sigma F$	Mean Rod Readings.		Approximate difference of elevation.	Mean temperature of rods.
						Σ B Meters.	Σ F Meters.		
			mm.		mm.				
65–66	F.	9	3 669	1.279	+ 5	10.6532	19.0087	− 8.3555	37
	B.	7	3 675		+ 9	15.1650	6.8087	+ 8.3563	26
66–67	F.	8	3 738	1.302	+12	17.6667	10.4370	+ 7.2297	32
	B.	7	3 739		−23	7.8823	15.0547	− 7.2314	23
67–68	F.	13	4 198	1.464	+ 4	15.5276	31.8222	−16.2946	33
	B.	8	4 206		−24	21.5524	5.2587	+16.2937	28
68–*G*	F.	5	1 697	.590	+ 7	3.9339	11.8957	− 7.9618	35
	B.	6	1 691		− 5	12.5587	4.5979	+ 7.9608	31
G–69	F.	11	5 126	1.785	− 2	28.4990	5.8171	+22.6819	30
	B.	11	5 120		+14	6.3550	29.0368	−22.6818	27
69–70	F.	12	4 589	1.602	−23	17.7855	22.7719	− 4.9864	22
	B.	9	4 607		− 9	17.5312	12.5410	+ 4.9902	22
70–71	F.	10	5 000	1.740	+ 6	6.9331	27.1772	−20.2441	25
	B.	10	4 987		− 5	26.9183	6.6720	+20.2463	24
71–72	F.	10	4 076	1.420	+ 2	10.5955	26.0830	−15.4875	33
	B.	8	4 073		+ 3	21.0375	5.5510	+15.4865	26

Abbreviations: S.=sunshine; C.=cloudy; S. & C.=alternate sunshine and clouds, or alternate sunshine and shade.
Abbreviations, strength of wind: S.=strong; M.=moderate; C.=calm.

distance along the rod. The next column gives the correction due to the expansion of the rod from zero to the temperature of observation, computed with the known coefficient of expansion of the rods, namely, 0.000004 per degree Centigrade. The sum of the quantities in the third and fourth columns in any line gives the correction due to the excess of length of the rod at the temperature of observation. For these particular rods, which are long, even at zero, the correction in each of these columns will always have the same sign as the measured difference of elevation.

The last four columns on this form are for use whenever special studies are to be made to determine if possible the sources of the principal errors of leveling. It is in order to note in the last column that the times of the backward and forward runnings of any section have no fixed relation to each other. The two runnings are sometimes made on the same day, sometimes on different days, and in some instances they both occur in the forenoon, at other times both in the afternoon, and frequently they occur in opposite halves of the day. Any long portion of the line will show corresponding forward and back measurements having all possible relations to each other as to the time of day.

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PRECISE LEVELS. *Right-hand page.*

Observer, W. H. B. Year, 1900.

CORRECTIONS.				DIFFERENCE OF ELEVATION.						
Curvature and refraction.	Level.	Length of rod.	Temperature of rod.	Each line meters.	Mean meters.	Divergence B-F.	Toward or from sun.	Sunshine or cloudy.	Wind.	Date and hour.
mm.	mm.	mm.	mm.			mm.				
.....	.....	−2.3	−1.2	− 8.3590	− 8.3593	−0.6	L.	S.	C.	8/28- 9.15
+0.1	+0.1	+2.3	+0.8	+ 8.3596	.....	.....	.....	C.	C.	8/29- 9.00
.....	+0.1	+2.0	+0.9	+ 7.2327	+ 7.2334	+1.4	.....	C.	C.	8/29-11.05
+0.1	−0.2	−2.0	−0.6	− 7.2341	.....	.....	.....	C.	C.	8/29- 7.45
.....	.....	−4.6	−2.1	−16.3013	−16.3008	+0.9	T.	S. & C.	C.	8/29- 1.30
+0.1	+0.1	+4.6	+1.9	+16.3004	.....	.....	.....	C.	C.	8/28- 5.00
.....	+0.1	−2.2	−1.1	− 7.9650	− 7.9645	+1.0	.....	C.	S.	8/29- 2.15
.....	.....	+2.2	+1.0	+ 7.9640	.....	.....	.....	C.	C.	8/31- 9.00
.....	.....	+6.4	+2.6	+22.6909	+22.6908	−0.3	R.	S. & C.	M. S.	8/29- 3.15
.....	.....	−6.4	−2.4	−22.6906	.....	.....	.....	C.	F. M.	8/31- 8.30
+0.1	−0.1	−1.4	−0.4	− 4.9882	− 4.9901	−3.8	.....	C.	C.	8/30- 7.15
.....	.....	+1.4	+0.4	+ 4.9920	.....	.....	.....	C.	C.	8/30- 4.30
.....	.....	−5.7	−2.0	−20.2518	− 20.2529	−2.2	.....	C.	C.	8/30- 8.15
+0.1	.....	+5.7	+1.9	+20.2540	.....	.....	.....	C.	C.	8/30- 3.30
.....	.....	−4.3	−1.9	−15.4937	−15.4930	+1.3	L.	S.	C.	8/30- 9.15
.....	.....	+4.3	+1.6	+15.4924	.....	.....	.....	C.	C.	8/30- 2.40

Abbreviations, direction of progress relatively to sun. $\left.\begin{matrix}\text{T}\\\text{F}\end{matrix}\right\}$ = within 45° of directly $\left\{\begin{matrix}\text{toward}\\\text{from}\end{matrix}\right\}$ sun. $\left.\begin{matrix}\text{T}_\text{R}\\\text{F}_\text{R}\end{matrix}\right\} = \left\{\begin{matrix}\text{toward}\\\text{from}\end{matrix}\right\}$ sun, but at an angle of more than 45° with sun to right. $\left.\begin{matrix}\text{T}_\text{L}\\\text{F}_\text{L}\end{matrix}\right\}$ = ditto with sun to left. $\left.\begin{matrix}\text{R}\\\text{L}\end{matrix}\right\}$ = sun to $\left\{\begin{matrix}\text{right}\\\text{left}\end{matrix}\right\}$ and nearly at right angles to line. The same abbreviations also apply with reference to the direction of progress relatively to the wind.

Table No. 29 is an abstract of results, and is essentially a summary and combination of the values derived on the computation form. The first form, Table No. 28, is discontinuous, showing results from separate sections, while this abstract, Table No. 29, is continuous, and is substantially the form in which the results are published.

The discrepancies between the forward and back lines are shown in the last two columns of the left-hand page. The accumulated discrepancy up to Bench-mark 72 is 45 mm. on 120 km., or at the rate of slightly less than $\frac{1}{3}$ mm. per kilometer of line run. It may be remarked that the total discrepancy had only increased to 53 mm. at the end of the line, 38 km. farther on.

Table No. 30 is a sample of the record and computation of the determination of C, the constant expressing the relation between the tangent of the level vial at its middle point and the pointing line of the telescope, and will be understood without difficulty by reference to the statement of the method of determination of C in the general directions for leveling, and by noting the way in which quantities have

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TABLE

ABSTRACT OF SPIRIT-

State, Tennessee. Instrument, Level No. 8. Rods, *V* and *W*.

Date.	From B. M. to B. M.	Distance, in kilometers.	Difference of Elevation.			Discrepancy.	
			Forward line.	Backward line.	Mean.	Partial.	Total accumulated.
			m.	m.	m.	mm.	mm.
Aug. 28–29	65–66	1.279	− 8.3590	+ 8.3596	− 8.3593	−0.6	+47.1
" 29	66–67	1.302	+ 7.2327	− 7.2341	+ 7.2334	+1.4	+48.5
" 28–29	67–68	1.464	−16.3013	+16.3004	−16.3008	+0.9	+49.4
" 29–31	68–G	0.590	− 7.9650	+ 7.9640	− 7.9645	+1.0	+50.4
" 29–31	G–69	1.785	+22.6909	−22.6906	+22.6908	−0.3	+50.1
" 30	69–70	1.502	− 4.9882	+ 4.9920	− 4.9901	−3.8	+46.3
" 30	70–71	1.740	−20.2518	+20.2540	−20.2529	−2.2	+44.1
" 30	71–72	1.420	−15.4937	+15.4924	−15.4930	+1.3	+45.4

been transferred from the right-hand to the left-hand page and *vice versa*. The correction of — 0.8 mm. for curvature and refraction is applied to the sum of the two distant rod readings, the correction for either one being — 0.4 mm. The curvature is not appreciable for the near rod readings.

Contrast of Methods.—A comparison of the present Coast Survey method, just described, with the author's method, indicated on pages 83 and 89, will show that there is just twice as much observing per station in the latter method as in the former, the three lines of the reticule being read against each of the rods twice in the author's method and only once in the Coast Survey method. It is still more important to note, however, that there is much less manipulation required in the Coast Survey method than in the other, the additional manipulation in the author's method being at each station the placing of the striding level in position and taking it off again after observation, reversing the striding level twice and reversing the telescope twice at each station. It is the writer's opinion that the additional time spent at the station for these additional observations and additional manipulations tends to reduce the accuracy of the observations by increasing the magnitude of all errors which are a function of the elapsed time at the station, the most prominent of which are errors due to changes of temperature in the instrument. In addition to this it is probably true, as a general proposition, that the more an instrument is handled during a set of observations the less accurate are the results obtained.

In the author's method reliance is placed upon reversals of the telescope and striding level to eliminate the instrumental errors. In the Coast Survey method reliance is placed definitely upon the near

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LEVEL RESULTS.

Observer, W. H. B. Computer, W. H. B.

No. of B. M.	Distance from B. M. No. 1.	Elevation above.	Locality.
	km.	m.	
66	110.585	423.8842	
67	111.887	431.1176	
68	113.351	414.8168	
G	113.941	406.8523	Stone post at Sunbright, Tenn.
69	115.730	429.5431	
70	117.332	424.5530	
71	119.072	404.3001	
72	120.492	388.8071	

equality of corresponding fore sights and back sights. The corrections which result from the small measured differences of corresponding sights are found to be so small that it is hardly worth while to apply them. The time spent in applying these small corrections and in determining the constant *C* each day in the field is probably little more than 1% of that required in the author's method for the thousands of reversals of the striding level and telescope in the field.

Other points of contrast between the two methods will be touched upon later in this discussion.

Reason for Change in Coast and Geodetic Survey Method of Leveling.—Beginning with the year 1899 the Coast and Geodetic Survey has used a radically different method of leveling from that which had previously been followed in that organization for many years. It is important, therefore, that the reasons for this radical change should be made known.

The old method of observation* fulfilled the author's doctrine that the method of leveling "becomes more complicated and requires more painstaking in proportion to the accuracy required" (page 1), and that "The most reliable results in precise leveling are undoubtedly to be obtained by careful and systematic observations designed to prevent and eliminate errors directly in the observations." The method involved the use of a target rod, and in that respect was radically different from the author's method, but it closely resembled it in that both the striding level and the telescope were reversed during the readings upon each rod at each station. The amount of observing at each station exceeded considerably that necessary in the author's

* This method is described in Appendix No. 15 of the C. & G. S. Rep. for 1879, pp. 202–211, and again in Appendix No. 8 of the C. & G. S. Rep. for 1898–99, pp. 416–418.

Mr. Hayford. method. In fact, the method advocated by the author may be said to be a mean, in degree of complication and the number of readings required at each station, between the old C. & G. S. method and the present C. & G. S. method.

TABLE No. 30.—DETERMINATION OF *C*, 8.20 A. M., AUGUST 28TH, 1900.

Left-hand page. *Right-hand page.*

Number of station.	Thread reading, back sight.	Mean.	Thread interval.	Rod and temperature.	Thread reading, fore sight.	Mean.	Thread interval.
A	1 515 1 528 1 542	1528.3	13 14 27	*W*	0 357 0 462 0 566	0461.7	105 104 209
B.....	2 252 2 357 2 462	2357.0	105 105 210	*W*	1 276 1 288 1 301	1288.3	12 13 25
		0461.7 2818.7	419 52			1528.3 2816.6	
Corr. for curv. and ref.		— 0.8	367			2817.9	

$$367) \quad -1.300 (-0.004 = C$$

As the number of connections of the level lines run by the old C. & G. S. method with tidal determinations, with other level lines of the same kind, and with level lines run by other organizations, increased, it became more and more evident that the real accuracy of such leveling had been greatly overestimated; and that the internal evidence of the observations furnished by the agreement of the two or more runnings of the same section with each other was an entirely fallacious indication of the accuracy. In 1898 so much evidence of this kind had accumulated that there could be no doubt of the desirability of a special investigation to determine where the difficulty lay.

On November 29th, 1898, the Superintendent of the Coast and Geodetic Survey appointed a committee of four* to consider the subject of precise levels and in particular to investigate "The accuracy of various methods, their relative freedom from systematic errors, and their relative quickness, cheapness and facility of reduction of the observations." This committee met for three or four hours on nearly every working day from that date until their report was submitted on February 9th, 1899. All the results of precise leveling published up to that date in the United States were examined, as well as much of the general American literature on the subject of precise leveling, and some of the principal European publications on the same subject. For their convenience all the levels illustrated in the author's paper were brought to Washington for direct examination. As a result of

* John F. Hayford, Isaac Winston, J. J. Gilbert and A. L. Baldwin.

this extensive study of the facts, in the form of published results and of the various opinions of individuals expressed in the literature examined, and of direct examination of the instruments advocated by various parties, the committee made almost unanimous recommendations which led to the abandonment of the old method of leveling and the modification of the instrument itself. Two of the main conclusions reached by the committee, and upon which their recommendations were based, were that the principal errors in the old Coast and Geodetic Survey leveling were systematic temperature errors following a certain law, which is stated more fully further on, and that the leveling by the Corps of Engineers with Kern instruments and a very simple method of observation was of at least as high a degree of accuracy as that of the C. & G. S., using a much more complicated and laborious method of observation and computation. Mr. Hayford.

Adjustment of the Precise Level Net.—It fell to the lot of the writer, as a part of his regular duties, during the first half of the present year, to direct the compilation of the results of precise leveling in the eastern half of the United States, and the adjustment of the complicated net-work formed by such lines. In the course of the work of compilation some new information became available in addition to what the committee had used. Before making the final adjustment of the net it was necessary to review carefully the work of the committee, to re-examine the evidence which they had considered, and to study the new evidence and test the conclusions of the committee in every possible way. It was of the highest importance in making the adjustment that the relative accuracy of the different kinds of leveling should be ascertained, and that the systematic errors of each kind of leveling should be specially investigated. The investigation thus made as a preliminary to the adjustment of the level net confirmed the committee's conclusion fully in regard to the systematic temperature error affecting the old C. & G. S. leveling, and showed incidentally that the work done by the Corps of Engineers was of a very much higher degree of accuracy than the old work done by the Coast and Geodetic Survey. It thus furnished new evidence that the recommendations of the committee in regard to change of method and instrument were correct. At the time the adjustment was made three parties had already completed a season's work with the modified level and the new method, and these results were also available.

The net-work of precise leveling resulting from the compilation, and which was adjusted, is shown in Plate VII. The line symbols indicate the organization which did the work and in a general way the kind of leveling involved. The figures inside each circuit show the closing error of the circuit and the circumference of the circuit not including water leveling along tidal waters. A plus value for the closing error of the circuit indicates that the elevation, carried contin-

Mr. Hayford. uously around the circuit in a counter-clock-wise direction, is too great at the closing point. The approximate closing error, expressed in millimeters per kilometer, can be obtained at a glance by dividing the upper value in each circuit by the lower.

No argument is needed to show that to secure the highest possible degree of accuracy in the elevations assigned to the 4 000 bench-marks connected with this net it must be treated as a whole, and not as an aggregation of separate lines, as it has been in the past. It is evident that the errors of closure should be distributed so as to make all the closures disappear and make all lines agree at junction points. The best method to be used in making such an adjustment is not self-evident. It is a matter of judgment rather than of fact. After an exhaustive study of the matter it was decided to use the least-square process of adjustment, but to supplement it and check the assumptions upon which it is based as frequently as possible by other methods of investigation.

In the Coast and Geodetic Survey Report for 1898-99 is given a full statement of the compiled facts and such a full statement of the processes of adjustment as will enable any engineer to judge for himself as to the trustworthiness of the results. This will be of especial value to the engineer who is interested in precise leveling, as such. For the engineer who is interested in the results rather than the processes there is also given a complete list of the adjusted elevations of more than four thousand permanent bench-marks connected with the level net and also a list of such short descriptions of these bench-marks as will enable the engineer to find them. These two lists are for convenience provided with an alphabetical index. This appendix* was, at the date of writing (Oct. 29th, 1900), in the hands of the printer, and it was expected that it would be ready for distribution in November, 1900. Any interested engineer may obtain a copy by writing to the Superintendent of the Coast and Geodetic Survey, Washington, D. C.

The Temperature Error in the Old Coast and Geodetic Survey Leveling.—The character and amount of the temperature error discovered in the old C. & G. S. leveling is of special interest in this discussion because the author concedes that the errors due to temperature and atmospheric conditions are among the most dangerous errors to be encountered in precise leveling (pages 29, 60 and 110) and because his method and instrument seem to leave the way open to such errors.

The committee report, confirmed by later investigation, indicates that a horizontal surface, as defined by C. & G. S. leveling previous to 1899, is always tipped up with reference to the true horizontal surface, the rotation being about a line 12° north of west or south of east, and the inclination being such that the southwestern part of the surface is

* No. 8 of the Report for 1898-99, p. 347 *et seq.*

PRECISE LEVEL NET OF 1900.
Old Coast and Geodetic Survey
Coast and Geodetic Survey 1899
Engineers Precise
Engineers Wye
Geological Survey and Van Orden
Pennsylvania Railroad
Closing errors are reckoned in the counter-clockwise direction.
Longitude West from Greenwich
GULF OF MEXICO
MEXICO
MONTANA
NORTH DAKOTA
SOUTH DAKOTA
WYOMING
NEBRASKA
COLORADO
KANSAS
OKLAHOMA
TEXAS
NEW MEXICO
MINNESOTA
IOWA
MISSOURI
ARKANSAS
LOUISIANA
WISCONSIN
ILLINOIS
INDIANA
MICHIGAN
OHIO
KENTUCKY
TENNESSEE
ALABAMA
GEORGIA
FLORIDA
WEST VIRGINIA
NORTH CAROLINA
SOUTH CAROLINA
LAKE SUPERIOR
LAKE MICHIGAN
LAKE HURON
LAKE ERIE
Bismarck
Pierre
Rock Creek
Cheyenne
Denver
Limon
Hugo
Roswell
Norfolk
Grand Island
Shelton
Lincoln
Sioux City
Des Moines
Ellis
Solomon
Salina
Abilene
Topeka
Hollidaysburg
Kansas City
Harrisonville
Pleasant Hill
Boston
Guthrie
Van Buren
Chester
Fort Smith
Little Rock
Friars Pt.
Arkansas City
Greenville
Jeters Landing
Farmerville
Parkville
Shreveport
Monroe
Rayville
Jonesville
Vicksburg
Smithland
Baton Rouge
Carrollton
Austin
Duluth
Aitkin
St. Paul
Madison
Savanna
Albany
Fulton
Keokuk
Springfield
Grafton
St. Louis
Odin
Cairo
Memphis
Corinth
Clarksdale
Wilkersons Ldg.
Greenwood
Jackson
York
Meridian
Demopolis
Montgomery
Birmingham
Atlanta
Decatur
Riverton Junc.
Pittsburg Landing
Nashville
Lawrenceburg
Frankfort
Cincinnati
Columbus
Indianapolis
Lansing
Chicago
Gibraltar
Lakeport
Port Colborne
Ft. Gratiot
Erie
Braddock
Charleston
Mobile
Biloxi
Tallahassee
Cedar Keys
St. Au
Marquette
Escanaba
-45 mm
1960 km.
-466 mm.
3000 km.
-392 mm.
1710 km.
-458 mm
2230 km
-237 mm.
750 km.
+769 mm
3020 km
+258 mm
220 km
+292 mm.
380 km.
-183 mm.
180 km.
+121 mm.
550 km.
-227 mm.
570 km.
-900 mm.
1280 km.
+131 mm.
460 km.
-26 mm.
340 km.
+533 mm.
1210 km.
+50 mm.
310 km.
+5 mm.
320 km.
L.L. POATES, N.Y.

PLATE VII.
TRANS. AM. SOC. CIV. ENGRS.
VOL. XLV, No. 888.
HAYFORD ON PRECISE SPIRIT LEVELING.

too low and the northeastern part of the surface too high, regardless of the direction in which the leveling actually progressed. According to this law a C. & G. S. level line running in a direction about 12° east of north will give elevations which run too low at a maximum rate; a line in the opposite direction will run too high at a maximum rate; and in the two directions at right angles, namely, 12° north of west and 12° south of east a line will not be subject to a systematic tendency to run either too high or too low. It may be well to note here that an error of this form will not be detected by re-running the line in the opposite direction, for if the forward line runs too high in going southwestward, say, the backward line running to the northeastward will run low by the same amount, and the two lines will show no divergence. Mr. Hayford.

The proofs for this law, brought forward by the committee, did not involve any use of the principles of least squares. They were based simply upon the discrepancies developed when lines were run between tide gauges or between points connected independently by other lines of levels run by some other method. The main reliance was of course placed upon the first line of evidence.

Several preliminary adjustments of the precise level net were made before a final adjustment was attempted. These preliminary adjustments showed that the C. & G. S. lines persistently needed corrections in accordance with the law of error stated above. The same tests were also applied by the committee, and again in the preliminary adjustments, to the leveling by the Corps of Engineers, and in every case indicated that there was no appreciable systematic error in the Engineer Corps leveling corresponding to that in the C. & G. S. leveling.

It might be contended that the evidence thus furnished by the preliminary adjustment was not valid, as it involved the assumption that the Gulf of Mexico is at the same elevation as the Atlantic, whereas it has been claimed that it is considerably above. It is desirable, therefore, to obtain some check on the reality of the systematic error which is independent of any such assumption. In the level net shown on Plate VII there are eighteen circuits, of which the C. & G. S. leveling previous to 1899 forms a part only, the remainder being either spirit leveling of some other kind or water levels. Only one of these circuits, namely, that given by the level lines from St. Augustine to Cedar Keys, Fla., and the water levels from Cedar Keys to St. Augustine through the Straits of Florida, has a closing error which is affected by the assumption as to the relative elevation of the Gulf and the Atlantic. The remaining seventeen closures are independent of any such assumption. Of these seventeen, fourteen have closing errors of the sign which would be accounted for by assuming that the closing error in each case is due entirely to the above systematic error in the portion of the circuit run by the C. & G. S. previous to 1899. An ad-

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ditional circuit has since been closed which also agrees with the fourteen, making in all fifteen cases out of eighteen. For example, in the circuit, Cairo—Odin—St. Louis—Cairo, the eastern and northern sides of the circuit were run by the C. & G. S. According to the law of the supposed systematic error, the line from Cairo to St. Louis, *via* Odin, should run considerably low, causing the closing error of the circuit to be negative. The actual closing error is —227 mm., and the circumference of the circuit 570 km.

The reality of the supposed law of systematic error having been satisfactorily established by a purely inductive method, without reference to any theory as to its cause, the next step in making the adjustment of the level net was to derive a numerical expression for the error and to apply the proper corrections to the various lines. It was found that the correction to the elevations on a line running 12° west of south was —1.25 mm. per kilometer, and that it was +1.25 mm. per kilometer in the opposite direction. For a line running in any other direction the correction required is proportional to the cosine of the angle between the line and the direction stated above. This maximum correction of 1.25 mm. per kilometer corresponds to a vertical angle of only 0.33 of a second of arc, less than one-sixth of a division on the level vials ordinarily used, one division being from 2 to 3 seconds. The minuteness of this correction should be kept in mind when considering the possible explanation given below.

It will probably be conceded, by all who are familiar with instruments of precision, and with the particular instruments herein described and the conditions under which they are used in the field, that unequal though small changes of temperature are continually taking place in different parts of the leveling instrument while in use; that these unequal changes of temperature produce continual small changes in the relative positions of the different parts of the instrument; that all portions of the instrument on the side from which the greatest amounts of heat are being received tend to assume higher temperatures than those on the opposite side, and that in general the instrument receives more heat from the side toward the sun at any given time than from other directions, even though the immediate source of heat is not the sun (from the direct rays of which the instrument is always protected), but the inner surface of the tent or umbrella, the surrounding ground, etc. Changes of the kind just outlined occurring in parts of the instrument which are below the line of collimation of the telescope can be shown to have but little influence upon the final results, since they affect both the line of collimation and the level vial alike, and may be considered as simply influencing the stability of the instrument.

On the contrary, similar changes taking place above the line of collimation of the telescope can be shown to have marked systematic and cumulative effects.

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One effect of unequal changes of temperature occurring above the line of collimation is to change the angle between the tangent to the level vial at its middle point and the line of collimation. As soon as the telescope and striding level are placed in a given position the parts which are toward the source of heat (the sun) tend to assume a higher temperature than the opposite parts, and the effect of the unequal expansion of the collars of the telescope and of the legs of the striding level, and of distortion of the vial itself, is to cause the bubble to crawl slowly toward the source of heat. Thus the bubble always tends to be nearer the sun than it would be if all parts of the instrument were always at the same temperature. The ultimate effect of this is to make the corrected rod reading too small when taken upon rods toward the sun from the instrument, and to make readings taken upon rods away from the sun too large. These errors combine in their effect upon the computed elevations, and tend to make the elevations carried toward the sun too great, and *vice versa*.

It may be remarked in passing that this explanation also applies to the author's method of leveling. If the bubble tends to crawl toward the sun on account of expansion of the sun-ward end of the striding level and of the sun-ward collar, and on account of direct distortion of the level vial, and is held back from such movement by use of the micrometer screw when taking readings on a rod toward the sun, the rod reading is necessarily too small, the line of sight having been inclined downward by the act of holding back the bubble. The reverse will be true in reading upon a rod away from the sun. These errors will combine as indicated above and make all elevations carried toward the sun too great, and *vice versa*.

Is this explanation quantitatively sufficient to account for systematic errors which were found in the old C. & G. S. leveling? Do the laws corresponding to this explanation agree with the actual results of the leveling? As to the first question it may be stated that the changes of temperature required are exceedingly small, differences of less than 0.1° Cent. between the two collars or two legs of the striding level being sufficient to account for 1 mm. per kilometer. The following paragraphs are given in response to the second question.

During the progress of the leveling along a given line the sun is steadily changing in azimuth during each day and the temperature error at present under consideration is necessarily undergoing corresponding changes. The temperature error remaining in the final result from a long line of levels should be that corresponding to the average direction of the level line and the average effective azimuth of the sun during the hours of observation. This average effective azimuth will be, for C. & G. S. work, a little west of south, both because the afternoon period of observation is upon an average slightly longer than the forenoon period and because the amount of

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heat reflected from the ground and other objects toward the instrument from the direction of the sun will be much greater during the comparatively dry afternoon hours than during the moist morning hours. This agrees with the fact stated above that the maximum systematic correction occurs on an average for a line 12° west of south or east of north.

These systematic temperature errors should be expected to be much smaller in cloudy than in sunny weather; much smaller on a line run in the shade of trees or over grass than on a line unshaded from the sun or over the bare ballast of the railroad track; should be expected to depend intimately upon the care taken to protect the instrument from the sun and from reflected heat from surrounding objects; and should be expected to increase as the length of time that the instrument stands at the station is increased. The resulting systematic errors on various lines due to this cause will, therefore, be expected to vary widely in magnitude.

On certain of the lines run by the C. & G. S. previous to 1899 the double simultaneous method was used. On these lines the two rods were usually known as P and Q. One of the double lines was run entirely with the P rod and the other with the Q rod, and at each instrument station all readings upon the P rod were taken before the corresponding readings on the Q rod. The observations on the Q line were, therefore, always taken later than those on the P line, and if the above theory in regard to the temperature errors is correct it should be true that upon an average the Q line should have a larger error than the P line, the differences of temperature having had more time to accumulate before the observations were made. It happens that here are four such lines, aggregating a total length of 1 000 km., for which the corrections are known with considerable accuracy. In each of these cases the Q line is farther from the truth than the P line, and the errors of both lines have the same sign, in accordance with the theory advocated.

If the Q line is always farther from the truth than the P line, the divergence between the P and Q lines should be systematic with respect to the direction of the line, just as the error of either one is systematic with respect to direction, that is, the sign of the discrepancy $P-Q$ should be plus for a line running a little east of north and minus for a line in the opposite direction, and should approach zero for a line running nearly at right angles. As a further test upon the theory, this supposition was examined on all the available lines leveled in this manner, nearly 2 400 km. in all, and 57% showed a discrepancy $P-Q$ in agreement with the theory, and 43% disagreed. The discrepancies of those lines which agreed with the theory were about three times as large as those which disagreed.

A still further test of this same matter of the relation of the discrep-

ancy $P-Q$ and the direction of the line was made by examining the same lines by separating them into sections at points where there were marked changes in the general direction, and comparing the rate of divergence of the P and Q lines on each section with the rate of divergence on the preceding and following section. If the theory advanced is correct, whenever the direction of the line changes in such a way as to approach the direction 12° west of south the minus rate of the divergence between the two lines should increase; and if the change of direction is such as to make it approach 12° east of north, the plus rate of divergence should increase. The examination showed that 82% of the leveling so examined agreed with the theory, and only 18% disagreed. Mr. Hayford.

It is important to note that the magnitude of this systematic correction has been determined by treating several thousand kilometers of leveling done under ordinary conditions, and that the above indirect tests of the theory are also based upon thousands of kilometers of such leveling. The conclusions just stated must, therefore, be given heavy weight in comparison with any deductions from a few experimental observations over short lines and covering but a few days or hours.

It is important to note that the same tests being applied to the leveling by the Corps of Engineers, in which a simple method was used involving but a short observing period at each station, failed to indicate any appreciable systematic temperature error. The author's method of observing is intermediate between the old C. & G. S. method and the Engineer method, and the writer, therefore, believes that his leveling is probably subject to a systematic temperature error which is considerably smaller than that in the old C. & G. S. leveling. The construction of the instrument used by him is also such as to warrant the supposition that a temperature error of the kind indicated must exist in the results.

The reason for the extraordinary efforts to eliminate if possible the temperature error, in designing the new instrument, should now be apparent. The substitution of the nickel-iron alloy, with a coefficient of expansion about as small as that of pine, in the place of the bell-metal and brass formerly used in the construction of the telescope and striding level, the increased protection of the level vial by countersinking it into the telescope and surrounding it by a dead-air space enclosed by heavy metal and glass walls, and the placing of the level vial as close as possible to the line of collimation, are all features designed to eliminate the temperature error in the 1900 instrument. The notes indicated in the last four columns of the form shown in Table No. 28, were taken for the express purpose of determining whether the temperature error was still appreciable in the new leveling. If the forward and backward runnings on a certain section of

Mr. Hayford. the line were made at such times that in both cases the direction of progress was directly toward the sun, the effect of the temperature error being to make both lines run high toward the sun, the discrepancy $B - F$ between the two runnings would tend to be of the minus sign. Similarly, if in both cases the leveling progresses away from the sun, the sign of $B - F$ should be plus. In short, if the temperature error still existed it should be possible to predict the sign of the divergence of $B - F$ for certain cases, and the actual signs and magnitudes of the discrepancy $B - F$ might, therefore, be used to detect the temperature error. Such a test was applied carefully and in detail to each of the three lines run in 1899. It indicated that even with the instrument of the intermediate type used during that year the temperature error, if it existed at all, was so small as to escape detection.

This particular form of test was not available in connection with the old C. & G. S. leveling, because the necessary notes as to time of day and direction of the line and weather conditions were not taken.

Minor Comments.—The writer agrees completely with the author in regard to the desirability of simultaneous reading of the rod and bubble when it is desired to obtain either accurate or rapid observations (page 41). The writer also agrees with the comment made on the French bubble-reading device (page 8) to the effect that it does not accomplish satisfactorily the primary object for which it was designed, namely, to enable the reading of the bubble and rod to be made simultaneously. It may be noted that the reading device on the 1900 C. & G. S. level makes the reading of the rod and bubble as nearly simultaneous as is possible, it being so constructed that the distance between the eyepiece of the telescope and the eyepiece of the reading device is adjustable to fit the distance between the eyes of the observer using it; the focus of the reading device is adjustable, and no movement of the eyeballs is necessary when the attention is changed from the rod to the bubble. In the opinion of the writer, the inclined mirror does not accomplish the purpose so well as the 1900 reading device, for the reason that the observer must change the direction of his line of sight in looking from the image of the bubble in the mirror to the rod, or *vice versa;* that he is likely to be troubled by distracting images of surrounding objects in the inclined mirror, and that the observer is forced to hold his head in such a position that the two eyes are nearly in the same vertical plane. He must, therefore, hold his head on one side, and is forced to set up the instrument several inches lower than is necessary when using a reading device which allows him to stand in a natural erect position.

The author's point, that the ordinary wye-level is very imperfect in principle and is not conducive to quick work, seems well taken (pages 4 and 14). The plan used on all precise levels, of mounting the telescope in such a way that it is controlled by a micrometer screw under

the eye end, is conducive to rapid work, and may be introduced to advantage on ordinary instruments. The same statement is true of the various devices for reading the bubble without moving away from the eye end of the telescope. The use of a direct-reading rod or "speaking rod" in the place of a target rod is also conducive to rapidity. The writer believes that any class of leveling can be done more rapidly with the C. & G. S. 1900 level, or an instrument of the same type provided with a less sensitive vial for rough work, and by using direct-reading rods, than can be done with the ordinary wye-level and target rod. The observer will usually read the three lines against the rod much more quickly than he can indicate to a rodman the point at which the target must be placed to obtain the same accuracy. The manipulations of the instrument can certainly be made more rapidly. In support of this opinion, attention is called to the statements of the actual performances of the 1899 and 1900 C. & G. S. levels given under the heading "Speed and Accuracy of Coast and Geodetic Survey Leveling, 1899–1900." Page 171. Mr. Hayford.

The author considers the fact that the horizontal axis in the Mendenhall instrument, about which the telescope rotates under the action of the micrometer screw, intersects the vertical axis of the instrument, an advantage, and implies (pages 6, 7 and 11) that when the horizontal axis is not so located there are appreciable changes in the height of instrument between corresponding fore sights and back sights on account of the re-leveling. This argument against placing the horizontal axis in the usual position has frequently been advanced by others. Is it a valid argument? The practice among C. & G. S. observers is to notice at the beginning of the day's work the reading of the micrometer head for which the telescope is at right angles to the vertical axis; in other words, such a reading for the micrometer that the bubble of the level will remain in a fixed position when the instrument is reversed in azimuth after being accurately leveled up. After once getting this reading in mind, and it will ordinarily remain constant within two or three divisions of the micrometer all day long, he places the micrometer at that reading when the instrument is set up and then finds that when the change is made from the back sight to the fore sight, or *vice versa*, that the reading of the micrometer head needs to be changed by something less than ten divisions and often not more than two or three. If the total change is even as great as eleven divisions, corresponding to about 30 seconds of arc, the height of the instrument is changed by about one-seven-thousandth of the distance from the vertical axis to the horizontal axis, or, in the case of the 1900 level, by about $\frac{1}{70}$ mm., a quantity which is negligible for a single station, especially as it is as likely to be of the contrary sign to the same sign at the next station. To guard against this very small and compensating error by introducing new difficulties for the instru-

Mr. Hayford. ment maker by requiring him to make the vertical and horizontal axes intersect seems to be an error of judgment.

Table No. 3 is given to show that the effect of solar attraction is indicated by the author's leveling in 1898 and 1899. If the solar attraction were appreciable there should be a large preponderance of minus signs in Group 1 of that table, and of plus signs in Group 2, while Groups 3 and 4 should each show no well-marked preponderance of either sign. As a matter of fact, the preponderance of signs in Groups 1 and 2 is no more marked than in 3 and 4, and it does not seem reasonable, therefore, to assert that the table contains any evidence of solar attraction.

The author maintains that a reversible striding level is an absolute necessity on a precise leveling instrument. He states, as his reason, that "no reliance should be placed on the capability of a level tube for holding its horizontal adjustment," and, that "The only rational elimination of this error can be accomplished by taking observations for each sight with the level tube direct and reversed." Such a statement is an indication of the quality of the particular instrument with which the author is most familiar. The C. & G. S. levels do not behave in the manner indicated. One of the 1900 levels was carefully put in adjustment at Washington in July, 1900, and shipped by express to a point in Kentucky. The observer there, on beginning work with it, found it to be still in adjustment. He began his observations on July 24th without having touched an adjusting screw and continued without making any adjustment until September 7th. During that period the value of C never exceeded 0.005 until September 7th, when it jumped to +0.015. The value 0.005 for C means that the line tangent to the level vial at its middle point makes an angle of approximately 3 seconds with the pointing line of the telescope. The instrument was adjusted on September 7th and from that time until October 20th C exceeded 0.005 only once, and then was found to be 0.006, only. The mean value (algebraic) of C during the period September 8th to October 20th was + 0.002, or but little more than 1 second of arc. C was determined regularly on every day on which observations were made. If C equals 0.005, the correction due to this error of adjustment when the difference of a fore sight and the corresponding back sight is 10 m., that being the maximum difference allowed, is only 0.1 mm. In the computation it is proposed to apply the corrections whenever appreciable, upon the assumption that the error is constant for one day at a time. When the total range of variation in the adjustment does not exceed 6 seconds of arc for weeks at a time, as in this instrument, and the test of the adjustment is made on every day and at various hours of the day, will the author contend that the accuracy of the leveling has been appreciably decreased by substituting such a fixed level for the striding level?

The other 1900 instrument behaved nearly, but not quite, as well as

this one. One of the 1899 C. & G. S. levels, which was still in use during the present year, shows a similar behavior. It was in use from May 8th to September 12th, during which time 225 miles of double line of leveling were completed. Its error of adjustment was determined twice during each day of observations. It was adjusted only once during the season. The error of adjustment exceeded 3.4 seconds only six times during the whole season. Mr. Hayford.

Especial attention is called (pages 45 and 46) to the necessity of accurately centering the objective and of insuring that the sliding tube carrying the eye end moves in a straight line. This is, of course, more important for a level telescope of the non-reversible type used in the 1900 C. & G. S. instrument than for the reversible type of telescope. It is necessary to insure within certain limits in the non-reversible telescope that the point in the reticule defined by the middle thread, or rather the mean of the three threads, moves in a straight line passing through the optical center of the objective during the process of focusing. A little investigation shows that it is not difficult to secure this condition, within the required limits, in the instrument shop. It is important, however, to note that when the reticule in such a non-reversible telescope has once been adjusted to the proper position, that this adjustment must not be disturbed in the field. All field adjustments must be made by moving the level vial and not by moving the reticule.

On page 55, a possible heaving or settling of the instrument or rods, as an explanation of the systematic differences often noted between forward and backward lines, is dismissed quickly without an adequate discussion. It is apparently assumed that if the turning point pins "are scarcely susceptible to any heaving or settling of sufficient magnitude to be readable with an instrument, even at a very short range," that it is proved that no appreciable systematic errors can arise from the cause under consideration. The systematic divergences between forward and backward lines, which have been accounted for frequently in the literature of leveling as having resulted from rising or settling of the rod supports or of the instrument, have seldom exceeded 0.25 mm. per kilometer of line run, and have more frequently been less than 0.1 mm. per kilometer than more. To account for an error of 0.1 mm. in a single kilometer, supposing that there are seven stations (average sight 71 m.), if it be assumed that each rod support (7) and each instrument support (7) had a systematic tendency to settle (or rise), requires that each movement should be on an average 0.007 mm., if they are all of the same sign. This quantity is not readable with a leveling instrument, but is sufficient to produce the error for which an explanation is sought. To assert that the rod and instrument supports are not subject to changes of that magnitude is to assert that they have an exceedingly high and improbable degree of

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stability. To account for an error of 0.25 mm. in a single kilometer by movements of the rod supports alone, the systematic movements of the instrument being eliminated by the method of observation, which is the case if half of the fore sights are taken before the corresponding back sights, requires a systematic settling (or rising) of less than 0.04 mm. at each point, a quantity which is still not readable on the rod with the leveling instrument. Is it not true that this source of error is sufficiently dangerous to make it imperative that all lines should be run in both the forward and backward direction and the mean taken, if the highest degree of accuracy is desired?

The requirement, stated without modification on page 74, that the recorder and the rodmen of a leveling party should be college graduates is at least open to question. The computation required of the recorder consists simply of adding and subtracting, and dividing by two or three. It is essential that he should be a man who is careful and conscientious, and who can make such simple computations as are required of him with accuracy and rapidity. The rodmen must be men, or youths, who will take an interest in the work and obey orders. All must be willing to hurry, and each must have the physique to stand the work and the stomach to stand the changing fare. These qualities can be found without difficulty among those who have never been to college, and to require such an education is surely arbitrary.

On page 77 the statement is made that "On some recent Government work, a system of single lines has been adopted with the expectation of checking or closing the work by leveling in large polygonal lines forming a net."

This statement is followed by a prediction as to the errors which would develop in such a case. The only recent work, within the writer's knowledge, which has been done in the manner stated is a portion of a line run in 1899 between Gibraltar, Mich., at the west end of Lake Erie, and Cincinnati, Ohio. Three different portions of this line were run as long loops by leveling continuously southward along one railroad and then returning to the starting point along another parallel railroad or canal towpath. In each case the leveling was carried continuously in a counter-clock-wise direction around the circuit as a single line. The lengths of the separate circuits were 82 km., 112 km. and 87 km., and the errors of closure were, respectively, −4.9 mm., + 9.0 mm. and + 9.3 mm. Moreover, these three circuits were broken into smaller circuits by six cross-lines at points where the two railroads, or the railroad and towpath concerned, drew near together. These cross-lines failed to develop the existence of any "serious local undulations," the nine small circuits showing the same absurdly small closures as the three large circuits. It is interesting to note that the observer on this line believes most thoroughly that the principal cause of a systematic tendency to run high (or low) is usually the settling or rising of rod supports.

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Rods.—It is objected that the paraffining of level rods causes the wood to become soft, prevents the perfect adhesion of paint and increases the coefficient of expansion 50% (page 12). The Coast and Geodetic Survey has had paraffined rods in continuous use since 1895, and its experience does not justify any one of these criticisms. The strongest objections to paraffined rods seem to come from those who have used them but little, if at all, or from instrument makers who are not disposed to take up a novelty.

The symmetrical cross-section of the rod which is used by the C. & G. S. has a decided advantage over the T-shaped cross-section, in that the rod is less likely to warp. It has the apparent disadvantage of affording too narrow a front surface for any complicated graduation.

The discussion of pins *versus* foot-plates is dismissed with a single sentence (page 17), upon the ground that the foot-plates "are considered entirely out of date." Foot-plates are still in use in the Coast and Geodetic Survey, and have been continuously in use for many years. The discussion of pins *versus* foot-plates will probably be an endless one, as there are good points on both sides of the argument. It is the opinion of the writer that but little error in the ultimate results can be credited to either the foot-plate or the pin, provided each section of the line is run in both the forward and backward directions independently and the mean taken. It also seems clear to the writer that the decision between the foot-plate and the pin should be made to depend upon the character of the ground over which the leveling is carried, the pin being probably the better support on some kinds of soil and the foot-plate on others.

It is claimed (page 14) that the design of the rod shown in Figs. 1, 2 and 3 "is based strictly on the self-reading principle, and thus enables the observer to read, practically without counting anything, from the coarsest down to the finest graduation." It may be seen by consulting the illustrations of the C. & G. S. rod, Plate VI, that in its graduation an attempt has also been made to eliminate the necessity of counting. On each decimeter the 2-cm. point is marked by the upper edge of a white rectangle containing a figure, the 4-cm. by the lower edge of a black rectangle, the 5-cm. is marked by a white diamond, the 6 by the upper edge of a black rectangle, the 8 by the lower edge of a white rectangle, and finally the zero and 10 by the center of a white rectangle containing a figure. It is not necessary to count higher than 2 to identify any centimenter, even when the distinction between the black rectangles and the unpainted portion of the rod disappears. It is important to note that there are no marks on the rods having a least dimension of less than 1 cm., and that all areas of color, with the exception of the 5-cm. mark, are rectangular. The markings will, therefore, remain distinct for greater distances

Mr. Hayford. and more furious vibration of the air than will those of any design having smaller areas of color or many sharp points.

The author indicates on pages 14 and 53 his reasons for believing that a 2-mm. graduation should be furnished in addition to the centimeter graduation. In this opinion he does not stand alone, as the same principle is advocated by some European authors. The C. & G. S. rod carries no graduation finer than single centimeters, because it is known that under the actual conditions of work the 2-mm. graduation will be readable only on short sights. On such short sights occurring at the end of a section of the line when a connection is about to be made with a bench-mark, or in climbing a very steep grade, the centimeter graduations can be read with greater accuracy than ordinarily. To add a 2-mm. graduation for use in such cases is to strengthen the strongest part of the line, and is, therefore, a refinement which is of little value, although it does no harm if there is room on the rod for the additional graduation. It is pertinent to note that the rods used under the direction of the United States Corps of Engineers carry no graduations smaller than 1 cm.

The writer fails to see the advantage of placing the zero of the graduation nearly 5 cm. above the foot of the rod. The C. & G. S. practice is to put the zero of the graduation at the foot of the rod, and no difficulty is found in maintaining that relation within narrow limits.

The author condemns the past practice of applying a constant rod correction for the whole season (page 51), and, going to the opposite extreme, advocates a direct measurement of the rod with a steel tape three times a day (page 17), or even at the beginning and end of each stretch (page 51). It would seem that the frequency with which the rod should be measured depends upon the accuracy with which the rod in question holds a constant length, and that the necessary frequency of measurement cannot be decided in the off-hand manner indicated on the pages cited. The various measurements of the C. & G. S. paraffined rods made between field seasons at the office during past years, and in the field once or twice a month during 1899 and 1900 indicate that after a paraffined rod has once been fully seasoned by actual use in the field or by exposure to temperatures as high as it will experience in the field, it will hold a constant length within a range but little greater than 1 part in 10 000, or 0.1 mm. per meter. This statement includes all variations due to changes in the humidity, or other causes, except changes of temperature, the measurements having been reduced to a fixed temperature by using the coefficient of expansion 0.000004 per degree Centigrade. The present C. & G. S. practice, of measuring the rods with great accuracy between field seasons and measuring them with a steel tape once or twice a month during the

season, limits the possible errors from the cause under discussion to much less than one ten-thousandth of the difference of height measured. Under these conditions, daily measurements of the rods seem to be unwarranted. Mr. Hayford.

During the process of seasoning the paraffined rods usually increase slightly in length.

Collar Inequality.—The author's statement, on page 60, that the commonly accepted method of determining the inequality in telescope collars is erroneous, and that "many of the constant errors frequently encountered in leveling operations and attributed to a great variety of causes, principally personal equation and settlement or heaving of instrument or rods," may be accounted for by this error, into which all who have dealt with this matter have fallen, is surely startling. The statement is made again, on pages 69 and 70, that the usually accepted method of determining the collar inequality "has already been explained and found erroneous." At various other points in the text the author indicates with emphasis the importance he attaches to this matter. Were the author's supposition true, not only would all who have dealt with precise levels be involved in confusion at the discovery of such a mistake, but so would several generations of astronomers who have used the same method, and set it forth in their writings, in connection with the determination of the inequality of the pivots of astronomical transits used for the determination of time. It is important, then, that the alleged proof of the error of the usually accepted theory should be carefully examined.

The first thing that attracts the attention of one who makes the examination is the startling array of assumptions which are stated to be necessary to prove the correctness of the ordinary procedure in determining collar inequality (page 47). The next matter that will catch the eye in a quick reading is the inaccuracy and confusion in the statements and formulas on page 48. In the figure, w is marked as a distance, and in lines 3 and 10 it is defined as an inclination or abstract number. ε is defined as an abstract number or inclination and t is stated to be a distance, and is so marked in the figure. The formula $w = \varepsilon + t$ is equivalent to a statement that a distance or abstract number is equal to the sum of an abstract number added to a distance, an absurdity. Similar absurdities occur at several points on the same page. Using this confusion as a clew, one would naturally look for some more serious error.

A careful examination will show that the author's whole argument is completely erroneous, and that the commonly accepted method of determining the inequality of collars is correct within attainable limits of accuracy. He has confused the measurement of the inequality of the radii or diameters of the two collars with a measurement of the effect of such inequality on the inclination of the telescope axis and

Mr. Hayford. the striding level. The quantity determined is usually referred to as inequality of collars, but by that is not meant inequality of the radii of the collars, but simply the effect of that inequality in producing an erroneous position of the telescope axis and the striding level.

The whole matter may be made clear by superposing Fig. 15 (*b*) on 15 (*a*), in such a manner as to make the two points *O* coincide, keeping both figures right side up. It must be kept in mind that these two figures represent in diagrammatic form a section at one collar through the striding level, telescope and cradle support in the plane of the contacts of those three bodies. The two circles representing the eye-end collar will coincide, after the superposition, *O* being the center of the eye end or larger collar in both cases. In the superposed figure the relative position of the "level tube" and cradle support will be that which they actually hold when the larger or eye-end collar separates them. The two small circles in the superposed figure, having centers at *O'* and *O'*, show the two possible positions in which the smaller collar might be placed, first in contact with the cradle support and next in contact with the "level tube," supposing these two to maintain the relative positions which they had when the larger collar was between them. It is evident that under the action of gravity the smaller collar will take the lower of the two positions shown in the superposed figure and its center will then be a distance w or $O\,O'$ below the position which the center of the larger collar had occupied. Similarly, as the distance $O'\,A$ [of Fig. 15 (*b*)], A being a point on the striding level, is fixed, A will occupy a position $2\,w$, equal to $O'\,O'$, lower than it did when the larger collar separated the cradle support and the "level tube." In the reversal of the telescope under consideration the point A will have fallen a distance $2\,w$, and the corresponding point on the other end of the striding level will have risen $2\,w$, and the total change in inclination of the striding level, which will be measured by the movement of the bubble, will, therefore, be $\frac{4\,w}{L}$, L being the length of the striding level between its supporting wyes. Similarly, the change in inclination of the axis of the telescope, the line joining the centers of the two collars, will be $\frac{2\,w}{L}$ and the error in the inclination of the axis, due to inequality of collars, will be half this amount, or $\frac{w}{L}$, which is one-fourth the change of inclination indicated by the striding level in accordance with the usually accepted theory, and in accordance with the method of computation shown on the upper half of page 47. It may be noted that this is a direct proof, dealing with the quantity of which the measurement is actually required, and involves no consideration of the difference of radii of the two collars. On examination it will be found that

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it involves implicitly the assumption that the angular opening of the wyes of the cradle support is equal to, or nearly equal to, the corresponding angular opening of the wyes of the striding level. For, unless this were true, the two distances marked w in the figures would not be equal.

The author's treatment of the same observations would give a correction for inclination of the axis of the telescope due to inequality of the collars which is nearly two and a half times as great as the ordinary treatment,

On pages 90 to 93 is given an example of the record of observations made at a river crossing, and on page 85 the claim is made that when these observations were reduced, by using a value 0.0635 for the inequality of collars, the results from the four crossings agreed very closely, within 2.4 mm., and that this would not have been the case if "the correction for collar inequality, found by the commonly adopted method of level tube and telescope reversals," had been used. This argument will not bear close examination. The author has failed entirely to apply any correction for curvature and refraction in his computation. It is essential to apply such a correction whenever forward and backward sights are very unequal as they are in the case in hand. The correction for curvature and refraction for a 215-m. sight (see Fig. 16) is 3.1 mm., for a 211-m. sight is 3.0 mm., and for a 22 or 23-m. sight it is 0.0 mm. Applying these necessary corrections to the observations, the results stand as shown in Table No. 31.

TABLE No. 31.

Line.	Excess of foresight.	Difference of Elevation.				
		Observed.	Correction for curvature and refraction.	Corrected for curvature and refraction.	Corrected, using 0.0635.	Corrected, using 0.0276.
	m.	mm.	mm.	mm.	mm.	mm.
1st direct D_1.....	+192.7	+123.8	+3.1	126.9	139.1	132.2
2d direct D_2.....	−188.6	+148.1	−3.0	145.1	133.2	139.9
1st reverse R_1.....	+188.6	+146.3	−3.0	143.3	131.4	138.1
2d reverse R_2.....	−192.7	+124.6	+3.1	127.7	139.9	133.0

The last column was computed by using 0.0276 mm. per meter as the correction for inequality of collars, that value being given on page 47 and stated to be the mean of twenty observations taken in the ordinary way. The last column but one shows the corrected values, using 0.0635, the author's value of the correction for collar inequality. It will be noted at a glance that the range of the four results is about as great in one case as in the other. The observations indicate that

Mr. Hayford. the true value of the collar inequality is between 0.0276 and 0.0635, as such an intermediate value would come closest to making the four differences of elevation agree. The remarkable agreement of 2.4 mm., shown on page 85, has now disappeared, and with it the force of the argument based upon that agreement.

It may possibly be contended that the correction for refraction should be neglected, as the refraction is a very uncertain quantity. The curvature, however, is not uncertain. It is positively known within very narrow limits. The corrections used herein would be increased slightly by neglecting the refraction, as that has been assumed to be one-eighth of the curvature and of the contrary sign.

It may also be claimed that, as the curvature and refraction are neglected in the determination of the inequality of collars, by the method advocated by the author and shown on pages 68 and 69, this neglect will produce errors of the same magnitude, and of the contrary sign, as those produced by the neglect just noted in the observations at the river crossing. Such an argument, however, is utterly fallacious, as the correction for curvature and refraction increases as the square of the distance, whereas the correction for collar inequality increases as the first power of the distance. Hence the curvature and refraction cannot be merged with the collar inequality by neglecting to correct for curvature and refraction when determining collar inequality. The true method is to correct for curvature and refraction wherever such correction becomes appreciable. If this had been done in Table No. 9 the derived value of 0.0624 for correction for collar inequality would have been decreased slightly.

With the exception noted, namely, the neglect to apply the correction for curvature and refraction, the method shown in Table No. 9 seems to be correct, and should give a value of the collar inequality in agreement with that of the ordinary method by reversal of the telescope, except for unavoidable errors of observation.

The discrepancy between the various values of the collar inequality determined by the two methods seems to the writer to be evidence of wear in the instrument, or poor observation, rather than anything wrong in the theory. The value + 0.0235 mm. per meter is given for this correction on page 85, and + 0.0276 on page 47, both being determined by the usual method by reversals of the telescope in the wyes. The value 0.0624 is shown on page 69, and the value 0.0635 is used by the author on page 85. Finally, the river crossing treated on pages 90 to 93 indicates that the true value lies somewhere between 0.0635 and 0.0276. The great range of these values, and the fact that they are unusually large, is an indication that the concentration of the wear on the telescope collars at four small areas has produced the effect which should be expected.

Table No. 6 and the Conclusions drawn from it.—The importance

ascribed by the author to the experiments indicated in Table No. 6 may be appreciated by noting the frequency and character of the references to it. According to his statement: Mr. Hayford.

"The important cumulative error is that produced by the warping effect of the sun's direct and reflected heat rays on the telescope (see experiments in Table No. 6). All other errors are generally of a compensating nature when good instruments are used" (page 103).

Again: "The error shown to exist in very warm weather (see Table No. 6) cannot be detected, except by the closure of a line, and the best and most careful work may be done under seemingly good conditions without yielding satisfactory results" (page 79).

On pages 60 and 102 important conclusions are also based upon this table as to the times of day and kinds of weather in which good work can be done. Finally, page 76, the author states that his conclusion, based upon Table No. 6, as to the times of day when double lines should be run, "supersedes all theoretical reasoning, being, as it is, the result of actual observation and experiment."

The experiments in question are confined to a single day and a single instrument station. Has the author shown good judgment in using these experiments as a foundation for the great superstructure of far-reaching conclusions which he has constructed? Or, if he has additional supports for his superstructure, is he not unwise in leaving the reader in ignorance of them?

When one examines the evidence carefully, with a view to answering the above questions, the first doubt is apt to arise in his mind as soon as he notices that all the experiments were made under one set of conditions. One of the primary conditions which must be fulfilled by a set of experiments, if the conclusions based upon them are to be thoroughly trustworthy, is that the conditions under which they are made must be varied as much as possible. In the case in hand, the direction of the line observed might have been varied; the observations might have been taken on different days; the character of the surface of the ground should have been varied, and the effect of the slope of the ground should surely have been taken into account as possibly affecting the refraction. As it is, all the readings are taken over the same line, and the only conditions which vary are the time of day, the temperature, cloudiness or brightness of the sunlight, and the azimuth of the sun.

The mean reading of Rod 5 varied during the experiments from 1529.2 mm. to 1529.9 mm., or a total of 0.7 mm. Similarly, the range of the readings on Rod 1 was 1.4 mm. The total change in the apparent difference of elevation of the two rod supports is from 287.1 mm. to 288.0 mm., or a total of .05 mm. All the author's conclusions are based upon these small changes. Is the internal evidence of Table No. 6 such as to make it unquestionably true that the accuracy of the observations was sufficient to warrant any conclusions drawn from such

Mr. Hayford. small variations. A side-light upon this question may be gained by noting the range of variation of the angle subtended on the rod by the upper and lower threads. Omitting the first values given in the table, because the instrument was moved, as stated in the text, between the first and second observations, the thread distance on Rod 5 varies from 151.6 mm. to 152.1 mm., or a range of 0.5 mm., and on Rod 1 varies from 151.8 mm. to 152.6 mm., or a range of 0.8 mm. It should be noted that the author's explanation of the changes in the mean rod readings and the apparent difference of elevation in the rod supports does not in the least account for any apparent change in the thread distances. If the eye end of the telescope curves downward during a part of the observations, as he assumes, all three threads will apparently move together on the rod, and the thread distances remain constant. The variation in thread distance which could be accounted for by a change in temperature of the reticule would be in the hundredths of a millimeter only. The range of variation in thread distances referred to, viz.: 0.5 and 0.8 mm., must be due either to errors of observation or changes in the difference of refraction between the upper and lower lines of sight. The errors from either of these causes are equally likely to affect the mean readings of the rods and the apparent difference of elevation of the rod supports. It seems, therefore, that to base any far-reaching conclusions on a variation of 0.9 mm. during a set of observations, when variations of 0.5 to 0.8 mm. remain unexplained, is unwarranted. The author's conclusions may possibly be correct, but Table No. 6, without support, hardly can be taken as proof of such correctness.

It is stated that the error of the determination of the difference of height of the two rod supports in this set of experiments became zero at 8 A. M. and 3 P. M. (page 59). An examination of the table indicates then that the author considers the true difference of elevation to be 288.0 mm. There is no evidence put forward by the author to determine which of the many measured differences of elevation was the true one. What evidence is furnished that the greatest of the measured values corresponds to the truth, rather than the mean of all the measures. A dogmatic statement is made, without any effort to support it. This statement is of especial importance as the conclusions, stated in various other parts of the paper, as to the time of day when good work can be done, all depend upon the correctness of this assumption that 288.0 mm. is the true difference of elevation.

The author states positively that to detect temperature errors of the class which he claims are indicated in Table No. 6 it is necessary to close each stretch of the line by making both the forward and the backward measures during the same half of the day (pages 60, 76 and 102). Even if the author's conception of the cause of such temperature errors be agreed to, and even if it be agreed that the measurements

shown in Table No. 6 are absolutely accurate, the conclusion stated does not follow. Let it be agreed that in running a line northward, as indicated in Table No. 6, the eye end of the telescope curves downward on the fore sight and thus inclines the line of sight upward, and that the elevations carried northward will be too small; it is equally true that, if the direction of the running were to the southward, under the same conditions, according to the author's theory the fore sight, which would then be toward the sun, would be undisturbed, while the back sight, taken from the sun, would be made too high on the rod, and the ultimate effect would be to tend to make the line run high, just as in the previous case it was supposed to run low. If the first measurement to the northward be called the forward measurement and the second the backward measurement, the forward measurement would carry the measures too low and the backward measurement would carry them too high in the direction of the measure, which is now reversed, and the two results would agree instead of showing a divergence. In other words, by the author's theory as to the manner of behavior of the telescope, it is necessarily true that elevations carried in the computation from one bench-mark to another which is in a direction away from the sun from the first will be too low, regardless of the direction in which the measurements actually progressed, and *vice versa*. To make the backward and forward measurements on the same half day on an east and west line is to insure a better agreement of the two measures than if they are made under differing conditions. This is the reverse of the claim made by the author, when applied to such a line. The logical conclusion from his theory would be that if the forward measure is made under one set of conditions as to the azimuth of the sun and cloudiness, the backward measure should be made under conditions as different as possible in these respects if it is desired to make the temperature error apparent by producing a divergence between the two lines, as he claims, and correctly, should be done. The correct conclusion is, for example, that on a line running to the westward, if the forward measurement is made in the forenoon with the sun behind, and the measures may therefore be expected to carry the elevations too low in the direction of the progress of the measurement, the backward measurement should be made in the afternoon with the sun again behind and with the line again running low in the direction of progress. Mr. Hayford.

A test of the validity of the author's conclusion that this temperature error may be as much as 15 mm. per kilometer (see middle of page 60) is available in the printed reports of the Coast and Geodetic Survey. The general custom from the beginning, on that Survey, has been for the level party to continue working throughout the day. In the various stretches of line comprised in hundreds of miles of leveling done by the forward and backward method it is necessarily true

Mr. Hayford. that all combinations of conditions as to the relative times of the runnings of the forward and backward line have occurred, but no divergence approaching 15 mm. per kilometer can be found in the observations.

It is interesting to note that the result from the experimental line run southward on a clear, warm day without any umbrella, stated on page 60, agrees with the theory of temperature errors stated, in connection with the leveling of the Coast and Geodetic Survey previous to 1899, earlier in this discussion. According to that theory a level line run toward the sun should always have a tendency to run too high, and, of course, this tendency would be greatly exaggerated when no umbrella is used.

The writer's opinion, then, in regard to Table No. 6 and the conclusions based upon it, is that the evidence of the table is too weak to support any conclusion, and that the author's theory of temperature errors must necessarily be supported by other evidence before it can be accepted; that the conclusion drawn from the table that good work cannot be done during the hottest part of the day is not valid; and, finally, that the conclusion that the forward and backward running on any section of the line should be made on the same half day is utterly contrary to the author's own statement of the nature of the error supposed to have been manifested in the table.

During What Hours of the Day should Leveling be Done?—The author is not alone in his contention that the hours of observing should be limited to the beginning and end of the day, and that the middle and heated half should be avoided. Such a proposition is put forward in many places in the literature of leveling, both by direct statement and by the indirect statement involved in the fact that the hours of leveling were actually so limited. The question as to whether the hours of work should be so limited is important, because it is one of the principal factors in determining the speed of the leveling per month and its cost. If the hours of work are extended say from 5 to 8 hours during each day, on an average, the speed will be affected in that ratio and the cost in the inverse ratio, since the main portion of the expense of the party is a fixed amount per month. In view of this it seems that no sufficient proof of the necessity for limiting the hours of work has ever been put forward in so far as the writer is aware.

In general, less vibration of the images of the rod as seen through the telescope may be expected during the early and late hours of the day, or at least the vibration will be less violent than during the middle portion of the day. Smaller errors of reading of the rods may, therefore, be expected during the early and late hours than in the middle of the day. But such reading errors are compensating in their nature, and are of minor importance. The most troublesome errors are those of the cumulative class.

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It can hardly be contended that there is less danger of errors due to large or rapidly changing refractions during the early and late parts of the day than during the middle, for it is a well-known fact that the refraction is varying most rapidly during the early and late hours, and that it is also much larger during those hours than during the middle of the day. It would seem that there is less chance of encountering errors due to refraction when observations are taken during the middle portions of the day, when refraction is nearly constant and is nearly a minimum, than during the early and late hours.

It might have been urged with considerable force that errors due to unequal temperature changes in the instrument might be expected to be smaller during the early and late hours than during the middle of the day, but, so far as the writer is aware, such an argument has never been put forward in the past, when it would have been valid. If brought forward now it will be of little weight, since it is reasonably certain that such errors can be kept within very narrow limits by proper construction of the instrument and proper methods of using it. Everything being considered, it would seem that the best plan of work is to continue the leveling throughout the day. Such a procedure will be very effective in increasing the speed and decreasing the cost. It will secure a much wider divergence of conditions than the plan of working only at the two ends of the day. The writer believes that it is true in leveling, as it is true in general, that the truth is to be reached by observing under all possible conditions and combining the results rather than by arbitrarily selecting one set of conditions.

The Principle of Least Squares.—The workman who does not understand his tool is apt to see more faults in it than one who has mastered it. The author condemns the principles of least squares as applied to leveling, except when used to compare the accuracy of different lines. The writer freely concedes that many absurdities have resulted from supposed applications of these principles. He believes, however, that the responsibility for these absurdities may always be traced to the users of the principles. The method of least squares is a most powerful instrument in the hands of an expert, but a dangerous plaything in other hands.

Certain palpable mistakes, in the application of least squares to the problem in hand, catch the eye at once. The formula given on page 95, $\pm r_o = \sqrt{\Sigma r^2}$, in which r_o is the probable error of the elevation of a point as found from the initial point, and the separate r's are in each case the probable error of the mean difference of elevation of the bench-marks at the end of a stretch. It is then stated that "This formula is applicable only when all stretches are of equal length, though it is generally used without regard to the actual weights resulting from unequal lengths." The statement is a positive contradiction

Mr. Hayford. of one of the fundamental principles of the theory of errors, that the probable error of the sum of several quantities is equal to the sum of the squares of the separate probable errors, regardless of the relative weight of the separate quantities. Again, it is stated (page 102) that:

"Three single lines would yield a theoretical accuracy of 1.4 times that obtained from two lines, * * * and four lines would give an accuracy of 1.7 times that from two lines."

The theoretical accuracy increases as the square root of the number of measures of single lines, and the foregoing ratios, therefore, should be $\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{4}{2}}$ or 1.2 and 1.4, instead of 1.4 and 1.7 as given.

The statement that, "When a line of levels closes on itself, thus forming a polygon, the resulting error of the polygon must be distributed over the entire periphery (either in proportion to the length, or, perhaps better, in proportion to the square root of the length, from the known starting point)" given on page 111, is evidently the result of a misconception of the principles under discussion.

To distribute such an error in proportion to the square root of the length from the known starting point would be an absurdity and would give two results which would vary widely from each other when the distribution was made first in one direction around the circuit and then the other. It is stated on page 106, that when the theory of error is applied to the comparison of groups of observations, "each group must contain the same number of observations."

In view of the foregoing quotations the condemnation of the theory of errors as applied to precise leveling (pages 106 and 107), and especially the condemnation of the method of least squares, as applied to the adjustment of a level net, should be given but little weight.

Probable Errors and Limits of Error.—The computed probable error of a single kilometer is a true measure of the relative magnitude of the errors belonging to the accidental or compensating class in different lines. Such comparisons are misleading, however, unless it is kept clearly in mind that the accuracy of the line of levels depends both upon the compensating and the cumulative errors in that line. The errors of the second class are usually much more serious than those of the first, in precise leveling. The probable errors on two lines being known, judgment should be suspended until the evidence as to the magnitude of the cumulative errors has been weighed. For example, the first two groups of lines shown in Table No. 21 have average probable errors which do not differ greatly. An opinion of the relative accuracy of these two groups based upon this fact would be grossly at fault, however, since the average cumulative error in one of these groups is many times that in the other. The best evidence of the magnitude of the cumulative errors in any class of leveling is furnished by connections with tide gauges, large circuits of leveling of that class, or numerous connections with leveling of some other class.

Similarly, to appreciate the true significance to be attached to a limit of error, it should be noted that it is essentially a specification that the probable errors must not exceed a certain value. As such, it limits the errors of the compensating class, and does not affect the far more serious errors of the cumulative class. Mr. Hayford.

Speed and Accuracy of Coast and Geodetic Survey Leveling, 1899–1900.—The probable errors of the leveling of a single kilometer, computed on the same basis as the values in Table No. 21, were, for three C. & G. S. parties in 1899 on three lines aggregating a total length of 1 300 km., or 800 miles, $\pm$ 0.86 mm., $\pm$ 0.8 mm. and $\pm$ 0.65 mm. The office computation of the leveling of 1900 has not yet been begun, and the 1900 values are, therefore, not available, but an inspection of the field computations indicates that the probable errors will be about the same as in 1899.

The foregoing values do not represent the best that the instrument and method will do, for the following reasons: Knowing that the probable error and the limit of error indicate simply the magnitude of the accidental or compensating errors, while the serious errors to be feared in the leveling are of the cumulative class and the compensating errors comparatively unimportant, it seemed that instructions to the observers should be to use all possible efforts to guard against cumulative errors, but that no more time or money should be expended upon the work than was necessary to keep the accidental or compensating errors within the limit indicated by a computed probable error of $\pm$ 1 mm. for a single kilometer. The limit 4 mm.$\sqrt{K}$ (in which K is the distance between adjacent bench-marks in kilometers) was chosen as being about right to insure this degree of accuracy, in so far as compensating errors are concerned. The observers were not, however, left free to do any class of work they pleased, subject only to the condition that they must keep inside of this limit. On the contrary, they were told to crowd the limit, so to speak, by lengthening the sights so as to secure increased speed. When two of the parties observed in such a cautious manner that little or no re-running under the limit stated was necessary they were promptly reminded that they were not following the spirit of the instructions, that too much caution was being used, and that the sights should be lengthened. The effect of the instructions was, therefore, virtually to make the probable error fall between $\pm$ 0.6 and $\pm$ 1.0 mm., and to require the observer to use his skill and judgment in increasing the speed of the leveling rather than in reducing the probable error. From the foregoing it will be seen that the probable error secured and the speed attained must necessarily be considered together.

The general experience of all organizations which have done much precise leveling indicates that the real accuracy of the leveling depends upon the success in eliminating the cumulative errors rather than in

Mr. Hayford. reducing the accidental errors. The question may be asked: Upon what is based the writer's opinion that the leveling of 1899 and 1900 is of a very high degree of accuracy, and that in it there are no large cumulative errors? Since, as indicated above, and elsewhere in this discussion, the probable error computed from the residuals on the separate sections is not considered a measure of accuracy, the writer does not feel absolutely certain of the grade of accuracy of the 1899 and 1900 leveling, and will not feel absolutely certain until such leveling has been subjected to severe tests by being involved in many circuit closures. Nevertheless, the confidence that this leveling, when subjected to such severe tests later, will not be found wanting, is based mainly upon four facts stated as follows, in increasing order of importance:

1st. A careful examination of the 1899 leveling by use of the notes as to weather, direction of sun, direction of wind and direction of line, taken by the parties during that season, shows that the temperature errors of the form discovered in the old leveling are exceedingly small, if present at all, in the new leveling.

2d. Three long loops of single line were run in Ohio in 1899 by this method, their circumferences being 82, 112 and 87 km., and the closing errors —4.9, +9.0 and +9.3 mm., respectively. This is a very severe test of the method, and was made still more severe by running cross-lines dividing these three loops into nine small loops.

3d. Two of the long lines run in 1899, namely, the line from Gibraltar, Mich., to Cincinnati, Ohio, and the line from Abilene, Kans., to Sioux City, Iowa (finished in 1900), are now involved in large circuits, and the results indicate the correction required by each to be exceedingly small.

4th, and most important: It must be noticed that the method of observation resembles very closely in all essential features that which has been used by the Corps of Engineers for many years, and of which the results, judged by the closing errors of large circuits, the most severe test which can be applied to any leveling, are unexcelled in accuracy up to the present time by any leveling in any country, so far as the writer is aware.

It may be noted that in the preceding paragraphs no reference has been made to the divergence between forward and backward lines as a measure of accuracy. The writer has failed to observe in the published results of leveling any relation between the magnitude of the divergence of forward and backward lines and the accuracy of such lines as indicated by the severe tests furnished by circuit closures, and he, therefore, believes that the magnitude of such divergence should be given little weight in measuring the accuracy unless the divergence is excessive. It is, perhaps, necessary to call attention especially to the fact that this statement is limited to lines which are run by the

forward and backward method, in which the effect of systematic set- Mr. Hayford.
tling or rising of the rod supports is given an opportunity to manifest itself and is eliminated from the results. It is not applied to double lines in which both the components are run in the same direction. The average divergence of the forward and backward lines for the three C. & G. S. parties in 1899 were, respectively, 0.20, 0.51 and 0.08 mm. per kilometer of double line. Divergences of the same magnitude as these may be found in many lines of the Corps of Engineers which have subsequently been shown by circuit closures to be unsurpassed in accuracy. It is also in point to note that one of the 1899 lines, having a divergence of 0.51 mm. per kilometer, is now involved in the circuit, Abilene—Kansas City—Sioux City—Abilene, which has closed so well as to leave no doubt of the accuracy of the line.

In stating the rapidity of observation, the number of minutes per station, speed per hour and speed per month offer convenient bases of comparison. Seven minutes per station, which is said by the author (page 112) to be the time required to walk between stations, set up the instrument and take the readings, under exceptionally good circumstances with clear steady air, seems to correspond to about the average speed for C. & G. S. parties, even though they work throughout the day. On June 20th, 1900, Mr. O. W. Ferguson, with a level of the 1899 model, observed between 7.10 A. M. and noon, and 1.45 and 6.15 P. M., total, 9 hours and 20 minutes, and during that time occupied 111 stations, or an average of 5.0 minutes per station. The length of sight on this day varied from a little less than 40 to about 80 m. On September 26th, 1900, the party under Mr. W. H. Burger, using a level of the 1900 model, was engaged in running a line over very rough ground, on which it was necessary to take very short sights. The period of observation was from 7.30 A. M. to noon and from 1.40 to 5 P. M. The total number of stations during the working period of 7 hours and 50 minutes was 120, making the average time per station 4.6 minutes.

The writer does not know that these are the best examples that could be given. A more careful examination of the record may show still more rapid work. These are especially interesting as indicating how rapidly the instruments may be manipulated and the observations taken, in the very simple method now in use, and will probably bear comparison with work done with the ordinary wye-level.

A speed of one mile per hour is not at all unusual, and the record shows many days in the season when a speed of 1½ miles, equal to 2 km., per hour has been kept up for several hours, and even throughout the day. For example, a party in charge of Mr. B. E. Tilton, ran 16.6 km., 10.3 miles, of single line on July 14th, 1899, during a working period of 7.4 hours, or at an average rate of 1.4 miles per hour. The length of sight on this day varied from 80 to 110 m. with an average of about

Mr. Hayford. 100 m. The best single mile of leveling which has yet come under the writer's notice was run in 34 minutes by this same party.

The speed, in terms of completed miles of leveling per month—and this is one of the important factors in fixing the average cost of leveling per mile—was as follows for the three parties operating in 1899: In Colorado and Wyoming 214 miles were completed in 5.3 months, or at the rate of 40 miles per month; in Ohio 285 miles were completed in 5.8 months, or an average of 50 miles per month; and in Kansas and Nebraska 334 miles were completed in 5.3 months, or an average of 63 miles per month. The average for the three parties is 51 miles per month. The period of work given in each case is that embraced between the first day on which observations were taken and the last. These figures make the rate of progress in completed miles for each day that the party was in the field, making no deductions for Sundays, holidays, days lost on account of rain or other causes, 1.3, 1.6, and 2.1, respectively, or an average for the three parties of 1.7. A completed mile of leveling necessarily means at least 2 miles of single line, and, upon an average, represents a little more than this, on account of re-running of sections on which the discrepancy fell slightly outside the limit 4 mm. $\sqrt{K}$. Hence, the foregoing values indicate that the number of miles of single line per day, making no deduction for lost time, was about 3.4, and, assuming that loss of time on account of Sundays, bad weather, etc., was 25%, this means that the rate per actual working day was, upon an average, about 4.5, even though the first runnings of sections which had to be re-run are not counted. The average rate of progress for 1900 is not yet known, as two of the parties are still in the field. The present indications are that it will exceed the average speed of 1899 by 10 to 20 per cent.

Cost.—The actual cost to the Government, of the leveling done in 1899, is as follows, in round numbers: For the party in Colorado and Wyoming, $17 per mile = $10 per kilometer; for the party in Ohio, $14 per mile = $9 per kilometer; for the party in Nebraska and Kansas, $11 per mile = $7 per kilometer. The average of the three parties for the whole season is $13.55 per mile or $8.40 per kilometer. These figures represent the actual cost of the leveling, including the establishment of the bench-marks, with the exception of the first cost of instruments and the cost of stationery supplied to the parties. It includes all transportation to and from the field, paid by the Government, the cost of inspection of the parties, and all wages and salaries, including chief of party and recorder. The salary of each member of the permanent force is charged to the leveling for the whole period during which he was engaged upon work incidental to the leveling. It includes the time spent in traveling to and from the field, and, in one case, the time spent in preparing for the field. The Government pays its permanent employees for twelve months' work per year and

actually secures, on an average, only eleven months, the annual leave granted being one month per year. To take account of this fact, one-eleventh has been added to the salary actually paid each officer during the time he was connected with the leveling work. The present indications are that the average cost per mile, of the leveling in 1900, will be considerably less than it was in 1899. If the salaries of members of the permanent force, which are included in the foregoing cost, be deducted, and this is not unusual in computing the cost of leveling, the average cost per mile for 1899 will be found to be $9.33; and per kilometer $5.79. The field expenses per month for each of these parties were about the same. The principal factors which vary are the speed per month and the salary of the observer. The conditions under which the work was done, for these three localities, also differed considerably. The three values of the cost are, therefore, given, not as a comparison of the parties, but as an indication of the extent to which the cost per mile may vary, though the instruments and methods are identical. Mr. Hayford.

The cost of the leveling as given may be compared with that given in the paper (page 114), namely, $16 per mile, or $10 per kilometer, not including the establishment of bench-marks; or, including the establishment of bench marks, $15 per kilometer or $24 per mile. It may also be noted that this estimate by the author does not include the traveling expenses to and from the field. A comparison may also be made with the values given in the paper* by H. M. Wilson, M. Am. Soc. C. E., namely, the cost of from $19 to $32 per mile for leveling by the Engineer Corps. Also on the same page it is indicated that 265 miles of C. & G. S. leveling cost $3 900, or an average of $14.72 per mile (not $10.94 as there printed). This estimate of the cost of the old C. & G. S. leveling, namely, $15 per mile, is probably considerably less than the average cost of that leveling, salaries of observers being included. Also, on page 416 of the same volume, it is indicated that 1 000 miles of the leveling by the Engineers along the Missouri River cost about $20 per completed mile. Throughout the preceding paragraphs the cost is given for a completed mile of leveling, comprising at least two runnings, and should not be confounded with the cost per mile of single line.

In each of the comparisons suggested above if the details are examined it will surely be found that the comparison is biased against rather than on the side of the new C. & G. S. levels. It will be found that in general some of the items involved in the total cost to the Government have been omitted, as, for example, transportation to and from the field, or salaries of observers, or other items.

W. S. Williams, Esq.† (by letter).—This paper presents every Mr. Williams.

* *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 377.
† Surveyor in charge of Precise Leveling, Mississippi River Commission.

Mr. Williams.

detail of precise spirit leveling with clearness, and points out in a scientific manner those features wherein some of the older methods of precise leveling are at fault and introduce errors into the work. The paper also gives to the profession many facts which good observers have known for some years but have kept to themselves. In some cases, however, the author has made deductions from too few observations, and made a few statements which, with a larger experience in this line of work, he would not have been likely to make.

Rods.—The author is undoubtedly correct in the reasons given for the superiority of self-reading over target rods, and the writer would add one other reason why a self-reading rod is superior to a target rod: An observer is inclined to take longer sights with a target rod than with a self-reading rod, because he is prone to think he can bisect a target at long distances; whereas, if he has to estimate millimeters on a rod graduated to centimeters, he will not take such long sights. The writer's experience has been that short sights are conducive to accurate work.

Foot-Plates.—Not many experienced observers will agree with the author that foot-plates are entirely out of date. As to whether pins or foot-plates are the better for rod supports depends on the nature of the ground over which the levels are run. In stiff clay soils, grass roots, wooded lands full of roots and leaf mould, pins are the better; but for very sandy soil, railroad embankments of sand, or sand and gravel, and on hard roads and pavements, foot-plates are the better supports.

Refraction.—The author seems to have underrated the errors due to refraction as he says (page 56):

"Since it changes slowly and with some degree of regularity, and is noticeable only on comparatively long sights (500 m. or more), little or no appreciable effect is produced thereby on level readings, unless a considerable time elapses between taking back and fore sight readings late or early in the day."

Under some conditions, changing refraction will affect the readings very materially, even on sights as short as 50 m. Late in the afternoon on a sunny day the earth sometimes cools very rapidly. At such times the refraction becomes very erratic, and changes rapidly. A rod reading will change as much as 2 mm. in a very short time. Under such conditions an observer must wait until the air steadies down, turning from one rod to the other until he is sure to get both sights under uniform conditions; or, it may be that he will have to quit observing altogether. A light drifting fog will cause large and sudden changes in refraction. Passing clouds on a warm sunny day cause sudden changes in refraction, which must be guarded against. When the line of sight passes over a ravine through which cool air is moving, the reading is very much affected by refraction. If the line of sight passes near the ground on one sight and high above the ground on the other, the error

due to refraction will be considerable if the sun is shining; and such conditions are to be avoided, or the sights must be very short. This applies particularly to leveling on a steep grade. Mr. Williams.

A change in the temperature of the air, from any cause whatever, will cause a change in the refraction, and an observer should be constantly on the lookout for such changes between the back and fore sights.

Collar Inequality.—The old method of determining the error due to collar inequality is undoubtedly correct, the author's attempted demonstration to the contrary notwithstanding; but the writer has found it necessary, in using this method with a Kern instrument, to firmly support the ends of the bar directly under the wyes, and not depend wholly on the vertical axes to keep the wyes perfectly fixed while the telescope is reversed. This can be done very easily and quickly by driving large spikes to the required height, into the top of a large stump or other stable structure.

The author's new method of determining the error due to inequality of collars would be all right if he had not neglected to take into account errors due to refraction and due to the curvature of the earth when taking unequal sights. In the example given, the error due to curvature is 0.55 mm., and makes the constant error due to inequality of collars + 0.0544 mm. instead of +0.0624 mm. per meter.

Methods of Observing.—There can be but one objection urged against the author's method of observing, namely, that it requires about one-half more time than other methods which are just as good, as far as eliminating instrumental errors is concerned.

Equalizing the sights eliminates all instrumental errors and errors due to curvature and constant refraction more completely than any other process; hence, there is nothing gained by reversing the striding level and inverting the telescope in order to eliminate errors of inclination and collimation a second time, especially as it is extremely doubtful if these manipulations can always be made perfectly with any precise level.

The author's method gives double the number of observations, as compared with the ordinary method, and should, therefore, more completely eliminate errors of pointing and estimation, and give better results, if there are not other errors introduced by increasing the time of observation.

It is generally conceded that there are certain small errors due to changes in temperature of the different parts of the instrument during observations and to the change in temperature of the earth, such as the rising and settling of the rod supports and the instrument. Now, these errors will be increased in proportion as the time at each instrument station is increased, so that the method which will reduce the time of observation at each instrument station to a minimum, and

Mr. Williams. thus allow the stretch to be run in the shortest time, is the best, so far as reducing the errors due to temperature is concerned.

Following the methods used by the Mississippi River Commission, the average time required to walk between stations, set up the instrument, and take the readings, is about 5 minutes, under average conditions; whereas, the author's method requires from 7 to 9½ minutes for each station. The progress, therefore, will be about 1½ miles per day by the former method and 1 mile per day by the latter, and the author's method would increase the cost per mile about one-half.

The author is undoubtedly correct in stating that all errors should be eliminated by the observations themselves as far as possible, as few errors remain constant, and, therefore, corrections for these cannot be applied to results with any degree of accuracy.

During the season of 1900, two precise-level parties ran 230 stretches on the Upper Mississippi River, and the sights were kept so nearly equal that a correction for errors in collimation, inclination and inequality of telescope collars was applied to only four results, and in each case the correction amounted to only 0.1 mm., so that the results would not be appreciably affected if these corrections had been neglected altogether.

Rapidity.—The progress made by the parties working under the Mississippi River Commission for the past three years may be of interest when comparing methods, accuracy of work, etc.

The limit of closure on the following lines was fixed at 3 mm. $\sqrt{2L}$, L being the distance run between bench-marks, in kilometers. There were two parties working together, taking alternate sections of the line, on all this work except the line from St. Paul to Aitkin, Minn., where the writer had a single party. The time given includes all the days the parties were in the field, Sundays and holidays excepted; days lost by rain or otherwise are not deducted. The distances given refer to completed work, and always represent a double line, and in some cases more, where the line, on first trial, failed to close inside the limit.

TABLE No. 32.

Stretch.		Year.	Number of miles run.	Time, in days.	Average number of miles per day.	Average number of miles per day for each party.	Probable error per kilometer, in millimeters.
From	To						
Baton Rouge, La.	Mouth of South Pass.	1897-8	235	89	2.64	1.32	±0.68
Ft. Adams, Miss.	Baton Rouge, La.	1900	170	62	2.74	1.37	±0.62
New Orleans, La.	Biloxi, Miss.						
St. Paul, Minn.	Aitkin, Minn.	1898	222	136	.....	1.63	±0.54
Brainerd, Minn.	Lake Itasca.	1900	204	67	3.05	1.52	±0.59
Cass Lake, Minn.	Grand Rapids, Minn.						

Mr. Williams.

The probable errors in the foregoing lines were computed by the formulas given in the paper (pages 108 and 109).

Cost.—The cost per completed mile of the foregoing lines, where the data are available, is as follows:

New Orleans to Biloxi	85 miles	$16.00	per mile.
St. Paul to Aitkin	222 "	13.85	"
Brainerd to Lake Itasca } Cass Lake to Grand Rapids }	204 "	16.73	"

This estimate includes all items of expense, traveling expenses, express on instruments, bench-marks, etc., except cost of instruments and outfit.

The tile and pipe bench-marks, with brass caps, used by the Mississippi River Commission, cost about $3 each, delivered.

As the traveling expenses and express or freight charges are determined by the locality of the work, and are the same in any given case, regardless of the number of miles run, and as the cost of bench-marks depends on the kind and number used, an estimate of cost per mile, leaving out these items of expense, would be of more value to the engineer in making estimates of cost for proposed lines of levels. Leaving out these items the cost per mile for the lines already given would be $15.50, $12.65 and $15.25, respectively, in the order given.

Mr. Ockerson.

J. A. Ockerson, M. Am. Soc. C. E. (by letter).—This paper is very comprehensive, and Mr. Molitor deserves great credit for the exhaustive manner in which he has treated the subject.

The Mississippi River Commission has carried a line of precise levels from Itasca Lake, at the source of the Mississippi River, to the end of the Eads Jetties, at the mouth of that river. Side lines have also been run from Fulton, Ill., to Lake Michigan, at Chicago, and from St. Paul, Minn., to Lake Superior, at Duluth—a total length of something like 3 000 miles. Many of the features now recognized as good practice have been developed in this work.

The surface and sub-surface bench-marks were devised by the writer in 1882, and their utility is best shown by the fact that no material change has been made in them since they came into use. The necessity of having durable marks at frequent intervals was soon apparent. A few years' experience demonstrated that massive buildings settled, and natural ledges of rock disintegrated from frosts and sometimes were quarried for various purposes. Stone posts, projecting a foot or more above the surface, are easily destroyed by a blow or by forest fires. If the stone projects but slightly above the surface of the ground it is soon obscured by deposits of mud or other debris.

With these difficulties developed by experience, it was endeavored to substitute a mark which would obviate as many of the foregoing defects as possible.

Mr. Ockerson.

The bench-mark adopted consists of a sub-surface vitrified tile 18 x 18 x 4 ins., its upper surface marked by a suitable inscription which identifies it beyond question. A copper bolt projecting slightly above the surface of the tile is leaded into the center. This tile is placed from 3 to 4 ft. below the surface of the ground.

On the tile, and concentric with the copper bolt, is placed a 4-in. wrought-iron pipe 5 ft. long. The lower end of the pipe is flanged or flaring to prevent its being pulled up; the upper end is covered with a bronze cap fastened with a bolt, or, sometimes, riveted. The surface of the cap is also marked with suitable inscriptions, and has a center projection defining the exact point of elevation or location.

This type of surface mark has been copied by the United States Geological Survey, which obtained detailed drawings from the writer, and also ordered a large quantity from the firm making them for the Mississippi River Commission.

The practice, with the Commission, is to place upon the cap of the survey mark the latitude, longitude and elevation above sea level; so that any engineer or surveyor who wishes to use one of these marks has all the information before him.

The advantages of this mark are, the practical indestructibility of the tile, its large bearing surface, as compared with its weight, and the ready manner in which it can be profusely marked before burning. The pipe serves as an admirable witness mark, which can be used at all times, unless it shows signs of disturbance, in which case the cap can be removed and the tile reached through the pipe.

In the earlier work of the Commission, the limit of error was fixed at 5 mm. $\sqrt{2L}$, in which L is the distance run between bench-marks, expressed in kilometer units.

The limit of 3 mm. $\sqrt{2L}$ was fixed by the writer in instructions to precise level parties in 1892, and has been the prescribed limit ever since that time. On the whole, all of the work lies well within even this limit.

The results of this very large experience in precise level work show that accurate work requires that the time interval between fore and back sights should be reduced to the shortest practicable limit, in order to avoid the effects due to temperature changes in the instrument, changes in refraction, in rod supports, etc. Hence, the trained observers on this work will hesitate in accepting the author's method of reversals in observing (which serve to more than double the ordinary time interval) for the purpose of eliminating slight discrepancies which are already practically eliminated by equal back and fore sights.

The author and several who have contributed to the discussion make frequent mention of the leveling done by the Engineer Corps, but fail to mention the work of the Mississippi and Missouri River

Commissions,* both of which have done a large amount of high-grade work. Mr. Ockerson.

It is only just to say that the Assistant Engineers of the Mississippi River Commission were the pioneers in the development of methods of precise leveling which are to-day recognized as the standard for high-grade work.

These methods, coupled with the improved instruments designed by the Coast Survey, will doubtless mark another step toward accuracy and economy in precise leveling.

C. L. CRANDALL, M. Am. Soc. C. E. (by letter).—In discussing leveling instruments the author objects to the French Government level, Fig. 2, Plate II, on account of the complicated method of reflecting the light from the ends of the level bubble to the observer by means of prisms, and on account of lack of delicacy of the instrument as a whole. Mr. Crandall.

He incorrectly states that in reversing the level the prisms over the level must be refocused. There is no adjustment for focus, and the frame carrying the upper prisms simply requires turning 180° about a vertical axis. This requires no more labor than rotating the mirror used with the other types, while both ends of the bubble are seen near together, so that they can be carefully compared by moving the eye about 1 in. to the left, and looking in the same direction as through the telescope.

The milled head shown at the right of the upper prisms on Plate II, is connected to a pinion which engages with a rack from each prism, so that the distance between them can be changed symmetrically, with reference to the center, for different lengths of bubble.

In computing the sensitiveness of the level tube the author has apparently made a mistake and drawn erroneous conclusions. In Table No. 4, he gives, as the result of Reinherz's experiments, the mean error of centering a bubble in a tube with 9-second divisions spaced 2 mm. apart, as 0.27 second, and for 2.7-second divisions, with the same spacing, 0.135 second, or one-half as much. He then computes the corresponding rod uncertainties at 100 m. as 0.5 and 0.1 mm., respectively (one being five times the other), and condemns the first as being beyond the allowable error of closure for the distance, and accepts the second as being about equal to the error of reading the rod.

The rod uncertainties, computed from the angles, would be:

$$0.27 \times \sin. 1 \text{ second} \times 100\,000 = 0.13 \text{ mm.};$$
$$0.135 \times \sin. 1 \text{ second} \times 100\,000 = 0.07 \text{ mm.};$$

the first double the second, as it should be, but fairly near the error of reading, as stated above. On page 13, however, the error of read-

*NOTE BY EDITOR:—Since this paragraph was written, the author has added, in Table No. 21, some statistics of the levels run by these Commissions.

Mr. Crandall.

ing the Kern rod, graduated to centimeters, is given at from 0.2 to 0.5 mm., for distances up to 80 m., with a magnifying power of 50, as a minimum seldom reached by the best observers.

The 8.26-second level divisions of the French instrument thus do not make so bad a showing.

It is claimed by Messrs. Claye and Lallemand,* in support of the constants adopted for the French instrument, that the small but continual changes of temperature produce in the different parts of the instrument alternate expansions and contractions which put the bubble in a perpetual condition of instability, and render its centering at any instant difficult and uncertain, especially when the radius of curvature exceeds 60 m.

They estimate that the probable error of a rod reading, due to uncertainty in centering the bubble, is 0.1 mm. at 50 m., or 0.15 mm. at 75 m.

As to magnifying power, they claim that the weight of the instrument increases about as the cube of the power, and that when the power exceeds 30, the weight easily reaches 17 to 20 kgr. Moreover, the atmospheric vibrations in the lower strata, due to thermal changes, frequently cause so much movement of the image of the rod as to render uncertain the estimates of the fractions of a division; an excess of magnifying power will only increase this difficulty. A power of 25 is used, as compared with 50 for most of the instruments of Table No. 1, but the object glass is nearly as large as any on the list.

With this power, they claim to be able to read the French rod, graduated to centimeters, with a probable error of 0.33 mm. at a distance of 75 m., or as accurately as the author states that the Kern rods can be read with a power of 50.

They also claim that experience has shown that the errors due to instability of temperature, in their total effect, are at least as great as those of reading the rod.

This makes the probable error for a sight,

$$r = \sqrt{0.15^2 + 0.33^2 + 0.33^2} = 0.5 \text{ mm.};$$

and for a set up for a mean of the two sets of readings,

$$r_1 = \frac{r}{\sqrt{2}}\sqrt{2} = 0.5 \text{ mm.}$$

On page 46 the author states that, in the commonly accepted method of determining the inequality of collars, the telescope and level are both reversed for the second reading; and that the difference of inclination as given by the bubble, whether the level is or is not in adjustment, will be four times the angle between the axis of the collars and the elements of the cone enveloping the collars. Fig. 14 is referred to in proof of the statement, but in the figure the level tube

* *Encyclopédie des Travaux Publics. Lever des Plans et Nivellement.* Paris, 1889.

is shown to be in perfect adjustment, *i. e.*, the zero point of the curved tube is the highest point when the top line of the telescope is horizontal. Had the figure been drawn otherwise it would not have been so convincing. In fact, it is difficult to see how, when the legs of the level are kept in position on the collars of the telescope, and all reversed, the shape or size of the upper portions of the collars can have the slightest effect upon the reading of the bubble, the collars simply forming extensions to the legs of the level as if permanently attached to them; the 3.5 divisions out of level, shown in the example on page 47, simply indicating that the level with its extended legs was one-half that amount out of adjustment. Mr. Crandall.

The ordinary method referred to consists in taking a direct and reverse reading of the bubble, before and after changing ends with the telescope, or, in taking all the readings without reversing the level, since only the difference in inclination for the two positions of the telescope is required.

The assumptions as to the shape of the collars, the objections to the method, etc., stated on page 47, have been so fully discussed by Mr. Hayford that no further mention will be made of them here. The paper, as a whole, is a valuable treatise upon the subject of precise leveling.

DAVID A. MOLITOR, M. Am. Soc. C. E. (by letter).—The writer is greatly indebted for the facts and opinions expressed by those who have participated in the discussion of his paper on Precise Spirit Leveling. It is significant, however, that the men of wide and varied field experience have not taken the initiative, but have left the discussion principally to those who have never done any extensive precise leveling themselves, and whose connection with the subject is generally limited to the study of work done by others. It is difficult to understand this, unless it would indicate that the criticisms offered are on theoretical and prejudicial rather than on practical lines. Mr. Molitor.

In many instances, errors, which are scarcely to be avoided in a paper of such length, have been pointed out, and for this the writer feels very grateful. Also, many unnecessary comments have been made, as a result of superficial reading of the paper, a feature common to most discussions.

As stated in the "Introductory" of the paper, it was the writer's object to give a comprehensive treatise on the subject chosen. To do this necessitated mentioning all possible sources of error, and deducing a method of practical field work, which would give the most accurate results attainable. This, the writer considers he has done, but, to infer therefrom that he would advocate following his method for all purposes and under all circumstances is entirely unreasonable.

It is likewise unjust to offer the criticism that the paper is too

Mr. Molitor. theoretical and voluminous, considering the simplicity of the subject. The subject is simple only to those who have not fathomed its depths.

The fact that very good work may be done with single sights (without bubble reversals and telescope inversions) is known to the writer as well as to those who criticise him for not having chosen this simpler programme of leveling. Where the highest attainable accuracy is required, however, the same reasons which compelled him to abandon the older methods, and which are fully set forth in the paper, may at some time cause a similar change in the methods still used by others.

The writer's conception of the term "Precise Spirit Leveling" is, not to run the greatest number of miles of levels for the least money, but to do the most accurate work possible at a reasonable cost.

Everyone is at liberty to simplify the general method advocated for the achievement of the most accurate work to any extent suitable for a given purpose, but to do this intelligently a complete knowledge of the subject is essential, otherwise the short cuts may prove very disastrous, even for work of ordinary quality.

The scope of the paper is very broad, and is intended to point out the manner of increasing the accuracy of all level work, of whatever class, by applying to common levels such features of the precise methods as may be accepted without material increase in cost. These are: Equalization of sights; use of an umbrella; use of self-reading instead of target rods; to have on the instrument a level tube (with mirror) which may be used as a striding or fixed level at will; and to supplant the rigid wyes with swinging wyes, thus doing away with the wye adjustment of the common level. A further addition of value to the instrument would be a watchglass level for approximate setting up. All this may be had at a small additional first cost, and may be used without any material increase in expense, while the advantages gained would surprise the most skeptical leveler.

For ordinary work, it would not be necessary to read more than the middle thread, or to use more than one instrument position. Sights up to 300 m. would be possible for a magnifying power of 40 diameters, and a 6 seconds per 2 mm. level tube.

The writer has done some work of this class at a rate of a mile in 35 minutes, with an error in millimeters of not more than $5\sqrt{\frac{L}{2}}$. However, the roads and atmospheric conditions permitting of such work were far better than the average, and these figures are cited merely to distinguish between common and precise levels, even though lines run in this manner frequently close with ridiculously small errors.

On the writer's line of levels along the St. Lawrence River, the average length of shot was 49 m., and single shots varied from 6 to

80 m. The practical limit was determined by the profile of the line, and no increased rapidity would have been attainable even if long shots had been permissible. Mr. Molitor.

It has been demonstrated quite conclusively that long lines of common levels can be run with comparatively small discrepancies, compared with precise level work, though this is due to a compensation of errors which, for short stretches, might be surprisingly large. For precise level work, any portion of a line must be as accurate as the entire line, and to accomplish this is quite a different problem from obtaining an accidental small closure at the end of a long line of common levels.

On the other hand, many lines of so-called precise levels have shown cumulative errors which, for a long line, would give poorer closures than lines of good common levels. Hence, the methods certainly are responsible for much of this, and when a certain accuracy, excelling that attainable by perfection of methods and instruments, is required, it must be obtained by a repetition of observations. The writer has introduced this latter refinement to the extent which is considered practical.

Several of the discussors have taken exception to the writer's method of observing, and Mr. J. A. Ockerson says for "the trained observers on this work" that they would hesitate in accepting it. It might have been as well to have allowed the "trained observers" to express their own views, and facts from this source would have been far more valuable than opinions. The principal objection is stated to be the increased lapse of time between back and fore sight readings, which it is claimed is twice as long as for the method without reversals, etc.

A careful reading of the paper (bottom of page 80), the substance of which is repeated in other places, would have avoided the necessity for the present comments. The above-mentioned paragraph shows clearly that the time interval between each pair of back and fore sight readings (during which no reversals, etc., are made) is exactly as in the commonly adopted method. The instrument positions are changed after taking the first pair of sights and before taking the second pair, which latter sights are read in reverse order. This virtually constitutes a direct and reverse line between successive turning points, without increasing the time interval between successive back and fore sights, while the four readings furnish the data for a single line only.

When, therefore, the atmospheric conditions or refractive changes become so severe that duplicate pairs of sights are prohibitive, then the same will be true of a single pair of sights, though this fact is not observable for single sights until a stretch is closed. With duplicate pairs of sights such deleterious influences are immediately de-

Mr. Molitor. tected by mere inspection of the notes. In this way it frequently happens that work which has been good up to a certain point can be saved by suspending operations as soon as the conditions are seen to become unfit. This can always be done when the steel turning point pins are used.

A further advantage of duplicate sights is the almost infallible check on the correctness of the readings which the recorder exercises by mere inspection (a feature which Mr. Marshall calls distinctly bad), and the additional accuracy in reading and pointing derived from two observations.

Mr. Hayford further states that the writer's method requires reversing the striding level twice and inverting (wrongly reversing) the telescope twice at each station. Apparently, this is not the case, and precludes the necessity for the further argument that "the more an instrument is handled, the less accurate are the results obtained." While the truth of this argument is freely granted, the question here is essentially one of repetition of observations, and not of handling the instrument.

To deny the foregoing statements of fact would be equivalent to asserting that the mean of two or more observations is not as good as a single one. The repetition of observations on triangulation and astronomical work is scarcely more justifiable than in the case of precise leveling.

The practical value of the writer's method may be further illustrated by the following: The level line St. Regis-Oak Point, Table No. 21, with a probable error of $\pm$ 0.61 mm. per kilometer, on which a slightly different programme of reading was used, necessitated re-running 10% of the work, in order to fulfil the specification $3\sqrt{L}$, while, with the same limit and using the method advocated in the paper, the line Oak Point-Tibbetts Point gave a probable error of only $\pm$ 0.48 mm. per kilometer, and not a single stretch failed to close on the first pair. No such record as this has ever been cited in support of the value of a method, and it might be well for the "trained observers" and others of less training, to give this method an impartial trial before casting it aside with a few prejudicial remarks.

Regarding the question of rapidity and cost, it will be seen, presently, that the writer's method of duplicate readings does not retard the work nearly as much as might be supposed.

For an average length of sight of 50 m., working 7 hours per day and counting 22 days per month for actual work, the average rate of progress was 100 km., or 63 miles, of single line per month, which, for the wages paid, would aggregate $10 per kilometer, or $16 per mile, of double line, as per Table No. 24, for the Lake St. Clair levels, where the cost includes all expenses of every description. The same work showed a probable error of $\pm$ 0.33 mm. per kilometer, and the allow-

Mr. Molitor.

able limit was $3\sqrt{\frac{L}{2}}$. Had the weather conditions been more favorable, thus permitting of longer sights (as the ground was quite level), this work could easily have been accelerated from 30 to 50 per cent. But, taking the figures as they stand, they compare very favorably with any of those cited in the discussion for levels where the single sights were used and work was carried on throughout the day.

It should be noted that confining the work to the early and late hours of the day does not necessarily mean a reduction in working hours and consequent increase in cost, as Mr. Hayford supposes.

This matter can be justly dealt with only when all the influencing factors are accurately known, especially the wages paid, allowable limit of closure, kind of territory covered, working hours, etc. The figures given in Tables Nos. 23 and 24 are actual, while those given by Mr. Hayford, on average rapidity, are, to some extent, estimated; and the extraordinary cases, while interesting, are not representative.

The levels by the Geological Survey are hardly comparable with those of the Coast and Geodetic Survey or those of the Engineer Corps, U. S. A., for very evident reasons. It cannot be expected that the most accurate work can be done as cheaply as that done under much less rigid specifications.

The figures cited by Mr. Hayford for cost of recent Coast and Geodetic Survey work show no greater economy than those produced by the writer. The difference in cost is readily explained by the longer sights permitted under the Coast and Geodetic Survey, a fact decidedly against the production of high-class work. The cost of bench-marks may also vary from almost nothing to perhaps $10 each. On the writer's Lake St. Clair levels, the cost of bench-marks was not separated, and Mr. Hayford incorrectly assumed it to be the same for the St. Lawrence River line.

The comparative figures for cost per mile of double line then stand as in Table No. 33.

TABLE No. 33.

Coast and Geodetic Survey.		Mississippi and Missouri River Commissions.		The Writer's Levels.	
Wyoming, 1899	$17.00	Carrollton-Biloxi, 1882	$31.93	St. Lawrence River*	$22.00
Ohio	14.00	Keokuk-Fulton, 1882	19.08	Lake St. Clair	17.44
Nebraska and Kansas, 1899	11.00	Blair, Neb.-Dewitt, Mo., 1893	23.00		
		1 000 miles, Missouri River, 1893	20.00		
Work prior to 1899 was much more expensive, being nearly double the above figures		New Orleans-Biloxi, 1900	15.50		
		St. Paul-Aitkin, "	12.65		
		Brainerd-L. Itasca, 1900	15.25		
		Cass Lake-Grand Rapids, 1900	15.25		

* Wages about as given on page 114, or 20% higher than for Lake St. Clair levels.

Mr. Molitor. The theoretical relation between rapidity and accuracy for two lines of levels, supposing both to be on an economical basis, is that the accuracies shall be proportional to the square roots of the rapidities. The best Coast and Geodetic Survey work was done at the rate of 63 miles per month, and the Lake St. Clair levels at the rate of 31 miles per month. Hence the accuracies should be as $\sqrt{63} : \sqrt{31}$, or 8 : 5.6. The actual relation is 8 : 4, or decidedly in favor of the Lake St. Clair levels.

Table No. 34 may add some comparative data on rapidity not yet exhibited. For other data relative to these lines, see Table No. 21.

TABLE No. 34.—RAPIDITY OF PRECISE LEVEL WORK.

Time of running single line by single party.

Items.	Grafton-Keokuk.	Keokuk-Fulton.*	Fulton-Chicago.	Grafton-Chicago.	Gibraltar-Port Huron.	St. Lawrence River.	Lake St. Clair.
Length of line, in kilometers	241.6	272	272	785.6	117.6	197.2	41
Number of days in the field	94	81	98	273	85	135	23
Number of days worked	73	58	67	198	71	105.5	12
Average run per day in field, in kilometers	2.56	3.36	2.72	2.88	3.70	3.21	3.63
Average run per day worked, in kilometers	3.36	4.64*	4.00	4.00	4.06	4.11	4.46
Allowable limit of closure, in millimeters	$5\sqrt{L}$	$5\sqrt{L}$	$5\sqrt{L}$	$5\sqrt{L}$	$3\sqrt{\frac{L}{2}}$	$3\sqrt{L}$	$3\sqrt{\frac{L}{3}}$
Probable error, in millimeters per kilometer	±0.74	±0.67	±0.84	.....	±0.48	{±0.61 / ±0.48}	±0.33
Observers	J. B. J. and O.W.F.	J. B. J. and O.W.F.	J. B. J. and O.W.F.	J.B.J. and O.W.F.	O.W.F.	D. M.	D. M.

* A hand car was used on this work, thus facilitating progress.

The figures in Tables Nos. 33 and 34 show that duplicate readings do not materially affect the cost or progress of the work, but that the differences which may exist must be explained by the many other factors mentioned. The writer feels safe in asserting that he can duplicate the speed of any of the examples cited in the discussion when working under the same conditions and specifications.

The reasons given by Mr. Wilson,* which required the "best effort" "to state the facts as nearly as they could be derived from published reports," were sufficient to cause the writer to dispense with further data on the above subject. In most cases it is doubtful

* *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 376.

whether the cost implies single or double lines, or whether the salaries of observers are included. Mr. Wilson's paper was not overlooked. Mr. Molitor.

The writer will now attempt to answer as briefly as possible the questions brought out by the discussion and which have not already been covered in a general way by the foregoing.

To remarks by Mr. Horace M. Marshall, the writer would reply that precise leveling is not as new in this country as he intimates, in fact, when it was taken up by the United States Lake Survey, in 1875, not very much had been done in European countries. The best work has since been done in this country, and the method still generally used in Europe is that by bubble readings, recently discarded by the Coast and Geodetic Survey.

Mr. Marshall has overlooked the fact that the errors which the writer attempts most to eliminate by double pairs of sights are those due to refraction, temperature and pointing, and these can only be eliminated by repetition of observations. In discussing Mr. Wilson's paper, Mr. Marshall * favors re-reading the back sight after each fore sight and taking the mean to give a reading simultaneous with the latter. But what guarantee has he that the fore sight does not change, and would this not give the back sights double weight?

This point and a number of others depend merely on a definition of terms. What satisfies Mr. Marshall's specifications is not necessarily called precise leveling by the writer.

In common with others, Mr. Marshall imagines many difficulties arising from the use of 2-mm. rod graduations. They can certainly do no harm, even for his class of work, because the centimeter graduations are present in practically the same form as on the Kern rods. The 2-mm. graduations materially increase the accuracy of work with less strain on the observer, a fact which can best be ascertained by practice, but never by inference.

No readings are supposed to be taken on the 2-cm. triangular figures, as Mr. Hayford supposes; these serve merely to facilitate counting, and replace the numerals commonly used.

There is nothing curious about finding the work of the Vicksburg Engineer Office omitted from Table No. 21, when the data are not obtainable in proper detail.

The data furnished in Table No. 26 show what a person can do for himself when he wishes to attribute "foolishness" to others. Although the writer does not consider the application of least squares to leveling as free from criticism (page 106), it is not very sensible to compare errors of closure of loops, expressed in millimeters per kilometer, with the probable error of a pair of lines, as Mr. Marshall has done (page 119). Had he computed the probable error of his lines he would not have made such an elaborate display, which is evidenced by the

* *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 400.

Mr. Molitor. result, ± 1.1 mm. per kilometer found by Mr. A. E. Kastl for the stretch Grand Bend—Smithland of the Red River levels.

In reply to Mr. Wilson it may be said that the paper will probably not "befog" the careful reader in the conclusions which he is to draw from the voluminous theoretical discussion. On the contrary, had this been omitted, the paper would have been decidedly wanting in substantial proofs of the methods advocated.

Regarding the comparative value of self-reading and target rods, Mr. Wilson stands alone in his favoritism of the latter. In all probability, the thread interval, in the instrument used, was altogether too large to admit of proper results on self-reading rods.

The criticism offered against the steel turning point pin is unfounded, and no trouble was ever experienced in keeping the sockets clean, even when working over roads which were impassable by vehicles on account of mud.

No recommendation was made regarding the concealment of benchmarks, and both exposed and concealed marks were used by the writer.

The fear expressed by Mr. Wilson "that the imposing array of mathematical formulas," etc., ... "will deter civil engineers . . . . from attempting work of this class" is not exactly flattering to the ability of engineers to read understandingly; for, besides demonstrating the complexity of the subject, the writer has very clearly shown and illustrated its simplicity in practice.

The information given by Mr. E. G. Fischer respecting the material used in the new Coast and Geodetic Survey leveling instruments is highly interesting and valuable, and on these lines the Buff and Berger Precise Level, No. 2 768, might be considerably improved. However, the criticisms relating to the design of this level are not all correct.

The agate points referred to did not wear the collars by any measurable amount during a whole season's work, but, for fear of such an effect, the writer had them replaced by soft brass points. The change could not be regarded as any improvement, so far as was noticeable.

Mr. Fischer's interpretation of the writer's motive in having the wye contacts in different planes from the respective level tube contacts on the steel collars is altogether wrong. This recommendation was made to prevent any deleterious effect on the level tube contacts as a result of worn collars, or, in other words, the relation between the axis of collimation (as defined by the centers of the collars) and the level tube would remain unchanged, no matter how much the collars were worn on the wye contacts.

With a little field experience Mr. Fischer would see the advantage of this, and would understand why the striding level can be relied on

for indicating the direction of the line of sight to a much greater extent than can reasonably be expected for a fixed level tube. The cork or blotting paper for mounting the level vial is not objectionable in practice, though, in principle, the elastic metallic supports appear to be better. Mr. Molitor.

Mr. John F. Hayford has contributed a very lengthy discussion, and, while he has given the subject a great deal of consideration and study, he views things almost entirely from a theoretical standpoint. It is not surprising, therefore, that he should dissent from many of the conclusions arrived at by the writer through practice of a most painstaking nature.

Mr. Hayford deserves credit for having pointed out numerous errors in faulty diction and in some of the theoretical reasoning, though, in several instances, he incorrectly builds up arguments on such errors, and attempts to disprove facts.

In regard to the 1900 Coast and Geodetic Survey Level and the high tribute paid thereto by Mr. O. W. Ferguson, who has always worked with Kern levels previous to 1899, and has never used a Mendenhall level, and probably not even seen one, the writer questions the right to apply to the Buff and Berger Precise Level, No. 2768, the arguments holding good for a comparison made between the Kern level and the 1900 Coast and Geodetic Survey instrument. The writer has used both the Mendenhall and Kern levels, and can say with authority that the disadvantages cited in opposition to the Kern level do not in any way apply to the former, for which reason (as stated in the paper) the Buff and Berger Precise Level, No. 2768, or the Mendenhall level, was chosen in preference to all other levels described. There could be no reason, other than true merit, for favoring this instrument now, because the firm, Buff and Berger, is out of existence, and, so far as known, this style of instrument is not offered for sale at the present time.

All the objections advanced by Mr. Ferguson against the Kern level are due to slight defects in design which have all been remedied in the Buff and Berger Level, No. 2768, even to the extent of producing a mirror which shows merely the level-tube graduation without surrounding trees and houses.

The Kern level tube is held in position by clamps, and cannot be readily reversed or removed. The writer's programme of reading, therefore, is not well adapted to this instrument.

Besides leveling up, nothing farther is necessary to put the Buff and Berger instrument in shape for observing, neither does it require any labor to prepare it for carrying. The level tube is not held, but rests freely on the telescope collars, and is guided laterally by guides of the wye frame. The observer always carries the level tube and the umbrella, and the umbrellaman carries the bare instrument and tripod.

Mr. Molitor. In the general directions for the observers of the Coast and Geodetic Survey, it is required that, whenever a pair of lines fails to close within the prescribed limit, both lines must be re-run. If a third line should happen to form an acceptable pair (direct and reverse) with one of the first lines, why would these not be as good as a third and fourth line. The first pair of lines must have been run at different times (or else be simultaneous, which is not permissible), and there is no reason why the errors of one should reflect on the quality of the other. It is the writer's experience that the errors nearly always belong to one line (see Table No. 37), and the experienced observer can generally identify the poor one by examination of the notes and weather remarks.

Reading the rods to the nearest millimeter will be found entirely inadequate for high-class work, especially for sights shorter than 50 m.

Section 6 of the regulations covers the idea carried out by the writer in duplicating readings in reverse order, so far as this is possible with single sights.

Mr. Hayford favors continuing the work throughout the day. However, the kind of work which the writer has done permitted this only during cloudy weather and during the spring and fall months, but not during the warm summer months. There may be different conditions in other parts of the country which would alter this circumstance, but, so far as known, this obstacle has been encountered whenever high-grade work was attempted. Most of the work done under the Mississippi River Commission during the summer was run early and late in the day. This point is clearly covered on page 78, and careful reading will not admit of much difference of opinion.

The importance which the writer places on the careful equalization of sights is seen on pages 80 and 84 (level notes), though this is done as an additional safeguard, and no absolute reliance attaches to this feature for the elimination of errors.

The resemblance between the writer's and the old Coast and Geodetic Survey method is not so striking when it is remembered that the latter method is based on bubble readings which are afterward reduced to the horizontal, while the former necessitates horizontal sights with the bubble always in the center.

It is difficult to understand wherein the writer's method and the Buff and Berger Precise Level, No. 2768, seem to leave the way open to systematic temperature errors, when Mr. Hayford freely admits that such errors are not manifested by work under the Corps of Engineers with the Kern level, which is designed on the same lines as the former instrument. This statement is entirely illogical. If Mr. Hayford's assumption is correct, why, then, is not the work done with Kern levels subject to these errors? The principal changes made by the Coast and Geodetic Survey in 1898, were in the methods of observing and not in

the instrument. Why, then, credit the improvement in results to the instrument alone and not to the new method? Mr. Molitor.

Mr. Hayford advances an explanation for the cause of systematic temperature errors which were inherent in the old Coast and Geodetic Survey levels. He then claims that the work of 1899, done with the intermediate type of level (with striding level, etc.), and that done under the Corps of Engineers, with the Kern level, was in each case free from systematic error, and then expresses the belief that the writer's levels, run with the Buff and Berger level of similar design to these two, are subject to such error. If the intermediate level was free from systematic error, why did he discard the striding level, and design the 1900 instrument? Also, why does he cast reflection on the validity of the writer's experiments given in Table No. 6, when the results there given confirm his own theory?

In attempting to discredit the observations in Table No. 6, Mr. Hayford attributes the observed changes in the difference of elevation between Points 1 and 2 to the method of observing and errors of reading.

The method of observing cannot be responsible for the variety of results there given, because the readings were not subject to sudden and erratic changes, and ten sets could be taken in quick succession without obtaining greater differences than 0.2 mm. The readings changed only when the atmospheric conditions changed. All this is contradictory to the view so liberally expressed, that the time required to take a set of readings affects the results. This is true for the time interval elapsing between a back and a fore sight, because of possible refractive changes, but it is not true for a duplication of pairs of sights. The mean of two pairs will always be better than either one by itself.

The range of differences in the readings in Table No. 6 is between 1 and 2 mm. for a 30-m. sight read on a 2-mm. rod graduation, where, for 50 diameters magnification the apparent magnitude of the graduation is about one-half natural size. If Mr. Hayford can afford to call such differences errors of reading, he might as well designate precise leveling as nonsense. The same applies to the readings in Table No. 9, where the observed differences are still larger.

Mr. Hayford makes several partial or contradictory statements regarding small errors, some of which he calls negligible and others equally small which he takes particular pains to emphasize.

In speaking of the non-importance of changes in height of instrument resulting from re-leveling, when the wye cradle is pivoted at one end, he says that this might amount to $\frac{1}{10}$ mm. per station for the 1900 Coast and Geodetic Survey level, and that this is a negligible quantity. For a coarser level it might be several times this amount, but assuming the small quantity as representative and that the back sights are always read first, then for 50-m. shots this would produce a

Mr. Molitor. systematic error of 0.14 mm. per kilometer, and for 20-m. shots the amount would be 0.36 mm. per kilometer.

Also, in discussing the effect of heaving or settling of the instrument or rods he claims that such errors seldom exceeded 0.25 mm. per kilometer and were generally less than 0.1 mm. He also figures that this would amount to an average movement of from 0.007 mm. to 0.04 mm. per station, and concedes that either quantity is too small to be readable. In another place it is required to read the rod only to the nearest millimeter on a 1-cm. graduation. In this instance, however, Mr. Hayford is not satisfied with his conclusion, for he then asks: "Is it not true that this source of error is sufficiently dangerous to make it imperative that all lines should be run in both....directions....if the highest degree of accuracy is desired?"

The writer nowhere asserts that settlements of this magnitude are impossible, but he states that these would be quite immaterial to work where readings are taken only to the nearest millimeter, and adds here that he would not run direct and reverse lines for the purpose of eliminating such errors, were there not many other much more serious errors to be considered.

However, if duplicate lines are justifiable for the sake of eliminating such a minute and doubtful quantity, why, then, should it be permissible to pivot the wye cradle at the end, where the error is equally large and probably several times larger?

In giving Table No. 3, no assertion was made respecting the conclusiveness of the evidence presented; in fact, the subject is treated with considerable reserve, and the reader is allowed to draw his own conclusion.

Mr. Hayford, in citing the excellent qualities of the 1900 Coast and Geodetic Survey level for holding adjustments, concludes that a lack of such qualities in the Buff and Berger level prompted the writer to state that no reliance should be placed on the capability of a level tube for holding its adjustment.

While it is not disputed that in point of material the latter instrument might be much improved, and such improvement was advocated by the writer in 1898, by proposing the substitution of a wooden or glass tube for the bronze case of the level tube, yet, with the instrument as it is, no difficulty was ordinarily experienced in keeping the level in almost perfect adjustment, as is shown by the readings (see Tables Nos. 12 and 13), where the combined bubble and collimation error is measured for every sight. Also, to cite extreme tests, this instrument was last used in 1898 at a temperature of about + 50° Fahr., and was in good adjustment. It was then put into the box, transported 4 miles by wagon and 60 miles by rail; it then stood in a warm room for several days, and, finally, was transported in a farmer's wagon (without springs) for a distance of 36 miles over a rough frozen

road and used to run a duplicate line of 1 km. at + 10° Fahr., all without adjustment. Mr. Molitor.

There were many cases, however, when the bubble was adjusted several times during a single day, but this was done merely to keep the error within small limits.

Collimation was adjusted only twice during a whole season, and when it changed it did so suddenly.

Still, with these comparative facts in plain sight, the writer would not dispense with the striding level or the invertible telescope, for the very evident reason that, without these, there is no absolute assurance of the condition of the instrument except at the times when the errors are measured. Besides, a careful determination of the instrument errors is quite time-consuming and frequently unsatisfactory, for which reason such determinations were entirely abandoned by the writer in his field methods.

The 1900 Coast and Geodetic Survey level is, in principle, not superior to a dumpy level, in fact, it is so simple that it ceases to possess the virtues of a precise level. In saying this, the writer does not mean that good work cannot be done with such an instrument, but that it is difficult to determine its errors of adjustment, and a great deal must be taken for granted, which the results of careful work have taught the observer to regard with suspicion.

In this new instrument the thread error cannot be measured accurately, and the thread adjustment should not be changed in the field. Also, the level-tube error is not directly measurable, and the resultant error of the instrument must be determined by the so-called peg adjustment, which is perfect in theory, but very inaccurate in practice, unless the mean of many observations (say, 10 to 20) be used. This instrument error is measured only once daily, and absolute reliance is placed on the constancy of such error throughout the day. Were this combined bubble and collimation error observed continuously, as is done by the writer's method, it would be found to undergo periodic changes, a fact which passes unobserved by the Coast and Geodetic Survey instrument and method.

If the levels run by the Kern instrument are free from systematic cumulative errors, as is freely admitted, even though this instrument has a striding level and invertible telescope, why, then, was it necessary to avoid these advantageous features in designing the 1900 level, and add others which are distinctly undesirable? Would it not have been proper to first establish the fact that the cumulative errors found in the old Coast and Geodetic Survey levels were due to defective instruments, and not to faulty methods of observing? It might be interesting for Mr. Hayford to run a stretch of levels, both by the old and new methods, to determine this fact.

While the least probable error of any work by the Coast and Geo-

Mr. Mólitor. detic Survey in 1899 is ± 0.65 mm. per kilometer, showing a marked improvement over the former work by this department, yet such a value is among the high ones for work with the Kern level, and would scarcely be comparable with the writer's best work, where the probable error was ± 0.33 mm. per kilometer.

The writer, therefore, contends that nothing has been gained by the 1900 Coast and Geodetic Survey level, that the design is a step backward, but that the mechanical execution and materials used are worthy of imitation and may be accepted as a decided improvement over anything yet produced.

The erroneous statement (page 12), relative to the coefficient of expansion of paraffined rods, was inadvertently made, and has been righted.

Regarding the question of foot-plates, the writer would add that he has used both plates and pins, and could under no circumstances give preference to the former. The only case where the pin cannot be used is on bed rock, and here the plate is quite as useless, and the rods are generally held on small knobs cut with a cold chisel. The writer has worked over all possible kinds of country, and has never found a case where the pin did not give the highest satisfaction. Like experience is shared by others. This again does not mean that good work cannot be done with foot-plates, but it is intended to convey the idea that the pin is safer and less likely to be disturbed, and, consequently, saves re-running of work. The use of the pin is regarded as a step in advance, but nobody is compelled to adopt it on this account.

Mr. Hayford fails to see the advantage of placing the zero of the rod graduation above the foot of the rod. No advantage is claimed, but it is good practice, because the end of the rod is subject to wear and does not afford a good point for making comparative rod measurements.

It was not the writer's intention to involve "several generations of astronomers" and others in confusion over the subject of collar inequality. So far as precise levelers are concerned, the writer believes he has discovered a disparity between the theoretical solution of this problem and its practical application. As to the astronomers, it may be added that a discussion bearing on this point has been published by Dr. W. Jordan.*

The writer is indebted to Mr. Hayford for pointing out some very glaring errors on pages 47 and 48, some of which have been corrected by explanatory foot-notes. However, imperfect as this reasoning may be, the fact still remains that the common method of measuring the error due to collar inequality does not give the true inclination between the axis of collimation (as defined by the centers of the collars) and the axis of the level tube, except under certain conditions.

* "Grundzuege der Astronom. Zeit und Ortsbestimmung," Berlin, 1885, p. 46.

Mr. Hayford states that when corrections for curvature and refraction are applied, "the method shown in Table No. 9 seems to be correct, and should give a value of the collar inequality in agreement with that of the ordinary method * * * except for unavoidable errors of observation." He then quotes the various values found for this function at different times by the two methods, and concludes that "the great range of these values, and the fact that they are unusually large, is an indication that the concentration of the wear on the telescope collars at the four small areas has produced the effect which should be expected." Mr. Molitor.

In the absence of further proof, such a conclusion is wholly unwarranted, and displays a very unscientific way of disposing of the subject. As a matter of fact, the collars show no appreciable wear, which is visible or measurable by either of the methods named, both before and after the instrument was used for over 100 miles of levels by the writer, and following the method of double sights with level tube reversals. This, Mr. Hayford might have seen by inspection of the values which he quoted and which are repeated in Table No. 35 with dates of observation.

TABLE No. 35.

Date.	ε by Common Method.	ε by Writer's Method.	
		As observed.	Corrected for curv.
	Millimeters per meter.	Millimeters per meter.	Millimeters per meter.
Aug. 11th, 1898....	+0.0235	+0.0635	+0.0555
May 5th, 1899.....	+0.0276	+0.0624	+0.0544

Hence, whether, or not, the writer has succeeded in theoretically demonstrating the reasons for this disparity, the fact still remains that the two methods do not give similar results. For different instruments these values would, of course, bear a different relation to each other. The close agreement of the values found on August 11th, 1898, with those found on May 5th, 1899, after running 100 miles of levels, shows the minute wear which took place on the collars.

According to Mr. Hayford, the collars might have any regular shape without affecting the validity of the common method, so long as the angles α and β [Figs. 15 (a) and 15 (b)] are equal.

While this is all true for a theoretically perfect reversal of the telescope collars in the wyes, yet, in fact, such reversals would not be possible, except for circular collars, for which reason the assumptions on page 47 are cited.

Mr. Molitor. The function sought is the angle ε between the axis of collimation through the centers of the collars and the level tube axis. Supposing, then, that the collars were elliptical, how, in practice, would the common method determine this angle? Also, if the contact points between a collar, the level tube and the wye support are not in a plane, how can the theory apply to collars which have become worn from use?

Referring again to Figs. 15 (a) and 15 (b), and calling all dimensions linear distances for collars 1 m. apart, in which case ε, p, x, y, t, q, w and w' could also represent tangents of angles, in millimeters per meter. Then $p = R - r$ and

$$t = p \cos. \frac{\beta}{2} \cot. \frac{\beta}{2}, \; w = \frac{p}{\sin. \frac{\beta}{2}} = \frac{t}{\cos.^2 \frac{\beta}{2}} = p \sin. \frac{\beta}{2} + t,$$

$$\text{and } y = w - p.$$

Also,

$$q = p \cos. \frac{\alpha}{2} \cot. \frac{\alpha}{2}, \; w' = \frac{p}{\sin. \frac{\alpha}{2}} = \frac{q}{\cos.^2 \frac{\alpha}{2}} \text{ and } x = w' - p.$$

By assigning values to α and β, and assuming $p = 0.0276$ mm., the other functions may be found as indicated in Table No. 36.

TABLE No. 36.

$\alpha = \beta = 180°$.	$\alpha = \beta = 0°$.	$\alpha = \beta = 20°$.	$\alpha = \beta = 90°$.	$\alpha = 180°$ and $\beta = 90°$.
Mm.	Mm.	Mm.	Mm.	Mm.
$w = w' = 0.0276$.....	Infinite.	0.1596	0.0390	$w = 0.0390$ $w' = 0.0276$
$t = q = 0.0000$.....	"	0.1543	0.0195	$t = 0.0195$ $q = 0.0000$
$x = y = 0.0000$.....	"	0.1320	0.0114	$y = 0.0114$ $x = 0.0000$

These figures illustrate very strikingly the effect which changes in α and β have on the telescope pointing when the level tube is always horizontal.

The total angle measured by the level tube in performing the ordinary determination of collar inequality is $4\varepsilon = 4p + 2x + 2y$, and the angle between the collimation axis and the level tube axis is $\varepsilon = p + x$, which is one-fourth of the total angle only when $x = y$.

Hence, when α and β are unequal, then x and y are unequal, and $\varepsilon = p + x$ cannot equal $\frac{1}{4}(4p + 2x + 2y)$, which would be one source of error, though small in comparison to the discrepancy found. However, the foregoing may serve to point out the chances for errors resulting from irregular collars and the other factors mentioned on page 47.

To avoid any doubt regarding the interpretation of the common

method, one set of the observations made May 5th, 1899, with Buff and Berger Level, No. 2768, is given here in full: Mr. Molitor.

Telescope.	Level tube.	Bubble readings from center.			
Eye end N.	Direct.	N. end.	17.0	S. end.	16.4
" " "	Reversed.	" "	16.9	" "	16.4
			33.9 (+ 1.1)		32.8
			+ 0.275		
" " S.	Direct.	" "	13.1	" "	20.2
" " "	Reversed.	" "	13.5	" "	19.9
			26.6 (− 13.5)		40.1
			− 3.375		

Mean N. end reading minus mean S. end reading $= 4\epsilon = +0.275 - (-3.375) = +3.650$
or $\epsilon = +0.912$ bubble division.
The value of the level tube was 6.51 seconds, which gives $\epsilon = 5.937$ seconds, $= 0.02884$ mm. per meter.

The method here followed is taken from "Theory and Practice of Surveying."* See, also, example on page 47.

The conclusion to be drawn from this rather lengthy discussion is that, so far as the subject of leveling is concerned, the better and probably more accurate method to follow in the determination of the residual error resulting from collar inequality and possibly other sources is the one given by the writer in Table No. 9; except that a smaller excess of sights (say from 20 to 30 m. instead of 70 m.) might be chosen, to reduce the error of refraction and curvature to a minimum, though correction for this should be made in every case.

The criticisms of Table No. 16 are not proper, and the final figures in Table No. 31 are entirely unjustifiable, because the value 0.0635 includes curvature and refraction for 70 m., and the value 0.0276 is shown to be erroneous, although correctly determined by the common method for finding the error due to collar inequality.

It is admitted that the data presented in Table No. 6 are not entirely conclusive, a fact well known to the writer before publishing them. However, the conclusions stated on pages 61 and 103 were arrived at prior to making the set of experiments given in Table No. 6, by observing the closures on lines run during June, 1899, and the special experiments were really made to give more weight to these conclusions.

The writer had intended to make a much more elaborate set of experiments, by observing on a set of 8 points around the horizon (all equidistant from the instrument), for a variety of weather conditions and different methods of shading the instrument. This fond hope was never materialized, owing to the fact that suitable conditions did not present themselves during continuance of the work, and after comple-

* By Professor J. B. Johnson, page 574.

Mr. Molitor. tion of the line (Lake St. Clair levels) the party was disbanded, and no permission to indulge in farther investigations of this kind was obtainable during the writer's connection with the service. It is confidently hoped that such work may still be undertaken, and much valuable information may be expected therefrom.

However, as to the validity of the conclusions cited, Mr. Hayford's attacks are not altogether warranted, even in the absence of other supporting evidence. For, if the facts here supposed to be proven are erroneous, then the systematic temperature error found to exist in the older Coast and Geodetic Survey levels is likewise unsupported, for both show apparently the same thing, viz., both "tend to make elevations carried toward the sun too great, and *vice versa*." Why, then, should Mr. Hayford reject these experiments which show this, and accept merely his theoretical proof?

Again, if Mr. Hayford objects to basing such "important conclusions" on changes as small as 0.9 mm. on a 30-m. sight, how is he warranted in drawing any conclusions from errors amounting to only 0.25 mm. per kilometer, as he has done; and what would he designate as a measurable error? He also attempts to destroy the weight of the observations in Table No. 6, by pointing out a change in the thread interval, which would naturally occur on such a set of readings.

Each thread taken by itself will constitute a line of levels wherein the collimation is not in adjustment, except possibly for the middle thread. An examination of the table gives differences of elevation for thread 1, varying by 1.0 mm. between maximum and minimum values. Likewise, the middle thread 2 gives 1.3 mm. and thread 3 gives 2.8 mm. The average of these is 1.7 mm. The range for the mean values given in the table is 1.2 mm. Hence, while the thread distances did undergo changes, the mean values give a range, 1.2 mm., almost identical with that of the mean thread, 1.3 mm., when taken by itself. Therefore, the conclusions which the writer has based on the observations as a whole are not vitiated by the variations in thread distance.

Mr. Hayford further states "that there is no evidence put forward * * * to determine which of the many differences of elevation was the true one." While this was not discussed at length, it hardly seems probable that anyone would exercise such poor judgment as to assume that the mean of all the values is the best, when there are such apparent evidences of disturbances as those recorded in the remarks column and shown by other comments in the text. The first pair of sights together with the last five pairs might readily be accepted, as indicated by the readings, their mean values and the remarks. This mean is 287.93 mm. The other values, however, indicate decidedly the presence of disturbances, and there is every reason to reject them.

The indications are that the direct rays from the sun constitute the disturbing element. Hence, the claim that the true readings are those

TABLE No. 37.—Lake St. Clair Levels, June, 1899.

B. M.	Determined from:	Distance.	Direction.	Difference of elevation.	Resid. from mean.	Length of shot.	Excess of F. S.		Time of Day.				Weather.		Magnetic bearing.	Closing error.
							Dir.	Rev.	Dir.		Rev.		Dir.	Rev.		
									From	To	From	To				
		m.		mm.	mm.	m.	m.	m.								mm.
106	105	881	*D*	— 29.69	−0.95	40	+0.14	.....	8 A.	9 A.	.....	.....	clear.	.....	.....	....
			R_1	— 34.39*	.....	31.4	.....	−0.24	.....	.....	10 A.	12 M.	.....	clear.	N.	4.70
			R_2	— 31.60	+0.95	40	.....	+0.02	.....	.....	3 P.	4 P.	.....	clear.	.....	1.91
			M	— 30.64	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	....
110	109	1 634	*D*	+ 203.40	+0.26	63	+0.02	.....	6 A.	8 A.	.....	.....	clear.	.....	.....	....
			R_1	+ 210.80*	.....	43	.....	−0.02	.....	.....	3 P.	6 P.	.....	hot.	N. 35° E.	7.40
			R_2	+ 203.92	−0.26	45.4	.....	−0.28	.....	.....	7 A.	10 A.	.....	clear.	.....	0.52
			M	+ 203.66	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	....
P. 40	112	1 429	*D*	— 178.19	+0.25	55	+0.16	.....	5 P.	7 P.	.....	.....	clear.	.....	.....	....
			R_1	— 172.04*	.....	40	.....	−0.88	.....	.....	8 A.	11 A.	.....	clear.	N. 35° E.	6.15
			R_2	— 177.70	−0.25	51	.....	+0.06	.....	.....	6 P.	8 P.	.....	clear.	.....	0.49
			M	— 177.94	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	....
117	116	805	*D*	+ 367.07	+0.27	45	−0.50	.....	8 A.	9 A.	.....	.....	clear.	.....	.....	....
			R_1	+ 361.93*	.....	40	.....	−0.46	.....	.....	9 A.	10.30 A.	.....	hot.	N.	5.14
			R_2	+ 367.62	−0.27	50	.....	−0.38	.....	.....	3 P.	4 P.	.....	clear.	.....	0.55
			M	+ 367.34	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	....
122	121	1 086	*D*	— 740.41	−0.34	54	−0.12	.....	6 P.	7 P.	.....	.....	clear.	.....	.....	....
			R_1	— 736.52*	.....	38.8	.....	+0.24	.....	.....	8 A.	10 A.	.....	hot.	N. 30° E.	3.89
			R_2	— 741.10	+0.34	54	.....	+0.06	.....	.....	7 P.	5 A.	.....	clear.	.....	0.69
			M	— 740.76	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	....
124	123	1 263	D_1	+ 280.33*	.....	52	+2.00	.....	4 P.	6 P.	.....	.....	clear.	.....	.....	4.26
			D_2	+ 282.86	+0.86	52	−0.62	.....	5 A.	7 A.	.....	.....	clear.	.....	N. 30° E.	1.73
			R	+ 284.59	−0.86	57.4	.....	+0.14	.....	.....	6.30 P.	8 P.	.....	clear.	.....	....
			M	+ 283.72	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	....
126	125	888	*D*	−1190.28	+0.74	55.5	+0.32	.....	5 P.	6.30 P.	.....	.....	cloudy.	.....	.....	....
			R_1	−1187.43*	.....	49	.....	+0.44	.....	.....	4 P.	5.15 P.	.....	clear.	N. 75 E.	2.85
			R_2	−1188.81	−0.74	55.5	.....	+0.14	.....	.....	6 A.	7 A.	.....	clear.	.....	1.47
			M	−1189.54	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

* Rejected.

A. = A. M. P. = P. M.

Mr. Molitor. taken in the absence of this disturbance (as before sunrise or under dense clouds), is correct within knowable limits. This must be acknowledged, if any true readings are considered possible, and it was taken for granted that this fact was shown directly by the observations without further proof.

Another fact, not so clearly shown by the observations, but substantiated by many of the writer's lines, is that the temperature error manifests itself only during midday hours, and on most cool days (as in spring and fall) it is entirely absent. This is probably not a new discovery, for nearly all the high-class work done under the Mississippi River Commission, mostly by Assistant Engineers J. B. Johnson and O. W. Ferguson (see Table No. 21), was done early and late in the day, and this is the work so generally recognized as free from cumulative errors. Hence, the practice of the Coast and Geodetic Survey to continue work throughout the day, together with the older method of reading the level tube off center and reducing to the horizontal, may be accepted as very conclusive evidence of the causes for cumulative errors in the old work.

The writer's conclusions on page 61 are in every way verified by experience and supported by the observations in Table No. 6, as explained on page 60. Mr. Hayford's statement to the contrary, "that direct and reverse lines run during the same half of the day would close well," is covered by Conclusions 1 and 2, on page 61, where the time interval between the runnings is either zero, as for simultaneous work in two directions, or the interval is too short to allow the conditions to change. Usually, this is not the case when both lines are run by one party.

A careful reading of page 60 will leave little chance for doubt as to the validity of the writer's conclusions in this respect, though a few examples of lines showing the practical results are given in Table No. 37, in order to strengthen these arguments.

These stretches are not particularly selected, but represent all those of wide closures where constant errors might be expected. The following facts are striking:

1.—The rejected or bad line of a set is invariably the one where the shortest sights were taken, a condition made compulsory by difficult reading. Ordinarily, the shortest sights would give the most accurate results.

2.—All the rejected lines are those run nearest to noon, where the temperature effect is greatest. This fact enabled the observer to identify the bad line of each pair, so that a third line would always supply the data for an acceptable pair.

3.—There is a marked tendency for lines to close well when run during symmetrical portions of the day with respect to noon. Six out of seven stretches show this.

4.—Lines run during the same half of the day generally close wide. Five out of seven show this. Mr. Molitor.

5.—Lines run during the same half of the day and closing well, were both run either sufficiently late or early in the day, so that the presence of deleterious temperature effects was very improbable.

All these facts substantiate the conclusions on page 61, though they do not all agree with Mr. Hayford's theory regarding the direction of the line with respect to the position of the sun. Therefore, it would seem more probable that the heat effect on the instrument resulted in a curvature of the telescope rather than in a distortion of the level tube, in which case the countersunk level vial of the 1900 Coast and Geodetic Survey level would be no advantage. The writer believes that the only superior qualities of this new instrument are due to the material used in its construction.

Mr. Hayford's remarks respecting work throughout the day, will not stand, either in theory or practice. The working hours rarely exceed 8 hours of actual instrument work, and are generally less. However, 8 hours of the day can be utilized whether the midday hours be included or not; and the amount and quality of work possible in a day when the midday hours are excluded will, with rare exceptions, exceed that producible in a day when the midday hours are included and short sights become compulsory. These midday hours may vary from 9 A. M.–4 P. M. to 11 A. M.–2 P. M., depending on the weather and season of the year.

In reality, then, there is a decided advantage in confining the working hours to those which are conducive to good results, and then working hard, rather than to "peg along" during the hottest portion of the day and afterward be compelled to reject much of the work.

The statement "that it might have been urged with considerable force that errors due to unequal temperature changes in the instrument might be expected to be smaller during the early and late hours," etc., and that "such an argument has never been put forward," is not correct for this, is one of the strong points brought out by the writer in various places, and especially on pages 59 and 60.

The question of rapidly changing refraction occurring early and late in the day is rarely ever a source of much annoyance, except during unusually warm days followed by very cool nights, However, the writer's method of observing always reveals these errors when they exist, and the celerity with which back and fore sights are taken does not permit of any great changes in refraction. Vibrations are much more annoying and dangerous. Hence, the seeming objections are not nearly as great in reality as in theory.

The writer does not concur in Mr. Hayford's statement that "it is true in leveling, as it is true in general, that the truth is to be reached

Mr. Molitor. by observing under all possible conditions." This is true when one wishes to ascertain the laws of level errors, but, these being once established, work should be done only under such conditions as are favorable to the exclusion of errors. There are numerous facts on record to prove that selected conditions are in many instances the only ones furnishing acceptable results. All base-line measurements as well as most astronomical and geodetic work must be confined to special conditions.

The best teacher in matters of this kind is experience, without which the theorist will often go astray.

The writer freely admits that he has made a few mis-statements relative to the theory of errors, as pointed out by Mr. Hayford. These have been rectified in the paper, except the one relative to the formula $\pm r_o = \sqrt{\Sigma r^2}$, on page 95, where it is stated that this formula is applicable only when all stretches are of equal length. Strictly speaking, this is incorrect; but the idea intended was that the probable errors of the individual stretches were absolutely unreliable, and hence, could not serve for the derivation of other functions as r_o.

Mr. Hayford says "the computed probable error of a single kilometer is a true measure of the relative magnitude of the errors belonging to the accidental or compensating class in different lines." This error involves the probable errors of the individual stretches found from the formula $r = \pm 0.674 \sqrt{\frac{\Sigma v^2}{m(m-1)}}$, where all observations are of equal weight. When allowance is made for unequal weights the formula becomes $r = \pm 0.674 \sqrt{\frac{\Sigma (p v^2)}{\Sigma [p] (m-1)}}$.

Referring now to Table No. 19, the probable errors of the two stretches, as found from the errors of the individual turning points, are $\pm$ 0.517 mm. and $\pm$ 0.32 mm. In Table No. 18, these same stretches have probable errors of $\pm$ 0.01 mm. and $\pm$ 0.11 mm., respectively, according to the usual interpretation of the method, as applied to leveling. Would Mr. Hayford accept the closing errors of these two stretches, and determine $r_o = \sqrt{\overset{2}{0.01} + \overset{2}{0.11}} = \pm 0.110$ mm., according to the process followed in Table No. 18, or would he prefer the value $\sqrt{\overset{2}{0.517} + \overset{2}{0.32}} = \pm 0.608$ mm., as would result from the probable errors in Table No. 19? Also, in the absence of the figures in Table No. 19, would he attempt to assign any weights to these observations by which acceptable values for the r's might be obtained?

In general, level observations are never of equal weight, and, frequently, both direct and reverse differences of elevation for a single stretch may be on the same side of the truth. Hence, by what

moral right could anyone discriminate between two observed values of a difference of elevation? Also, what, then, constitutes the worth of the theory of error in connection with the subject of leveling? The writer and many others fail to see any, though they may use it for want of something better. Mr. Molitor.

The foregoing stretches are fairly representative, and may safely be classed with work of the highest attainable accuracy, yet it is impossible, by any process of reasoning, to assign values for their probable errors. An outright guess would be entitled to as much weight as any theoretical value assignable.

Hence the real question, "what is the probable error of a pair of lines over a given stretch," cannot be answered; from which it follows that r_0 must remain a conundrum.

The writer does not wish to convey the idea that he is opposed to, or even finds fault with, the theory of errors in general, but it is merely to the special case of two observations that the application of the theory is not considered of practical value.

In response to Mr. W. S. Williams the writer would say that a more careful reading of the paper would have precluded the necessity of saying more on the subject of refraction errors. The paper abounds in references to this source of error (pages 55, 56, 57 and 78), and one of the principal reasons for the duplication of readings is to detect the presence of such errors, which single sights fail to do until a stretch is closed.

Professor C. L. Crandall has pointed out an error on page 42, in comparing the effect on rod readings by using a 2.7-second and a 9-second level tube. What the writer meant to show was the effect of a 0.1-division error, in these respective levels, on rod readings at 100 m. It would not have been fair to apply the mean error, as given in Table No. 4, to a single rod reading, where the bubble might readily be 0.1 division out of center.

Very sensitive levels have long been used by observers in Germany, Switzerland and the United States, and have given no such trouble as predicted by Messrs. Claye and Lallemand. Also the 50-diameter magnifying power of the Kern, and Buff and Berger levels has not increased the weights by such enormous proportions as claimed.

In his criticism of the writer's treatment of collar inequality (pages 46 and 47), Professor Crandall is correct, to the extent that the level tube should be reversed before and after changing ends with the telescope. However, his further conclusion, that the example on page 47 simply measures twice the error of the level tube, is entirely wrong, because, in this case, the level tube was in perfect adjustment. When there is an error in the level tube, this process would measure the combined error of level and collar inequality.

Professor J. B. Johnson, though not participating in the discussion,

Mr. Molitor. has pointed out a few errors, mostly in Table No. 21, all of which have been corrected in the paper as now printed.

It was the writer's intention to present the facts relating to precise leveling, and he hereby expresses his thanks and warm appreciation to those who have contributed their knowledge, and assisted him in achieving this purpose.

With the same motive, the writer felt called upon to write a comparatively lengthy closing discussion to set aright those facts which had been assailed by seemingly plausible arguments of doubtful weight.

A very valuable collection of the results of "Precise Leveling in the United States," by Mr. John F. Hayford, Assistant, Inspector of Geodetic Work and Chief of the Computing Division of the United States Coast and Geodetic Survey, will be found in Appendix 8 of the Report of the Survey for 1898–99.

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No. 889.

CANALS BETWEEN THE LAKES AND NEW YORK.

By JOSEPH MAYER, M. Am. Soc. C. E.

PRESENTED NOVEMBER 7TH, 1900.

WITH DISCUSSION.*

The reports of the New York Canal Commission on a barge canal between Buffalo and Albany, and of the United States Commission on ship canals 21 ft. deep and 30 ft. deep between Buffalo and New York, giving estimates of cost and other information, enable us, by gauging the probable benefits to be derived from them, to judge the merits of each of these enterprises.

There are advocates who sincerely believe that railroad transportation is cheaper than transportation in narrow waterways. Their opinion is based on the undoubted fact of the abandonment of many canals, and on State subventions received by others which are not abandoned but are doing an increasing business.

The notion that the free play of economic forces would induce private enterprise to build canals, as railroads are built, if they would really furnish a cheap means of transportation, has still great influence on many minds, and creates at least a prejudice which must be

*This paper was discussed with that of Mr. Wisner (page 224). The joint discussion will be found on pages 240, *et seq.*

overcome before they are willing to vote in favor of taxes for their construction.

There are also various enthusiasts, anti-monopolists, and other enemies of railroads, who are in favor of anything that they believe will destroy the monopoly of the latter, regardless of any careful weighing of costs and advantages to be gained.

One erroneous argument, often used by advocates of water transportation, consists in a direct comparison of average freight rates on the Lakes and ocean with the average freight rates of railroads. The average freight rate on the railroads of the United States was in 1899 0.724 cent per ton-mile, while the average freight rate on the Lakes and Atlantic was until recently about one-tenth as much. This comparison, however, is misleading, since a very much larger percentage of the lake freight consists of crude materials than is the case with railroad freight. On the Lakes, ore, grain, coal and lumber form the bulk of the freight, and it would be more correct to compare the average freight rate on the Lakes with the average freight rate on railroads for these articles only.

The average haul on the Lakes and Atlantic is very much more than on railroads; this is another reason why ton-mile rates are lower by water than by rail, since the average rail rates include expensive short-haul freights as well as through freights. If allowance is made for these differences it is found that the freight rate per ton-mile on the Lakes and Atlantic is about one-fifth and not one-tenth of the ton-mile rate charged by railroads for similar freight transported over similar distances, with favorable grades, which latter are always obtainable along water routes.

To obtain the average rates for water transportation it is necessary to take a number of years into consideration, since the variations from year to year are much larger than on railroads. Recently, all freight rates by water were excessively high, from transient causes, but it is certain that they will shortly recede to the level of the last few years, and probably lower, in consequence of the larger and more economical steamers which are now being built. This ratio of charges of 5 to 1, however, does not quite represent the ratio of cost, since the railroad charges include interest and profit on capital invested in line and terminals and the cost of their maintenance, while the terminal facilities of waterways, and the interlake channels with their lighting,

are partly provided by the public without charge, and therefore, the cost incurred ought to be added to the freight rates by water in the proper proportion to obtain a fair comparison with railroad rates.

The endeavor will be made in this paper to show that the arguments based on past experience with canals are inconclusive, and to give the real reason why private capital cannot safely undertake the construction of the canal here considered and count with certainty on receiving the benefits derived from it. It is a necessary condition for the success of human intercourse that he who confers a benefit on others receive an equivalent benefit in exchange. Wherever there is free competition between those providing the different kinds of benefits or goods, and between those desiring them, this offers the means of ascertaining what are equivalent benefits or what is the relative value of different goods.

Where such competition is absent it is extremely difficult, if not entirely impossible, to ascertain what would be an equivalent benefit for a service rendered, or, in other words, what would be a fair charge for the same.

On account of this impossibility of ascertaining what is a fair charge for services which in consequence of their nature exclude competition, public opinion is fluctuating on this point. In any business which is a monopoly seriously affecting the welfare of a very large number of people who can easily co-operate politically, interference with the liberty of charging what the traffic will bear must always be feared, and the probable profits can therefore not be estimated. All those benefits which can be rendered at very much reduced cost, if manufactured on an extremely large scale, limit or entirely exclude competition. All means of transportation belong to this class. A highway will facilitate transportation, and the cost of the unit of benefit rendered by it falls rapidly with the amount of traffic. A fair toll on a highway privately owned would be such as would give an equal return to the capital invested in its construction as that obtained by capital invested in competitive enterprises of equal risk, and managed with the same amount of skill.

This needs only to be stated to show the impossibility of ascertaining what is a fair toll when the highway is constructed without sufficient public control to show the amount of capital really expended,

not to speak of the difficulty of judging the risk and the skill used in the enterprise.

So many interests are affected by the amount of the tolls collected on highways, and the usefulness of a highway is so much reduced by the collection of uniform tolls of any size, that highways, where they have been built by private enterprise, have mostly been expropriated and the tolls abrogated.

Private railways are not quite as precariously situated as either highways or canals.

In a rapidly developing country, if a State regulates the railroad charges so that they pay no fair return on the capital invested, railroad building or improvement of facilities is stopped, and this is soon felt to be such an obstacle to further growth that public opinion becomes more favorable to railroads, and permits them to profit by the increase in traffic. It is further evident that competition very largely influences many railroad charges; they are therefore not arbitrary, and in consequence there is a strong element in public opinion advocating that they be left alone, which succeeds in greatly tempering public interference.

It is felt more and more that the only fair way to fix the compensation of the various factors contributing to the creation of public benefits outside the range of competition is to induce the capitalists to furnish their contribution, not by promising them the net earnings which they might get by managing the enterprise and charging what the traffic will bear, but by promising the smallest rate of interest for which the capital is obtainable in the open market, and compensating the various grades of labor by wages and salaries in the same manner as the capitalist would have to do. The *entrepreneur* is in this case the public, which is then at liberty to charge for the services rendered whatever is thought most advantageous for the public welfare.

A private company which would propose to build a canal between the lakes and New York would, in the present state of public opinion, probably be unable to obtain a charter, and, should it obtain one, it would be so entirely at the mercy of the public, in respect to the charges which would be permitted, that it probably could not raise the necessary capital.

The proposed project, if we consider now that recommended by the United States Commission, of a canal 21 ft. deep and 215 to

250 ft. wide at the bottom, through the Niagara River from Buffalo to Lasalle, thence by eight locks to Lewiston, then through Lake Ontario to Oswego, from there to Lake Oneida and through the Mohawk Valley to the Hudson, is a waterway that would permit free access from all the Lakes to the ocean, for vessels 550 ft. long, 58 ft. beam and 19 ft. draft; it would make them available in winter, and would thereby reduce cost of local transportation on the Lakes; it would connect the harbors serving the most productive regions of the United States with each other by an unobstructed waterway, avoiding transfer of freight from lake vessels to canal boats, and from these to ocean steamers.

There are only three waterways which approach this one in the nature and magnitude of the services rendered—the Suez Canal, the Sault Ste. Marie Canal and the waterway between Lakes Huron and Erie. While the Suez Canal saves thousands of miles of distance, or the two transfers and a short railroad haul, which would be necessary in its absence, the three other canals of this class save transfers at each end which, with the attendant charges or costs, the delays in loading and unloading, and the risks of damage to goods, are economically considered equivalent to thousands of miles of distance.

Isolated canals suffer from the disadvantage that their cheap transportation is overbalanced by the necessity of transfers for all the freight either coming from or going to points which they do not reach. They might be called transfer canals, while the four canals above grouped together are transit canals.

The canals of Europe, where they are successful, are parts of large systems, thereby increasing the number of points accessible without transfer; they stand, therefore, more nearly on an equal footing with railroads in this respect. They form an intermediate link between the isolated canals creating transfers and the canals connecting large systems of water transportation which save transfers.

This difference is sufficiently important to vitiate all inferences drawn from the failure of canals of one class to the prospects of those of another.

The cost of running a vessel though the proposed ship canal between Buffalo and New York is no mystery. The practicable speed is known by experience with other canals of similar dimensions. Leaving for later consideration the interest on cost of canal and its

operating expenses, it is sufficiently accurate to say that the cost per ton-mile of running a vessel through the canal is larger than that on the lakes in the proportion that the time required to pass through the canal is larger than that for an equal distance on the Lakes. The speed of the newer freight steamers on the Lakes is about twelve miles per hour; the speed through the St. Clair flats canal, of dimensions similar to those of the canal proposed between the Lakes and the Hudson, is over eight miles per hour, as found by actual measurements by the United States Canal Commission. To estimate the increase in cost of transportation due to this reduction in speed it is important to separate the total cost of transportation into two parts, the one due to the running of the vessel, which increases as the distance, and the other due to the expenses at terminals, which is dependent on the number of trips, and is not materially affected by an increase in the time spent while running. For trips between the Upper Lakes and Buffalo not more than two-thirds of the expenses of vessel owners are incurred for running the vessels between terminals. The ship canal between Buffalo and New York, proposed by the United States Canal Commission, has 102 miles of canal of standard section and 96 miles of canalized river in which the bottom width varies between 250 and 1 000 ft. If we assume for the whole length of restricted channel, amounting to 198 miles, a speed of 8 miles an hour, which can be exceeded in some parts of it, the time required for passing through this channel would be 24.75 hours, or the same as would be required for a distance of 297 miles on the open Lakes, an increase over actual distance of 99 miles. The delay caused by the 37 lockages would be equal to 23 hours, which would be sufficient to cover 276 miles in the open Lakes. The time required for going from Chicago to New York, an actual distance, *via* the proposed canal, of 1 367 miles, would be the same as that required for covering a distance of 1 742 miles on the open Lakes.

The average charge on the Lakes for low-class freight between Chicago and Buffalo was, in the years before the recent abnormal rise, below 50 cents a ton, of which 33.3 cents only is due to the distance to be covered, and 16.7 cents is due to terminal expenses incurred by carrier. If these terminal expenses of carrier remain the same, the only increase in total freight charge to New York over that to Buffalo will be due to the increased time required in going from Chicago to New York instead of to Buffalo. The increase is as 890 miles to 1742

miles, and the charge for transportation proper would, therefore, be 65.3 cents per ton instead of 33.3 cents. If we add to this the charge of 16.7 cents due to terminal expenses of carrier, we obtain 82 cents per ton as the probable future charge for low-class freight from Chicago to New York. This is at the rate of 0.60 mill per ton-mile for the actual distance of 1 367 miles.

This rate is nearly identical with the rate per ton-mile between Chicago and Buffalo ruling before the recent rise in lake freight rates, and corresponding to 50 cents a ton between the latter points.

This illustrates the great advantage of the absence of transfers; the terminal expenses, being distributed over a greater number of ton-miles, decrease the total rate per ton-mile as much as it is increased by the slower speed on the canal.

Any comparison of actual ocean, lake or railroad rates between points of widely varying distances will show, other things being equal, that the ton-mile rate decreases with the distance. Between San Francisco and Europe the ton-mile rate falls sometimes to 0.03 cent for grain and 0.02 cent for coal returning; while between New York and Europe ton-mile rates twice as high prevail. This clearly shows that a large percentage of the freight charge is due to terminal expenses.

The above rate of 82 cents per ton, Chicago to New York, is the charge toward which the actual freight rate might be expected to gravitate, after the profits in the business, which would at the beginning probably be excessive, should have fallen to a normal amount. The same rate, increased by 4 cents per ton for every hundred miles distance from New York, might be expected to rule between Chicago and places beyond New York.

Six days would be the time required for the transport of through freight between Chicago and New York.

This quick time, and the absence of transfers, would have the effect that a large amount of high-class freight could be secured, for which the present charge is, by rail, $6 to $15 per ton, according to class.

The freight charge for wheat, Chicago to New York, corresponding to the above rate of 82 cents per ton, is 2.46 cents per bushel. Through the present Erie Canal the charge for wheat, Chicago to New York, was in 1898, 4.82 cents per bushel. The saving would therefore be 2.36 cents per bushel in comparison with the cheapest and slowest

method of transportation now available. Since the freight tonnage of the New York State canals is only about 5% of that of the railroads crossing the State, it is more reasonable to compare the charges for freight through the new canal with the lake and rail and with the all-rail rates now existing between Chicago and New York. The lowest average lake and rail rates for wheat ruled in 1898, when they were 5.4 cents per bushel. The lowest average all-rail rate is given at 10 cents per bushel; 6 cents per bushel has, however, sometimes ruled from Chicago to the seashore.

In comparison with lake and rail rates, the saving on the new canal, offering far better service, would be 2.94 cents per bushel.

For wheat going to points on the Atlantic coast which cannot be reached by present canal boats, or to Europe, the transfer in New York could be avoided by the use of ocean steamers loading at upper lake ports. Not only the transfer charges in New York would thereby be saved, but the costs due to the loss of time of both carriers engaged in the transfer, and the total saving in comparison with present charges for transport from upper lake points to all points beyond New York and immediate vicinity, would be over 5 cents per bushel, or $1.67 per ton.

Advocates of a barge canal between New York and Buffalo lay much stress on the low cost of barges and lake vessels per ton of carrying capacity in comparison with ocean steamers, and by assuming that an ocean steamer or lake vessel would have to go at the rate of 4 miles an hour between New York and Buffalo, and would remain as long at terminals as a canal boat, they figure out advantages for a shallow canal with the present system of transfers. Their assumptions are, however, not justified by the facts. The disadvantage of ocean steamers arising from the necessity of a stronger hull than is required for lake steamers, and from the use of surface condensers has been greatly exaggerated by comparing the cost of fast and durable ocean steamers with that of much slower and short-lived lake steamers, without making allowance for these differences. The difference in cost that does exist in this respect is overbalanced by the fact that the ocean steamer can run for 12 months in the year, while the lake steamer is limited to an open season of 8 months. The interest charge per month of running time is therefore less for an ocean steamer than for a lake steamer of equal carrying capacity, equal speed and durability.

A lake steamer can give only transient employment to its *personnel*

during a season of the year when wages are high, while the ocean steamer can give permanent employment, and obtains, therefore, a cheaper service.

Ocean freight rates are, in consequence, rather lower than freight rates on the lakes. Canal boats costing only $8 per ton carrying capacity could not safely go beyond the canal. They would obtain in a barge canal a speed of only 4 miles per hour; the thereby increased cost of labor per ton-mile, and the delays due to transfers of freight more than balance the advantages gained by lower interest charge.

Should a canal of the kind proposed by the United States Canal Commission be built, the through freight business between the lake points and transatlantic countries would be done largely by medium-sized ocean steamers, while lake steamers and coasting vessels would do the through business between lake points and the Atlantic coast harbors, and barges might be used for local business. Even the local business done in barges could be done much cheaper in a ship canal than in a barge canal, because the practicable speed in the former would be twice that in the latter.

In regard to the amount of business which could be expected for such a canal, it need only be mentioned that were it finished now, 200 000 000 bushels of wheat and its products, and 250 000 000 bushels of corn, or 12 250 000 tons in these two items alone, would probably be transported through it in an average season. Adding to these, other grains, ore, coal, lumber, salt, stone, cement and the large amount of high-class freight which might be expected, it is evident that 20 000 000 tons is a low estimate of the probable freight per year shortly after its opening.

The traffic of the Sault Ste. Marie Canal has doubled every six years during the last twenty-four years; it was 25 000 000 tons in 1899. If we compare the productivity and population, or the tonnage visiting the harbors at both ends of the Sault Ste. Marie Canal, with those at both ends of the canal between the lakes and New York, we can hardly believe that the latter will have a smaller amount of freight than the former.

The Detroit River has at present an estimated traffic of over 40 000 000 tons. There can be little doubt that the more eastern canal, having the whole lake region at one end and the coasts of the Atlantic at the other, would soon reach and exceed this amount.

A comparison of the traffic of 8 500 000 tons of the Suez Canal, connecting a population of 700 000 000 in the East with one of 400 000 000 in the West, with the traffic of the interlake waterways, which mainly serves the trade of a population of only 40 000 000, gives a striking illustration of the small amount of trade between distant old world nations in comparison with a fraction of the internal trade of a part of the United States.

The internal trade of the United States is more than ten times its foreign trade. The internal transportation costs about twenty times as much as that for the foreign trade. This explains why these internal waterways secure such an enormous traffic in comparison with the population they serve. By far the largest part of the traffic through a canal between Buffalo and New York would be traffic between the Atlantic coast harbors of the United States and the interior of the country.

There can therefore be no doubt that the saving in cost of transportation would benefit the United States principally. The only benefit Europe would derive from it would follow after the higher prices for grain in the West had so stimulated production as to largely increase exports, and reduce European prices in consequence, and after the decreased prices of European products in the West had increased the demand for them, so as to produce a rise of prices in Europe.

Since the canal cannot be finished before six years have elapsed, any estimates of its traffic based on the present business of the interlake channels would have to be increased at least 50%, since this increase may reasonably be expected in their traffic during this time. It would therefore be advisable in construction to make provision for duplicating the locks in the near future.

The capacity per season of the canal proposed by the United States Canal Commission is 36 000 000 tons; its estimated cost, inclusive of deepening of interlake channels to 21 ft., is $200 000 000.

Allowing one-eighth of the cost for interest during construction, and taking interest at 3%, the yearly cost would be $6 750 000, plus the cost of maintenance, which latter is estimated at $2 350 000 per year.

A very moderate toll, in comparison with the saving in cost of transportation, would cover all expenses incurred by the nation. The

saving to the public in freight charges would probably average $1.50 a ton in consequence of the advantages which uninterrupted and quick transportation would offer to high-class freight.

The whole district, not alone along the canal, but the whole lake and Atlantic coast of the United States, would thereby become unequaled in its accessibility to raw materials and markets. The lake basin, having the cheapest steel of any shore, would then become the principal shipbuilding center of the world.

The rapid increase of business on the Lakes, as well as through this canal, would undoubtedly create a demand for greater depth in the lake harbors and larger cross-section in the interlake channels and in the canal, and this prospect should be kept well in view during the planning and construction of the same.

It might even be advisable, considering the improvements in lake channels and harbors which may be expected during the next twenty years, to build the canal 24 ft. deep, instead of selecting dimensions which will be just sufficient for accommodating the vessels which will exist in the Lakes at the time of its completion. This would give, from the beginning, the advantage of permitting greater speed than would be possible on the smaller canal. The larger locks to accommodate the vessels on this larger canal would also give a larger carrying capacity, which will almost certainly be needed before twenty years of its operation have expired.

To sum up: The ship canal between the Lakes and New York proposed by the United States Commission is a transit canal connecting without transfer two large systems of important harbors; it is therefore in the class of canals which have been most successful in the past.

It connects the two most productive regions in the world, and will therefore soon have a traffic equal to the proposed capacity of 36 000 000 tons a year.

The yearly cost of the canal will be $9 100 000, which is about 25 cents per ton of freight which will soon pass through it.

The saving in cost of transportation will average about $1.50 per ton in comparison with the cost of equally good service at present available. Most of this saving will accrue to the United States, which should therefore undertake its construction.

A toll adequate to cover all the cost will not materially interfere

with its usefulness, and will serve to distribute justly the burden of the expense on those who gain by its construction.

There is at present under way a survey for a barge canal of 12 ft. depth, the cost of which has been estimated at about $59 000 000 by the New York Canal Commission. It is proposed that the State of New York shall build it for the purpose of securing its commercial supremacy. The route is that of the present Erie Canal, with some deviations; the estimated capacity is 20 000 000 tons per season. It is hoped that the transfer charges at Buffalo and New York will be largely reduced in consequence of the building of such a canal. It is estimated that the saving in cost of transportation through the whole length of this canal, in comparison with the cost through the present Erie Canal, will be 61 cents per ton, which would, in the opinion of the Commission, amply compensate the State for its expenditures for construction and maintenance. It is not proposed to levy any tolls.

The difference in the cost of transportation on the present and improved canal is calculated by estimating the cost, running expenses and carrying capacity per season, of a new style of barge, with which there is no practical experience, and comparing this with the actual cost of transportation through the present Erie Canal.

This method is evidently subject to a great many errors, even if used by investigators who are not in any way prejudiced in favor of a particular result.

An illustration of the amount of difference between facts and estimates of this kind is furnished by the estimate made by Major Symons, M. Am. Soc. C. E., of the New York Canal Commission, on page 81 of a report to the Chief of Engineers of the United States Army in 1897, on a canal between Buffalo and New York. He there gives ten as the practical number of round trips per season between Buffalo and New York for a vessel of 20 ft. draft, in a canal 24 ft. deep, on the Oswego route. The United States Canal Commission gives sixteen as the practicable number of round trips per season between Chicago and New York, for a vessel of 19 ft. draft in a canal 21 ft. deep, on the Oswego route. This latter estimate is based on careful observation of the actual speed of similar vessels in a canal of the same dimensions as the one proposed between Buffalo and the Hudson, on the observed speed in the open Lakes, and on the actual performances of lake vessels. This gives

15 days 7½ hours for one round trip, of which 6 days are required for the forward and return passage between Buffalo and Chicago. This leaves 9 days 7½ hours as the time required, according to their estimate, for one round trip between Buffalo and New York, and would permit twenty-six round trips per season between these points, with 3 days to spare in the average open season.

The practicable number of round trips is the most essential element in all the estimates of cost of transportation in various kinds of canals between Buffalo and New York, made by Major Symons.

This throws considerable doubt on the accuracy of his calculations of the saving in cost of transportation to be expected from an improved barge canal.

But let us assume, for the sake of argument, that the estimate of the New York State Canal Commission, of the difference in cost of transportation in the present and the improved Erie Canal, is correct, and let us investigate whether the conclusion drawn therefrom, that it is advisable for the State of New York to execute the improvement, is justified by the premise.

If the public provides, at its own cost, a new means of transportation, or an improvement in an old one, cheapening cost of transport by a certain amount, various freight will be transported that would not have been transported at all in the absence of the improvement, and other freight is deflected from other means of transport, though the advantage thereby secured may be very trifling, but just sufficient to turn the balance in its favor. It is therefore a serious error to estimate the advantage to the public achieved by an improvement in the Erie Canal, by multiplying the difference in cost of transportation per ton in the old and new canal with the number of tons passing through the new canal, and subtracting therefrom the expenses incurred by the public for construction and maintenance. This method is only correct for freight which would pass through the Erie Canal in its present condition. This tonnage of the present Erie Canal is constantly decreasing, in consequence of the continually falling railroad rates.

The average saving on freight, either newly created or diverted from other transportation lines, will be liberally estimated if assumed to be 0.6 of the amount of 61 cents, or 36.6 cents per ton.

If the taxpayers of the State of New York are expected to provide

an improvement, it is reasonable for them to ask whether their benefit from it is likely to compensate them for the expense. An improved Erie Canal, lowering the cost of transport and the transfer charges, would undoubtedly largely increase the amount of freight passing through it, originating at, and destined for, points outside New York, but the saving on this part of its freight traffic would only slightly benefit New Yorkers.

Of the freight, either exported from New York State or imported into the State through the canal, half the saving in cost of transport might be expected to represent the benefit to New Yorkers. The following method will, therefore, give the balance of advantage and expense of an improved barge canal from the standpoint of New Yorkers.

Take the whole saving on the present amount of local freight of canal originating at, and destined for, points in New York State. The average distance traversed by this freight on canal is not known, but will not be underestimated if taken to be three-fourths of its length. The saving on this freight would therefore be three-fourths of 61 cents, or, say, 46 cents per ton.

Take half the saving of cost of transportation on the present canal business originating in, or destined for, points in New York State. If we estimate the distance traversed by this freight on canal as three-fourths of its total length, the saving to New Yorkers on this freight would be 23 cents per ton. For new business created by the improvement take 0.6 of the amounts above given.

Neglect the through business originating at, and destined for, points outside the State. Then compare the sum of advantages so obtained in one year with the yearly expenses incurred for maintenance and interest on cost of improvement. If there is a large surplus on the right side the State may be justified to commit the unavoidable amount of injustice which lies in collecting taxes in proportion to assessed value of property while the benefits are very unequally distributed among the whole population.

Should the total freight carried by the canal be equal to its capacity of 20 000 000 tons, the freight with origin and destination outside New York would be at least 10 000 000 tons; the remainder, not more than 10 000 000 tons, would partly have both origin and destination in New York, and partly only one of them. We can only

guess at the relative amount of these two parts; we shall therefore take them to be equal.

Of these latter two parts about one-eighth would have to be taken as the successor of the corresponding freight on the present Erie Canal.

The saving for New Yorkers would, therefore, be as follows:

On 625 000 tons of freight corresponding to the freight on the present Erie Canal, with origin and destination in New York, 46 cents per ton, equal to	$237 500
On 625 000 tons of freight corresponding to freight on present Erie Canal, with either origin or destination outside New York, 23 cents per ton, equal to	118 750
On 4 375 000 tons of new freight, with origin and destination in New York, 0.6 of 46 cents, or 27.6 cents per ton, equal to	1 207 500
On 4 375 000 tons of new freight, having either origin or destination in New York, 0.6 of 23 cents, or 13.8 cents per ton, equal to	603 750
The total saving to New Yorkers, due to the cheapening of transportation on the improved Erie Canal, if the tonnage on the same were equal to its capacity, would therefore be	$2 167 500

We have estimated that 10 000 000 tons would be through freight, saving two transfers, or benefiting by reduction in cost of two transfers, probably partly the one and partly the other; and 5 000 000 tons would be freight benefiting by reduction in cost, or avoidance of one transfer. Half of the benefit from reduction in transfer cost on these 5 000 000 tons would accrue to New Yorkers.

The present cost of these transfers is 70 cents per net ton for two transfers; if one-half of this cost were saved the New Yorkers would benefit to the extent of $437 500.

If we add this amount to the saving in the cost of transport, we obtain $2 605 000 as the possible saving which might be obtained for New Yorkers through the improved canal, should its business be equal to its capacity.

Against this stand the interest charges on the money spent on the improvement and the cost of maintenance, which latter may be estimated at $1 500 000 per year. The estimate of cost of the improve-

ment is $59 000 000. This does not include interest during construction and the first years of use, when business would be much smaller. If we add 12% to the estimate of cost on this account, we obtain $65 600 000 as the actual cost to the taxpayer; 3% interest on this is $1 968 000.

The yearly cost would therefore be about $3 468 000, against a benefit to New Yorkers of $2 605 000.

Should the business of the canal be only 10 000 000 tons, which seems to be the estimate of the New York State Canal Commission, the total benefit to New Yorkers would be about $1 500 000, at a cost more than twice the amount.

The above does not pretend to be an accurate estimate, as that is impossible; it is only intended to prove that even if the difference in cost of transportation on the present and improved Erie Canal were 61 cents per ton, if the improved canal would secure all the freight it could carry, and if it would be the means of reducing transfer charges one-half, none of which favorable assumptions is either proved, or even probable, the State of New York would not be justified in undertaking the improvement at its own cost.

This whole question must be looked at, not from the narrow point of view of the State of New York and its particular interest, but from that of the interest of the United States. Modern trade is regardless of State boundaries, and it is impossible to benefit the State without benefiting the Union, or to benefit the Union in this matter, without benefiting New York State.

By the fact that every point along the canal is practically a harbor, or can easily be made one, the State must always profit, as a center of manufactures, by any improvement in its waterway, more than either the shores of the Lakes or the Atlantic, since only comparatively few points along these shores, namely, those with harbors, are put in direct communication by this waterway.

The main service of this waterway does not consist in cheapening transportation along its course, though this also will be accomplished more efficiently by a ship canal than by any other, but in removing the necessity of transfers at both ends.

The present transfer charges are only the smaller part of the total cost incurred on account of these transfers. Transfer or transit means all the difference between slow and inefficient service, excluding all

except the lowest class freight, and in great danger of losing even this on account of the rapidly falling rail freight rates, and quick, safe and reliable service, by through transportation lines, at such low cost as to be beyond the reach of railway competition, either in quality of service or price for the same, between all the points which can be directly reached.

A ship canal between Buffalo and New York means the creation of one great system of water transportation out of two of half the size; it means that each harbor of each of the two systems will have on an average twice as many points to which it can ship directly by water as it had before. Its opening will be the signal for a renewed period of rapid increase in the shipping of the Atlantic coast as well as the lakes.

A toll of 20 cents a ton for each transfer saved by the freight passing through this canal would not amount to more than one-quarter of the benefit derived from this saving, if the improvement in quality of service made possible thereby is properly valued. It would cover all expenses incurred by the nation.

Since the main reason for the large cross-section of canal, and the great expense caused thereby, is the saving of transfers, it would be just to charge tolls in proportion to the number of transfers saved.

The proper course for New York State is, therefore, to abandon its policy of desiring a canal all by itself, and to join in hearty co-operation with the other interests desiring a ship canal.

If it refuses to do so, there is serious risk that the various interests which are principally concerned in having the cheapest possible communication between the West and Europe, will join forces to build a ship canal from the Lakes to Montreal, which could be built at a cost quite commensurate with the services rendered thereby; though these would not compare with those rendered by a canal through New York, which would also serve the much larger and more important trade between the West and the East of the United States.

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THE ECONOMIC DIMENSIONS FOR A WATER WAY FROM THE GREAT LAKES TO THE ATLANTIC.

By GEORGE Y. WISNER, M. Am. Soc. C. E.

PRESENTED NOVEMBER 7TH, 1900.

WITH DISCUSSION.*

The economic depth for canals of large traffic was discussed in the *Transactions* for June, 1898,† but as the data did not exist at that time on which to base comparative estimates of cost of canals of different dimensions for the routes under consideration, it was impossible to fix the limiting depth where the fixed charges for maintenance and interest on the cost of construction would make the actual transportation rate greater than for lesser depths of channel.

Plans and estimates have been made since then for regulating works to control the level of Lake Erie, and for constructing waterways 21 ft. and 30 ft. deep through the connecting channels of the Great Lakes, and from the Lakes to the Atlantic.

A preliminary estimate of a barge canal 12 ft. deep from Buffalo to New York has been made, and while the best location and exact cost

* This paper was discussed with that of Mr. Mayer (page 207). The joint discussion will be found on pages 240 *et seq.*

† *Transactions*, Am. Soc. C. E., Vol. xxxix, page 273.

cannot be stated until the surveys now being made under the direction of the State Engineer are completed, it is safe to say that the estimate made by the New York State Committee on Canals will not be changed sufficiently to materially affect the present discussion.

In previous discussions of the proposed waterway from the Lakes to the seaboard the depth of channel has been made the important factor and but little consideration given to the economy in transportation rates due to the different speeds which may be maintained by steamers in waterways of different dimensions.

For instance, in Major Symons' able report on a Ship Canal from the Great Lakes to the sea it is estimated that lake freighters of 16 ft. and 20 ft. draft would make only four miles per hour in a waterway 24 ft. deep, and that the time required at terminal ports would be from 11 to 12 days, or about the same as the sailing time of round trip; while in a barge canal 12 ft. deep, with a cross-section of 1 200 sq. ft., steam canal boats with three to five consorts in tow would be able to make practically the same number of round trips per season.

To anyone familiar with the speeds maintained by steamships in the artificial channels of our lake waterways, and with the rapidity with which cargoes are loaded and discharged at terminal ports, it seems like begging the question to draw conclusions from transportation rates based on such assumptions.

The St. Clair Flats Canal is 7 200 ft. long, 20 ft. deep at mean stage of Lake St. Clair, and has a cross-section of about 5 700 sq. ft., or 200 sq. ft. greater than the cross-section of the proposed 21-ft. waterway from the Lakes to the Atlantic for which plans and estimates have recently been made.

It has been found from actual observations that the large freighters on the Lakes can safely navigate the canal at speeds of from 7 to 11 miles per hour without slackening speed when passing other steamers in the waterway.

There is an average current of about 1.7 miles per hour in the canal, which makes navigation much more dangerous than it would be in a canal with no current except that required for the necessary water supply, yet no serious accident has ever occurred in the waterway.

Lake freighters, capable of steaming 12.5 statute miles per hour in the open lake, make from 23 to 24 round trips per season between Duluth and Buffalo, or an average of one round trip every ten days, three days

of which are due to time consumed in loading, unloading, coaling and making repairs at terminal ports.

If the detentions at the terminals of a ship canal between the lakes and New York should amount to ten or twelve days per round trip, it is safe to say that but little freight would be discharged at New York except that consigned directly to the city, a condition which would unquestionably develop methods of handling cargoes such that the detentions need be but little greater than at lake ports.

The Suez Canal had a cross-section of 3 700 sq. ft., previous to the enlargement now being made, through which the speed for large steamers was restricted to 6.2 statute miles per hour.

In the Amsterdam Canal, with a cross-section of 4 000 sq. ft., the speed is limited to 5.6 miles per hour.

In the Kiel Canal, having a cross-section of 4 100 sq. ft., a speed of 6.2 miles per hour is allowed, but the larger ships only average about 5 miles per hour.

Speeds of 6 miles per hour are allowed for large ships in the Manchester Canal, where the cross-section is 4 400 sq. ft., but, owing to the sharp curves in the canal, tugs are used for towing the larger steamers.

The limit of speed for these canals has been fixed with reference to preventing injury to the banks, but is not very different from the economical speeds which could be maintained by large steamers in waterways of such dimensions.

The retardation of the speed of a ship in a restricted waterway is due to the resistance of the sides and bottom of the channel to the movement of the water displaced by the steamer, to the velocity of the back flow of the displaced water, and to the back pressure of the water piled up in the channel in front of the moving ship.

It has been found from actual observations of the speeds of steamers on the Great Lakes that a ship capable of averaging 12.5 miles per hour in the deep water of the open lakes is retarded about 16%, or 2 miles per hour, when steaming with the same boiler pressure and only 2 ft. of water under the keel.

When the lake channels are improved so as to allow the passage of ships of 19 to 20 ft. draft, the cross-section of the St. Clair Flats Canal will be about six times that of a loaded freighter, and the mean back flow due to the movement of the ship will be about one-fifth of

the speed of the steamer. It is apparent, therefore, that where the cross-section of the waterway is less than six times that of the passing ship the speed will be retarded at least one-third of that which could be maintained in open water.

The dimensions which should be given to a waterway between the Lakes and the Atlantic depend upon the economic depth which can be obtained and maintained in the water routes to be connected.

The depth of channels at the entrances of lake harbors before improvement was only about 3 to 10 ft., and across the shallows of the connecting waterways about 9 to 13 ft.

The project of improvement now being carried out contemplates channel depths of about 21 ft. for mean lake stage, but as yet only about 18 ft. has been secured. The question to be settled is, what is the depth which can be economically given these channels such that ships able to safely weather the lake storms may be passed at all times, and that the fixed charges for maintenance and interest on cost shall not exceed the benefits to be derived from the reduction in rates of transportation and from developing new industries and commerce?

During the heavy storms which are of frequent occurrence in the fall of the year on the Lakes, large freighters drawing less than 17 ft. when loaded, have not been found well adapted for safe navigation, which fixes the minimum limit of depth of channels at about 20 ft.

For greater depth of channels than 20 ft. the advisability of improvement will depend upon whether the benefits to be derived will be equivalent to the fixed charges arising from maintenance and cost of construction.

At the head of Lake Erie, through Lake St. Clair, and at the foot of Lake Huron and Lake Superior there are extensive flats where the water is only about 21 ft. deep at average stage, and if the connecting channels should be made a greater depth, nearly 60 miles of the route through the rivers and lakes would be excavated channel.

The cost to the Government to deepen the lake channels for depths of over 21 ft. will be about $3 000 000 per foot, and to improve the lake harbors and entrances correspondingly will cost about $4 000 000 per foot, or a total of $7 000 000 for each additional foot of depth in harbors and waterways. Assuming that the Government pays 3% for the money expended on improvements and one-half of 1% for maintenance,

the annual fixed charges for each additional foot of depth would be approximately $245 000.

The actual freight carried on the Lakes in 1899 was not far from 40 000 000 tons, and if the annual fixed charges arising from making and maintaining deeper waterways should be chargeable to the entire amount of traffic, the saving in transportation rates would have to exceed six mills per ton for each foot of improvement to make the accounts balance.

The actual saving in the cost of transportation on the Lakes by using deeper draft vessels is only about one-half the above estimated fixed charges, and therefore the improvement would be advisable only on the supposition that the indirect benefits from developing new industries and new commerce will be of greater value to the country than the total decrease in the cost of transportation. This could hardly hold true as the decrease in rates would be permanent, while the effect of the improvement in developing new commerce would likely be of only a few years' duration.

It is true that the fixed charges arising from interest on the cost of waterway improvements and from expense for operation and maintenance are paid by the Government, but whether paid from Government revenues or from toll on traffic the net result is the same, and should be considered in the comparison of transportation rates, especially with those on railroads where such expenses are paid directly from the earnings.

If we take into consideration the fact that ships of more than 500 ft. length cannot be safely handled in the sharp curves of the connecting waterways of the Lakes, that with a draft of 18 to 19 ft., ships not exceeding such length can safely navigate the Lakes at all seasons, and that for greater depths of channels than 21 ft. the interest and maintenance account will exceed any expected returns from lower transportation rates, there seems to be no legitimate reason why the ultimate depth for lake waterways should not be definitely settled at once.

While it is true that the small freighters of the Lakes must go out of business and make way for the more economical type carrier, it is also true that a continual change of dimensions of waterways renders the smaller ships practically obsolete long before worn out, and since the rate at which freight can be carried depends upon the probable

useful life of the ships, continual changes in the type of freight carriers will materially increase the cost of transportation.

If the freight business on the Lakes can be conducted as economically with waterways 21 ft. deep as with channels of greater depth, the waterway from the Lakes to the Atlantic should not exceed such depth unless it can be shown that the benefits to be derived from through transportation between the Lakes and foreign ports in deep draft ships will be of sufficient importance to warrant the larger expenditure required to construct a canal of corresponding dimensions.

Tables Nos. 1 and 2 give the details of the length of channels of different dimensions, the amount of lockage, and the estimated cost of waterways 21 ft. and 30 ft. deep from Buffalo to New York *via* the Oswego-Mohawk and St. Lawrence-Champlain routes, and are abstracted from the report of Board of Engineers on Deep Waterways.

TABLE No. 1.—BUFFALO TO NEW YORK.

	*OSWEGO-MOHAWK ROUTE.		CHAMPLAIN ROUTE.
	High-level plan.	Low-level plan.	
Total distance, miles	477.04	476.94	685.21
Fall, regulated stage of Lake Erie to mean tide, feet	574.5	574.5	574.5
Down lockage, feet	742.6	705.6	547.2
Up " "	170.6	133.6	0.0
Total lockage, "	913.2	839.2	547.2
Number of locks	39	37	19
" " guard locks	1	1	2
Standard canal, miles	102.56	102.42	102.35
Canalized river—			
200 to 250 ft. bottom width, miles	.....	.....	1.51
250 to 300 " " " "	20.38	20.38	.....
300 to 350 " " " "	12.37	12.37	38.97
350 to 400 " " " "	2.59	2.59	.....
400 to 450 " " " "	13.90	13.90	8.08
450 to 500 " " " "	8.99	8.99	11.59
500 to 1 000 " " " "	39.15	37.65	73.65
Open lake and river	277.10	278.64	449.06
Total.....Miles	477.04	476.94	685.21

*Note: In the high-level plan for the Oswego-Mohawk Route the water supply is to be obtained from reservoirs in the Black and Salmon River Valleys, and in the low-level plan Oneida Lake is to be used as a reservoir, with a deep cut for the waterway between Oneida Lake and the Mohawk River.

TABLE No. 2—Estimated Cost of Waterways.

	30-Ft. Channel.	21-Ft. Channel.
Lake Superior to Lake Erie	$33 539 869	$6 961 818
Lake Michigan to Lake Erie	16 226 548	1 466 439
Lake Erie to Lake Ontario	73 435 350	42 393 203
Lake Ontario to New York—		
Oswego-Mohawk Route (High-Level Plan)	206 253 553	155 324 968
Oswego-Mohawk Route (Low-Level Plan)	210 309 129	157 003 082
St. Lawrence-Champlain Route.	213 123 864	141 027 415
Duluth to New York—		
Oswego-Mohawk Route (High-Level Plan)	313 228 772	204 679 989
Oswego-Mohawk Route (Low-Level Plan)	317 284 348	206 358 103
St. Lawrence-Champlain Route.	320 099 083	190 382 436
Chicago to New York—		
Oswego-Mohawk (High Level-Plan)	295 915 451	199 184 610
Oswego-Mohawk (Low Level-Plan)	299 971 027	200 862 724
St. Lawrence-Champlain Route.	302 785 762	184 887 057

The Committee on Canals of New York State, in its report to the Governor of New York, estimate that a barge canal from Buffalo to the Hudson River, 351 miles long, 12 ft. deep, with a cross-section of 1 200 sq. ft. will cost $58 894 668.

The Canadian government has expended about $65 000 000 upon the construction and maintenance of the St. Marys, Niagara and St. Lawrence canals, or about $13 per capita for the entire population of the country. This amount, however, does not represent the cost to construct the canal system as now completed, as nearly the entire system on the Niagara and St. Lawrence Rivers has been enlarged from two to three times. The canals are 14 ft. deep, with locks 270 ft. long and 45 ft. wide, and were intended to develop through transportation between the Lakes and Montreal.

The expectations relative to the volume of traffic that would be

developed have not been realized, and it is extremely improbable that the small type of steamer which can pass the locks will be able to compete with the large lake freight carriers, even when handicapped with excessive transfer charges at Buffalo.

It will be noted that the length of standard canal is practically the same by the Mohawk or Champlain routes, and that the 208 miles greater distance by the latter route is mostly through the wide channel of the Lakes and rivers.

The Champlain route is a down-grade waterway from the Lakes to the seaboard, and has 366 ft. less lockage than the Mohawk high-level route, and for a 21-ft. channel will cost $14 300 000 less than the latter.

It is fair to assume that in a 21-ft. waterway with an area of cross-section practically the same as the St. Clair Flats Canal, the speed of ships in straight reaches will be fully as great as that of the larger lake freighters in the St. Clair Canal. Making the necessary reductions for slower speed on curves and in passing ships *en route*, and for detentions at locks, it is found that a freighter capable of steaming 12.5 miles per hour in the open lake should make the round trip between Chicago and New York in 11 days and 8 hours *via* the Mohawk Route, and in 12 days and 9 hours *via* the Champlain route.

The navigation season on the Mohawk route will average about 245 days and for the Champlain route 230 days, and if the detention at New York is not more than twice the time required at Buffalo, a lake steamer should make 16 round trips per season between Chicago and New York *via* the Mohawk route and fourteen by the Champlain route. The average sailing time between Chicago and Buffalo for steamers capable of running 12.5 miles per hour in the open lake is about 72 hours, and the estimated time for the same type of steamers between Buffalo and New York based on the actual performance of freighters in similar restricted channels of the lake waterways is 64 hours.

This estimate is based on a 21-ft. waterway having a cross-section of 5 500 sq. ft. area or about six times the midship section of ship, which ratio will allow safe speeds of 8 to 10 miles per hour with the same consumption of fuel required in open water.

In the previous discussion* the Ship Canal is assumed to be 24 ft. deep, with a section of 4 200 sq. ft., and that ships having midship

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sections of 1 000 sq. ft. would average about 4 miles per hour, not including detentions at locks and terminals.

In the case of the barge canal the proposed cross-section is 1 200 sq. ft. or four times that of the barge, a ratio which will not admit of average speeds of over 4 miles per hour with ordinary barge fleets.

From these elements it was estimated that steamers in the ship canal, and barge fleets in the barge canal, will only average ten round trips per season between Buffalo and New York, whereas with suitable dimensions for the waterway and proper dispatch in loading and discharging ships at terminals at least 14 to 16 round trips may be made between Chicago and New York, or about three times the annual volume of business estimated in the 1898 discussion for steamers on the ship canal from Buffalo to New York.

The expense account of lake and ocean freighters is so much greater than that for canal barges that an accurate estimate of practical speed in canals and of detentions at terminals is essential in making any comparison of rates at which freight can be carried on the two types of waterways.

In the report of the Committee on Canals of New York State, it is estimated that a ton of freight can be transported from Buffalo to New York on barges for 26 cents, but in obtaining such a result the Committee omits from the expense account the items for interest on cost of construction, annual expenses for repairs and maintenance, and profit for the shipowner.

The interest and insurance expenses are based on a valuation of $28 500 for a fleet of one steamer and three barges, yet in a foot-note on page 19 of the report it is shown that if constructed of steel the fleet would cost $70 000.

With the increase in the cost of lumber which is sure to occur in the near future, the cost of barges constructed of wood will approximate that of steel; but even if this should not be the case, it is probable that the greater cost of towing wooden barges would make the steel barge more economical for transportation at the values given.

At the Brussels International Congress on Navigation in 1898, Captain C. V. Suppan, Chief of the Steam Navigation Service on the Danube, gave the result of some experiments in towing steel and wooden barges of similar models on the Danube, from which it appears that the power required for the wooden barges was nearly twice that required

for the steel barges, and that the cost for towing per ton-mile for the wooden barge was about double that for the steel.

If these results are correct, the wooden barge will not be economical, even at a lower value than that given in report.

Correcting the estimate on page 64 of the Report of the Committee on Canals for omissions and for change in values, the expense account for barge fleet, with a total annual traffic on canal of 15 000 000 tons, will be as follows:

TABLE No. 3.—EXPENSE ACCOUNT, BARGE FLEETS.

Wages and subsistence	$4 000
Fuel, oil, waste, etc	3 300
Ordinary repairs	300
Insurance on fleet, 4½ per cent	3 150
Insurance on cargo	3 900
Miscellaneous expenses	200
Interest on investment, 5 per cent	3 500
Deterioration, etc., 5 per cent	3 500
Fixed charges for maintenance, repairs and interest on cost of canal (3 per cent.)	8 800
Shore expenses and commissions	1 000
Profit to ship-owner	1 500
Total for 52 000 tons	$33 150
Cost of transportation per ton between Buffalo and New York	0.64
Rate on lake, Chicago to Buffalo	0.50
Transfer at Buffalo	0.25
Total rate per ton, Chicago to New York	$1.39
" " bushel, " "	0.042

The rate per ton for both the barge and the ship canal (Tables Nos. 3 and 4) will vary from these estimates with any variation of the volume of traffic from that used. West-bound freight, which has been estimated at one-third of the eastward movement and at the same rate per ton, would, under normal conditions, command a higher rate than the east-bound freight, but with the volume of westward traffic only sufficient for one-third cargoes for the ships in the service, competition would be

TABLE No. 4.—Dimensions, Capacity and Expenses of a Steamship Adapted for the Traffic of a 21-Ft. Waterway, and Capable of Running 12.5 Miles per Hour in the Open Lake.

Length over all	480 ft.
Breadth	52 "
Draft	19 "
Horse-power, (I. H. P)	2 200
Coal consumed per hour	3 850 lbs.
Net carrying capacity	8 600 tons
Cost of ship for lake and canal business only	$360 000
Cost of ship adapted for ocean and lake business	387 000
Pay-roll and subsistence, per day	60
Fuel, oil and waste, per day, when running	117
Fuel, oil and waste, per day, in port	30
Insurance	4½%
Repairs and depreciation	5%
Expenses for season of 245 days:	
Wages and subsistence	$14 700
Fuel, oil, waste, etc	23 400
Insurance on ship	16 200
Insurance on cargo	22 900
Miscellaneous expenses	1 000
Interest on investment, 5 per cent	18 000
Deterioration, etc., 5 per cent	18 000
Fixed charges for maintenance of canal, repairs and interest on cost (3 per cent.)	73 400
Shore expenses and commissions	2 000
Profit to ship owner	10 000
Total for 183 470 tons	$199 600
Rate per ton, Chicago to New York	$1.09
Rate per bushel, Chicago to New York	0.033

likely to force the rate fully as low as for east-bound traffic. The interest on the cost of the canal during construction would also increase the amount of fixed charges above those used in the estimates, and correspondingly increase the rate per ton.

These variations will have relatively the same effect on the rates for both the barge and ship canal, and therefore would not materially change the comparative value of the two types of waterways.

The items from Tables Nos. 3 and 4, for wages, subsistence, fuel and miscellaneous expenses, amount to between one-fourth and one-fifth of the respective total expense accounts, showing that the actual cost of moving a ton of freight from Chicago to New York will be less than one-fourth of the rate necessary to transact the business, and therefore any material reduction in transportation rates can only be obtained by constructing waterways on which quick trips can be made and all unnecessary transfer and terminal charges eliminated.

The comparative transportation rates given in Major Symons' report differ from the present estimate for the reason that the latter includes fixed charges for maintenance and interest, and is based upon speeds in the restricted portions of the waterway which have been demonstrated to be safe and practical in the connecting channels of the Lakes, and upon dispatch at terminal ports similar to that now obtained at Lake harbors.

The average time in port for freighters running between Duluth and Buffalo is about three days per round trip, but to make allowance for the different conditions in New York Harbor four days per round trip have been used in the present estimate.

It is also assumed that if the volume of traffic on the barge canal should amount to 15 000 000 tons annually, a ship canal of dimensions suitable to allow the lake tonnage free passage to the seaboard would develop at least 20 000 000 tons per year.

The record of the transportation rates and volume of traffic on canal and railroads between Buffalo and New York indicate that any waterway which does not admit of steamers passing direct between lake ports and the seaboard is not likely to prove a successful competitor of the railroads.

The freight carried on the Erie Canal:

In 1837 was	667 151	tons.
In 1850 "	1 635 089	"
In 1860 "	2 253 533	"
In 1870 "	3 083 132	"
In 1880 "	4 608 651	"
In 1890 "	3 303 929	"
In 1899 "	2 414 084	"

Since 1880, when the traffic on the Erie Canal reached a maximum, the rate per ton-mile on the railroads from the Lakes to the seaboard has been nearly double that on the Erie Canal, yet during that time the business of the canal has diminished one-half, while that on the railroad has nearly trebled, showing beyond question that the volume of freight which will be shipped by any given route does not depend entirely upon the relative transportation rates, and that unless the causes which have produced the decline of traffic on the Erie Canal are eliminated in the construction of new waterways no better results need be expected.

Two of the principal benefits to be expected from the construction of a waterway of proper dimensions between the Lakes and the Atlantic, are the development of ship-building industries at lake ports, and to make it possible for individual ship owners to do business in competition with combinations, neither of which could be realized from the construction of a 12-ft. barge canal.

The only possible way in which the traffic of a barge canal requiring transfer of freight at Buffalo can be increased over that of the present Erie Canal, will be by organizing large transportation companies to operate fleets of barges in connection with freight steamers on the Lakes, so that through bills of lading can be made from lake ports to the seaboard.

Such an arrangement would also require the control of sufficient elevator capacity at Buffalo to take care of the surplus grain which at times would be greatly in excess of the carrying capacity of the barges.

With economical water transportation only possible for organizations controlling large amounts of capital, it is by no means improbable that the canal and railroad companies would combine to maintain rates, and thereby destroy about the only benefit to be expected from a canal of such dimensions.

In a waterway having dimensions to correspond with the depth of the water routes connected, individual ship owners would have equal opportunities with the large organizations and practically make the control of freight rates impossible.

Such a waterway would also make it possible to increase largely the steel and shipbuilding industries of the country, and probably be of great assistance in restoring the Merchant Marine of the United States in foreign trade.

It is of interest to note that the distance from Buffalo to New York by railroad is 440 miles, by Barge Canal 496 miles, and by ship canal *via* Lake Ontario and the Mohawk River is 477 miles, making the barge route 19 miles longer than the ship canal, with about three times greater amount of canal section through which boats will have to move at minimum speed.

Referring to the estimated cost of 21 and 30-ft. waterways from the Lakes to the seaboard it will be noted that the latter will cost about $100 000 000 more than the former, not including the cost of improving terminal harbors. The difference in expense for operation, maintenance and repairs will be about $550 000 per year, making the fixed charges for a 30-ft. waterway, including the improvement and maintenance of lake harbors, nearly $5 000 000 greater than for one of 21-ft. depth.

It has been shown that taking the cost of improvements into consideration the commerce of the Lakes can be transported more economically with channels of 21-ft. depth than with deeper ones, and therefore whatever advantages are to be derived from a 30-ft. waterway from the Lakes to the Atlantic must come from developing new industries and commerce by direct trade between the Lakes and foreign ports.

With a traffic of 20 000 000 tons annually in such a waterway it is not probable that more than 6 000 000 tons would be foreign commerce, and if carried in deep-draft vessels the 14 000 000 tons for domestic consumption would have to be discharged at New York for use in the city, or for reshipping to other ports on the Atlantic Coast. This would necessitate two-thirds of the ships trading direct between the lake ports and New York, or sailing for Europe with light loads.

With a 21-ft. waterway the ships adapted to the service could discharge domestic commerce at New York or at any other port on the coast to which consigned, but would probably transfer all foreign freight to ocean steamers at New York, except on the last trips before the close of lake navigation, when it might be found advantageous to load for through passage to Europe.

Taking into consideration the annual expenses for interest and maintenance of the two waterways, the cost of transportation for domestic traffic will be 6 cents per ton greater for a 30-ft. waterway than for one 21 ft. deep, from which it would appear that any benefits

to be derived from canals of greater depths than necessary for economical transportation on the Lakes must result from direct foreign trade between the lake ports and Europe.

If the difference of the annual fixed charges for the two depths of waterway between Buffalo and New York be charged to the probable volume of foreign traffic, the transportation rate will be about 20 cents per ton greater with a 30-ft. waterway than with a 21-ft. canal.

The amount of transfers at New York for the combined domestic and foreign traffic would be fully as great when transported in deep-draft ships as when carried in steamers adapted for safe navigation of the lake waterways, and therefore the benefits to be expected for the deeper waterway must be wholly of an indirect nature.

Since the interests of the steel and ship-building industries would be equally well subserved by either waterway, it is difficult to conceive of conditions which would warrant the large expenditure necessary to construct a waterway of greater dimension than required for safe and economical navigation on the Lakes.

Almost any transportation rate may be deduced for the routes of different depths, by varying the assumed speed in the restricted channels and the time lost by detentions at terminal ports. If, however, the dimensions of channels be such as to allow economical speeds for the type of vessel adapted to their use, there need be no trouble in establishing these elements of the problem within reasonable limits, as the average speed at which canal barges can be towed is pretty well known, and the safe speeds of steam ships in the restricted waterways of the Great Lakes and in the Ship Canals of Europe furnish data from which the ratio of cross-section of channel to that of ship may be determined, which will give a minimum rate of transportation.

Conclusions.

1. A waterway from the Great Lakes to the Atlantic, to produce the best results, should have dimensions which will permit the passage, at economical speeds, of ships best adapted for the traffic of the water routes connected.

2. Safe navigation of the Great Lakes requires a minimum depth for the connecting channels of at least 20 ft. and for depths of over 21 ft. the interest on the cost of construction will exceed the decrease in transportation rates due to the use of deeper draft freight carriers.

3. With any waterway requiring the transfer of freight at Buffalo, independent canal boats cannot compete with existing lake and railroad lines, and, unless the waterway is operated by rich transportation companies controlling lines of lake steamers and canal barges, the business developed will probably not exceed that of the present Erie Canal.

4. The decline of traffic on the Erie Canal since 1880, and the failure of the 14-ft. Canadian canals to divert commerce from the lake and railroad lines, indicate that a waterway of less depth than required for the passage of the best type of lake freighters cannot materially modify the transportation rates over existing routes.

5. The indirect benefits to commerce to be derived from a 21-ft. waterway will be nearly as great as can be expected with greater depth of channel.

DISCUSSION.

Mr. Symons. THOMAS W. SYMONS, M. Am. Soc. C. E. (by letter).—Without entering upon a discussion of the general canal proposition, the writer feels compelled to take exception to some of the statements in Mr. Wisner's valuable paper, and particularly wishes to dissent from his fourth conclusion, which is as follows:

"The decline of traffic on the Erie Canal since 1880, and the failure of the 14-ft. Canadian canals to divert commerce from the lake and railroad lines, indicate that a waterway of less depth than required for the passage of the best type of lake freighters cannot materially modify the transportation rates over existing routes."

The decline of traffic on the Erie Canal is due to very many causes, the principal ones being that it is not in any manner up to date, that the towing is still being done on it largely by horses and mules, and that the business has never been organized and conducted on modern lines. It has been seriously handicapped by the fact that improvements have been undertaken and discussed which would render the old type and size of boats obsolete; rendering it inadvisable as a business proposition for people to build new boats for canal business or to replace those outworn, with the possibility before them of a larger canal in the near future.

That the traffic on the Erie Canal has declined, it is true, but it would immediately spring up and grow to enormous dimensions if a proper barge canal were built in its place, if legislative restrictions on the capital of operating transportation companies were removed, and if the business were organized in a thoroughly up-to-date manner. It must not be forgotten that if the Erie Canal has stood still, or worse than still, for the last thirty years, the competing railroads have improved their transportation facilities enormously in ways which it is unnecessary to recount here.

But the most palpable error in this fourth conclusion of Mr. Wisner is his acceptance and statement as to the "failure of the 14-ft. Canadian canals to divert commerce from the lake and railroad lines." As a matter of fact, the Canadian 14-ft. canal system has not yet been fully completed, and it has not yet been used by any boats drawing 14 ft., and probably will not be for some time. Considerable work remains to be done in the canals and the St. Lawrence River before this transportation scheme is fully completed. In addition to this, the Canadian Government and private parties have only just begun work at Port Colborne and Montreal to prepare for the traffic which they expect to go through this commercial highway formed by the Welland Canal, Lake Ontario, and the St. Lawrence River and Canals. According to the papers, quite a number of boats are being projected and a number are being built to run on this route as soon as it is com-

plete, and Mr. Wisner has certainly jumped at a conclusion which is in no manner justified by the facts and present data. The 14-ft. Canadian canals may be determined hereafter to be failures, but it certainly has not been proven yet, in advance of their completion. Mr. Symons.

His premises being entirely in error, it follows that his conclusion "that a waterway of less depth than required for the passage of the best type of lake freighters cannot materially modify the transportation rates over existing routes" is unfounded and unjustifiable. In fact, even granting the premises, the conclusion is by no means justifiable. The barge canal proposed for the Erie Canal route has a capacity for boats four times the size of those now in use. To say that they cannot materially modify transportation rates is just as absurd as it would be to say that 8 000-ton steamers on the Lakes cannot materially modify rates suitable to 2 000-ton steamers.

To aid him in his contentions, Mr. Wisner is willing, apparently, to stand sponsor for the statement that in the case of wooden barges and steel barges of the same size and model, the power required to tow the wooden barges is about twice that required for towing steel barges, and that "the cost for towing per ton-mile for the wooden barge was about double that for the steel." This, he states, is based upon some experiments in towing by Captain C. V. Suppan, Chief of the Steam Navigation Service on the Danube. This alleged fact in regard to the difference in the cost of towing steel and wooden barges is so contrary to common sense and experience that it is difficult to understand how an engineer of standing would be willing to use it in an argument in favor of anything. The only difference there could possibly be would be in the skin friction on the outside of the boat. Assuming that the steel boat and the wooden boat were in equally good condition, the difference in the amounts of this skin friction would necessarily be very slight. This is one of the reasons why Mr. Wisner, in his comparative tables of cost of transportation, insists upon the transportation through the barge canal being done in expensive steel vessels, instead of cheap wooden ones. As a matter of fact, one of the strongest arguments in favor of the barge canal, following the general route of the Erie Canal, is that business upon it can be done in cheap light wooden boats, as these boats would not, at any part of the route, be subjected to the storms which prevail at times on the Great Lakes.

Mr. Wisner gives 4½% as the rate of insurance on a fleet of barges operating on the canal. As a matter of fact, the prevailing rate is 2% for steamers and 1% for ordinary boats, making the average insurance rate on a fleet of one steamer and three plain barges less than 1½%, or less than one-third of the amount stated by Mr. Wisner. He has also increased the cargo insurance in some unexplained manner.

By swelling the cost of the necessary boats for operating on the

Mr. Symons.

canal in an entirely unnecessary manner, which largely increases the charge for insurance, interest, and deterioration, Mr. Wisner makes his showing favorable to the ship canal.

The steel boats estimated on by Mr. Wisner would be able to navigate the open Lakes, as the Cleveland steel boat fleets are doing today, and could run between the Lakes and the sea in competition with the large steamers on the proposed ship canal, and, because of their much lower cost per ton of carrying capacity (about one-third), and lower operating expenses, could furnish transportation cheaper than could the large steamers.

Mr. Wisner also predicates his findings on the lake ship going to New York and getting as quick dispatch, or nearly as quick, as it gets at the Lake ports. Now, this cannot be admitted as a reasonable probability. The only way in which it could get a quick dispatch would be to go to an elevator and discharge its cargo and return with little or no cargo. This would mean another handling of the cargo in New York and an added expense of about as much as the total cost of transportation on a barge canal. The conditions under which business is done in New York are such as to preclude the reasonable probability of a lake ship delivering its cargo into a foreign-bound ship directly, except occasionally. Nearly all the foreign-bound commerce of New York leaves the port as berth cargoes in the large ocean liners. These ships do not leave their docks, freight has to be brought to them, and there is not room at the dock for the ocean ship and the lake ship, while there is room for the canal's boats and the necessary means of transfer, such as floating elevators, etc. To get anything like the dispatch, at the Port of New York, on which Mr. Wisner figures, would require a very complete upheaval and reversal of the methods of doing business in this great port.

There would certainly be serious delays in the ordinary transaction of business, in getting rid of the cargo of a great ship, which delays could only be obviated by going to large expense in extra handling of the cargo.

Even if the port delays to the barges of a barge canal were as great in New York as the delays to the ship, these delays would not be so serious, as the cost per ton capacity for the barges is very much less than the cost per ton capacity of a great ship, and the operating expenses of the barges are very much less than the operating expenses of a ship, either moving or in port.

There are decided reasons for believing that it would be more convenient, and better for commerce, to have nearly all the western products arrive in New York in lots of about 1 000 tons, in place of having them arrive in lots of 6 000 to 10 000 tons. Nearly all our exports are shipped as berth cargoes in vessels carrying diversified loads, not more than 10 to 15% being shipped in full cargo lots. A cargo of

8 000 tons arriving in eight different loads would go to eight different ships and be transferred, as is the custom now, directly into the ship without its leaving its own berth. The advantages of this in economy and time saving are obviously very great. That it is sometimes more advantageous to divide a large amount of freight into several lots than to carry it all in one, is exemplified in the transportation of coal by water along the Atlantic seaboard. The enormous business of transporting some 30 000 000 tons per year is practically all done in barges of 1 000 to 2 000 tons, three or four of them being towed by one tug. The business of transporting this coal was formerly done in single steamers and sailing vessels, but the tow barge system has supplanted this method almost entirely, and the writer cannot but regard it as a lesson fraught with very great value to any student of the problem of connecting the Great Lakes with the ocean by a water route. Mr. Symons.

The data furnished by Mr. Wisner are not at all sufficient to prove that the Great Lake freighters could with safety make a speed of 8 miles per hour in the canal proposed by the Board of Engineers on Deep Waterways. There is much to be said on the other side. The fact that boats habitually pass through the short straight St. Clair Flats Canal at a speed of 7 to 11 miles per hour is not conclusive that it would be safe to maintain this speed with full-sized ships in a comparatively long canal, with more or less curvature, and rocky bottom and sides. If the ship canal has a commerce upon it in large ships sufficient to justify the enormous expense to be incurred in building and maintaining it, it must necessarily be fairly well crowded with boats at all times, day and night, in windy and stormy weather as well as quiet and pleasant weather, and under all circumstances. The writer cannot conceive of these ships being driven at a speed of 8 miles per hour and meeting and passing other ships without picturing to himself very great danger of accidents.

Experience demonstrates that accidents are bound to occur to ships navigating such a narrow channel, and that it will be necessary for them to move slowly and with the utmost caution. Any accident which leads to the sinking of a ship in such a narrow canal might, and probably would, in many cases completely block the canal until the ship could be raised and gotten out of the way. During the season of 1899 there were three incidents in the history of transportation on the Lakes which illustrate this danger and the serious consequences which are likely to be incurred through an accident. In the early part of September, 1899, one of the Great Lake boats, the *Douglas Houghten*, laden with ore, broke her steering gear, and went aground in the channel of the St. Marys River. In sinking she swung around across the channel and completely blocked it. It took five days of great effort to get this vessel afloat and out of the channel, and in the meantime 332

Mr. Symons vessels were blocked and unable to get by. This caused a great loss to the Lake carrying interests, probably over $500 000.

In the latter part of November another blockade of the St. Marys River Channel occurred, due to a collision in which three vessels, two steamers and a tow barge, were mixed up and stranded, and this caused a delay of nearly four days to some 167 vessels, and involved a very great loss.

In the early part of December a tow barge went aground in the channel leading to the St. Clair Flats Canal, blocking the canal for nearly two days and delaying 35 vessels.

Besides these greater accidents, delaying many ships, there are constantly being reported other accidents of a less serious character in these inter-lake channels, such as the sinking of individual vessels, collisions and damage, the stranding of vessels, etc., due to breaking of steering gear, mistaken signals, or boats taking a sheer. Besides, there is the likelihood of accidents to vessels at the locks themselves, causing detentions and blockades.

There is no getting over the fact that the home and place for large ships is in the wide free waters, and that when they enter the narrow waters they are subject to grave dangers, and must move with extreme caution. So great are these dangers on the narrow channels of 300 to 400 ft. width connecting Lakes Erie, St. Clair, Huron and Superior that the shipping interests of the Great Lakes are making strenuous efforts to get the Government either to widen them to 600 or 800 ft., or to build duplicate channels, one to be used by up-bound and one by down-bound vessels.

It does not do to shut our eyes to the perils of great ships in narrow channels. In spite of all precautions, such accidents are likely to happen in narrow channels traversed by ships of 7 000 to 10 000 tons capacity. The longer the channel, the greater is the likelihood of accident, and it is easy to imagine that in a ship canal, from Lake Erie to Lake Ontario, and from Lake Ontario or the St. Lawrence River to New York, such accidents, blockades and detentions would be of frequent occurrence, if used by many vessels. These would all tend to reduce greatly the average speed attainable in the canal. Of course, accidents and detentions are likely to occur on a small canal also, but, owing to the greater ease with which the smaller locks and boats can be managed, the dangers here are not so marked; besides, the results of accidents on a small canal are not so likely to be disastrous as in the case of a greater canal and larger boats. It is a much easier and quicker matter to lighter a small vessel carrying a few hundred tons than it is to do so with a great freighter carrying several thousand tons.

There is another point of view from which this problem must be looked at, and which should receive very serious consideration. The canal, if built, will be built by the United States at a cost of about

$200 000 000. If built by the United States the benefits should all accrue to the United States, but would they? It is the writer's very decided opinion that they would not. If a ship canal be built from Lake Erie to New York by the Ontario-St. Lawrence-Champlain route it would be more apt to redound to the disadvantage of American commerce than to its advantage. Such a ship-canal route would follow down the St. Lawrence until within about 50 miles of the seaport of Montreal. It would then branch off to New York, which would be reached only after traversing some 350 miles of canal, lake and river, and when at New York, commodities destined for most foreign markets would be further away from these markets than they were when they left the St. Lawrence. Such a canal would be of some benefit to our local and strictly domestic business, but it would just as certainly be a detriment to our inbound and outbound foreign commerce, tending strongly to take it away from New York and our other North Atlantic cities and build up the commerce of Montreal. Of course, to reach Montreal, Canada would have to add a short section to our ship canal, but this she would certainly do, and then it would be inconceivable that grain, lumber, iron ore, manufactured steel or other articles destined for foreign markets, reaching within 50 miles of Montreal, would retrace their steps through 350 miles of contracted waterway back to the Port of New York. Mr. Symons.

The same is true, though to a somewhat less extent, in regard to the ship canal projected by the Oswego-Ontario-Oneida-Mohawk route. If the Great Lake freighters, carrying from 7 000 to 10 000 tons of produce, should go into Lake Ontario, they could continue on down the splendid waterway of Lake Ontario and the St. Lawrence River as far as Prescott and Ogdensburg, only about 120 miles from Montreal, and then transfer into 2 000 or 3 000-ton barges to be taken through the Canadian canals to Montreal. All foreign-bound commerce would undoubtedly take this course rather than the route from Oswego to New York through 320 miles of contracted and dangerous waterway with its 590 ft. of lockage. It is altogether certain, too, that in this event Canada would soon enlarge her St. Lawrence canals so as to permit the Great Lake vessels to run directly to Montreal, and the resultant benefits, in that event, would be far greater to Canada than to the United States.

Whatever canal is built should be so located that its benefits would only accrue to the United States, and there is only one canal route between Lake Erie and the ocean of which this is true, and that is along the line of the present Erie Canal. This route is not available for a ship canal, but is available and well suited for a barge canal, and is the only route which should ever be considered by the people of the United States.

The point made by Mr. Wisner against the barge canal, that it

Mr. Symons. could only be fully and properly utilized by the organization of large transportation companies to operate boats on the Lakes and canal with elevators at Buffalo, and that these transportation companies and lake and rail lines would probably combine to maintain rates and thereby destroy the benefits from such a canal, is not, in the writer's opinion, well taken. Such a canal would be free to any number of transportation companies, just as free as the Lakes themselves, and while such a combination might be made, it is also equally probable that in that event other companies would be organized and do business at the lowest profitable rates. The charge of a possible combination to maintain rates would lie almost as well against the ship canal advocated by Mr. Wisner as against the barge canal. In fact, in some respects it would lie more strongly, for it would take less capital to break into a barge canal combination than into a ship canal combination.

Mr. Wisner says:

"Since 1880, when the traffic on the Erie Canal reached a maximum, the rate per ton-mile on the railroads from the Lakes to the seaboard has been nearly double that on the Erie Canal, yet during that time the business of the canal has diminished one-half, while that on the railroads has nearly trebled," etc.

While it may be true that railroad freight rates are double those on the canal if there be included all kinds of freight carried, and all the year round, it is not true if there be included only the heavy coarse freight ordinarily carried on the canal, and during the period of canal navigation. For many years the canal rates and railroad rates on grain, etc., have very closely approximated to each other during the period of canal navigation, the railroad rates going up at the close of this period. As an illustration it is to be noted that the prevailing rate of 3 cents per bushel, which the railroads have been charging all summer, has just been lifted by the railroad combination to 4 cents, an arbitrary increase of 33⅓%, to take effect November 1st, about the close of the season of canal navigation.

The writer is glad that he can so completely agree with Mr. Wisner in one respect, and that is in his belief that a canal for deep sea ships of 30-ft. draft is not advisable or justifiable. The writer investigated this matter and reported to this effect to the General Government in 1897. He has not yet seen reason to doubt the correctness of the conclusions reached in that report—that there is one type of vessel best suited for use on the ocean, another on the Lakes, and another on narrow canals and waterways; that economy of transportation demands that the vessel be suited to the waters which it traverses, and that no single type of vessel has been, or probably will be, devised, which is suitable to all three or any two of the waters mentioned. He also cordially agrees with Mr. Wisner that the depth

to be aimed at for the inter-lake channels and Lake harbors should be limited to that suitable for vessels of 20-ft. draft. Mr. Symons.

Mr. Wisner and others who advocate a ship canal, lay great stress on its influence in expanding the ship-building industries of the country and "restoring the Merchant Marine of the United States in foreign trade." The writer is unable to see much in this argument. Ships are going to be built on the seashore and on the Lakes just as fast as they are needed, and ships of the same character will cost nearly the same in one place as another. As an instance of this it may be mentioned that a ship of Canadian canal size has just been launched in Buffalo, destined for ocean trade, while at the same time the City of Buffalo is having a large steel fire tug built on the seacoast, the contract for the latter having been awarded after full competition. The mere transference of an industry from one section of the country to another is not in itself a benefit for which the whole country should be called upon to pay a large sum of money.

The foregoing is intended as an answer to a good deal that appears in Mr. Mayer's interesting, but rather transcendental, paper. Some of his theoretical deductions are based on entirely erroneous data. For instance, he bases some of his theories on an amount of tonnage tributary to a ship canal that is far beyond any reasonable probability at any time in the near future. If he were more familiar with the business of transportation on the Lakes he would know that the total tonnage passing through the Detroit River and the Sault Ste. Marie has little relation to the amount of tonnage that would pass through a ship canal from Lake Erie to the sea. The Detroit River and Sault Ste. Marie tonnage is made up principally of coal, ore, lumber and grain. Practically none of the coal would be apt to pass through the ship canal, but little of the ore, while possibly half of the lumber and two-thirds of the grain might be conceived as using the canal. Mr. Mayer's estimate of 36 000 000 tons as a reasonable amount of tonnage on which to base his estimate is far beyond reason. Even Mr. Wisner, strong advocate of the ship canal as he is, only uses 15 000 000 tons in his estimates. It is nearer the mark, for some years to come, to figure on 10 000 000 tons, with a possible increase to 15 000 000 or 20 000 000 tons in the not distant future. With an annual cost of about $9 000 000 and a tonnage of 36 000 000, Mr. Mayer computes an interest and maintenance charge of 25 cents per ton. With the tonnage reasonably to be expected, this would be increased to 90 cents, gradually diminishing to perhaps 50 or 60 cents. And it must not be overlooked that if the tonnage approaches anywhere near Mr. Mayer's 36 000 000, the alternative suggested by him of additional locks and facilities must be met, which will largely increase the interest and maintenance charges. At the Sault Ste. Marie there are now three locks: The American Weitzel lock, 60 x 515 ft.; the American Poe lock, 100 x 800

Mr. Symons. ft., and the Canadian lock, 60 x 900 ft. The tonnage passing through them all is something like 25 000 000 tons per annum. The American locks are now at times taxed to their full capacity to take care of the tonnage demanding passage, and it is one of the strong demands of the Lake-carrying interests that a new lock, larger even than the Poe lock, be built to accommodate the traffic. It is understood that the Board of Engineers on Deep Waterways proposes a single lock on the line of its ship canal about 60 x 600 ft., about half the size of the Poe lock at the Sault Ste. Marie, and only large enough for the passage of one large steamer at a time.

Experience at The Sault would indicate that as the Lake carrying business is distributed throughout the year, such a lock cannot be expected to handle much over 10 000 000 to 15 000 000 tons annually.

In regard to the number of trips on which the estimated cost of transportation is based, the writer has now only this to say:

In the transaction of a transportation business by great vessels on such a ship canal as advocated by Mr. Mayer and Mr. Wisner, there is a vast difference between a reasonable probability and a theoretical possibility. The writer conceives that his estimate of 10 trips per year between Buffalo and New York is the first, and the estimate of 16 trips between Chicago or Duluth and New York is the second.

Mr. Randolph. ISHAM RANDOLPH, Esq.* (by letter).—The writer lacks the data upon which to base a discussion sustaining his preconceived ideas in favor of a channel deeper than 21 ft. This question passes beyond the problem of construction and cost into the realm of political economy, and statesmen must decide how much the Nation should expend upon a channel designed to connect the mid-continent with the World's ocean highways and give the producers and consumers of the Middle West and Northwest access to the World's markets upon terms nearly equal with those of the Atlantic States. Of one thing, however, the writer feels sure, that any plan for connecting the Great Lakes with the Atlantic Coast should be executed in such a manner as to admit of progressive development to meet the maximum needs of a deep waterway. Locks and all permanent structures should be constructed for maximum depths even though the intervening channels be excavated at the outset for the 21-ft. depth advocated by Mr. Wisner. That is a concession to the greater project which seems to be justified by American faith in America's future.

Mr. Schenck. ARCHIBALD A. SCHENCK, M. Am. Soc. C. E. (by letter).—These valuable papers open up two important inquiries:

(1) Are Lake vessels for commerce on rivers and canals more profitable ultimately than small barges and canal boats?

(2) Should our "seaports" be located at the sea or in the interior?

* Chief Engineer, Sanitary District of Chicago.

Mr. Schenck

There is no doubt that the free ocean freights can be handled for less than ½ mill per ton-mile for long voyages. On the "half-way free" Great Lakes, freight can be handled for less than 1 mill per ton-mile, where the voyage is the longest possible—Chicago or Duluth to Buffalo. This voyage may include travel through short stretches of canal connecting large open lakes. Owing to the lack of intermediate large basins (such as are the Lakes), in the proposed long canal and canalized river through New York State, no freight rate can be positively given. No actual test has been made for such long distances of narrow canal crowded with deep-draft vessels so thickly, without relieving basins. A comparison by highest speeds is hardly a safe financial operation. A railway train on one road can make 10 miles per hour on long stretches of track—on another road it will make 40 miles. Therefore, does the latter road carry freight for one-fourth the rate of the other? The consumption of time and money on most railways is not in transit, but while waiting to get into transit. On the canal, crowded with numerous vessels, the passing of opposing and overtaken vessels will reduce the speed to one-half, often to one-fourth, occasionally to one-tenth of the maximum speed. The vessel must run "on caution card" most of the time, even when having the right of canal; all this, apart from the mechanical difficulties in a cramped waterway, and with poor steering power, which are well-known causes of retarding. The premise of Mr. Mayer—tested speed over measured lengths—leaves out of sight all the difficulties of transit over which the railway man grows sad, even on a double-track railway, and which will exist largely on a canal. Such difficulties and delays on a long and narrow canal would lessen his 8-mile speed to a small fraction of it as average speed; will multiply by several times his 0.6 mill per ton-mile, and will cut into several small pieces his $1.50 per ton saving.

With these allowances, he is then in competition on river and canalized river with the immense tows of small boats, and on the ocean with the immense ocean carriers. His boat has neither great divisibility nor great mass.

The cheapness of these great tows of shallow boats is seldom noted by those who urge deep-water canals. On the Mississippi about 30 000 000 tons have been carried annually at a rate of 0.5 mill to 2.0 mills per ton-mile, with trifling investment in vessels. The tonnage reaching Hamburg by the Elbe is on water from 2 to 6 ft. in depth. On the Rhine, the Elbe and the Oder, the "strings," or tows, reach 10 000 tons each. On the Hudson they run from 10 000 to 15 000 tons, with draft of 6 ft. The towage rate is about $\frac{1}{10}$ cent per ton-mile. This low rate is not the only, or, perhaps, greatest, advantage of these tows. Their divisibility enables them to secure return cargoes, or cargoes of various sorts for many uses and many ports. Their light

Mr. Schenck. draft enables them to enter a vastly greater number of ports, ports for which the expense of construction and maintenance is trifling compared with deep-vessel ports; to secure an immensely greater variety and number of customers and shippers; to gather their freight nearer the producing point and deliver it nearer the consuming point, with little intermediate tribute paid to middlemen. These tows more nearly resemble the railway trains, in their divisibility and power of reaching close to consumer and producer, than any other form of water transportation that exists or can exist. It is doubtful whether the cheapening on canals and rivers in freight of moderately deep-draft Lake vessels would equal in cash value the gain those light-draft vessels have. At the ocean shipping port they can gather about and get closely "in touch" with large or small vessels, in deep or shallow docks, as no deep-draft Lake vessel can. They are the convenient, adjustable unit, perhaps never to be supplanted in inland narrow navigation for freights.

Again, the Lake vessel is also at a loss in competing against ocean carriers. It is dwarfed, stunted by its inland channel and ports and its effort, so to speak, to run on both sea and land. The tendency of ocean carriers to greater tonnage and draft can readily be noted by examining a few well-known trans-atlantic steamers.

TABLE No. 5.

Steamer.	Length, feet.	Beam, feet.	Draft, loaded, feet.	Tons.
Umbria	502	57	28	8 128 Gross.
Teutonic	582	57	29	9 685 "
Majestic	582	57	29	9 861 "
City of Paris	580	63	27	10 500 "
City of New York	580	63	27	10 500 "
Kaiser Wilhelm	648	.....	.....	.....
Oceanic	704	68	.....	17 040 Gross.
New Liner, *Harland-Wolff*	750	.....	.....	12 000 Net.

The function of each class of vessel is essentially different. The proper style of inland vessel, small, penetrating everywhere; the ocean carrier, as large as the greatest ports will permit. The 24-ft. seaport entrance of former days, the 30-ft. entrance depth of to-day, are already outgrown; a 40-ft. entrance is now demanded. The increased tonnage is mostly clear profit, and the Lake vessel can never get that profit. It must go down before such ocean competition. It is neither one thing nor the other, neither truly inland nor truly oceanic. Our inland lakes, too large and dangerous for river vessels, too small and with ports too shallow for ocean vessels, are an inconvenient intermediate. A canal to connect two bodies of water having similar ves-

sels is reasonable. A canal to connect two bodies of water much different, not adapted to the same class of vessel, is not desirable, or likely to be profitable. It is partially like trying to connect a narrow-gauge road and a broad-gauge road for running through trains. Lake vessels will not fit an ocean—ocean vessels proper cannot get upon the Lakes or use Lake ports. The effort to extend these Lakes seaward, as if they were deep-draft ocean waters seeking similar waters, is likely to be costly and unprofitable. Hence, "let the Government attempt it, it can afford to lose." Mr. Schenck.

The pressure for this waterway appears to come from four directions:

(1) The great steel-producing plants, the Carnegies, Rockefellers, etc.: They can exterminate the middlemen, be producers and reach to the foreign consumer. Their gain comes, not so much in decreased transportation cost, as in annihilated middlemen. Only immensely wealthy concerns can thus act as producer, carrier and foreign merchant. The usual blocks of distance, into which commerce divides itself, must be made in all but such cases, and take place where bulk can most advantageously be broken—at the coast.

(2) From small shippers, to reach inland consumers: For these, the majority of the boats on canal and river should be small, receiving from the Lake boats as now, and delivering to them. The remedy of present cost lies in lessening transfer costs rather than in deepening waterways.

(3) The great mid-continental railways that reach from Pacific tide water to the Lakes: These are anxious to make seaports of their Lake terminals, and be independent of railways further east. Each wishes to unite Europe and Asia by a single line—its own.

(4) The numerous Lake cities, each eager to become a Greater New York:

All these efforts to transfer seaports to the heart of the continent are unnatural. Millions and hundreds of millions of dollars have been wasted, in recent years, in attempts to draw ocean commerce far inland. A few more hundreds of millions are now to be added, perhaps. The civil engineer, manager of transportation and capitalist, can find no larger or more important problem. It is worth the most painstaking study, with millions in it, or to be lost by it. It cannot be taken up as light reading or for temporary amusement. A correct, generally accepted decision as to whether our ocean ports should be far inland or far oceanward, would tend to save and make more millions than any other question the civil engineer encounters. This canal project is only one more struggle in the expensive contest of ports.

One of the causes of this contest has been that mere shipping ports and great marts of trade have been supposed to be one and the same thing. All that each town has aimed for has been to be a shipping

Mr. Schenck. port, a mere transfer table (with very little profit dropped in the transfer, it is found). All that railways have sought to reach has been a shipping point at which to dump their freight as near at hand and with as short a road as possible. The effort to ascertain the natural mart of trade and interchange, and to reach it has seldom been made.

The two alternates of location have been at the "head of navigation" and at the coast. Navigation is generally hydra-headed, and "the" head of navigation varies and dwindles in size and importance with draft of vessel until it becomes microscopic and unascertainable. An "immutable law" is claimed to put the great shipping port and mart of trade at this "head of navigation." The real test of commercial supremacy, such as the Lake ports are seeking by this canal, lies at the foot of inland navigation, always a distinct thing. This is the outermost point where inland vessels and great tows can safely go and transfer into the great ocean carriers. It is where the two can unite without sacrifice to either in respect to deep draft for ocean or light draft for small waterways.

At such a point the ocean vessel, the river vessel, the railway, can all get together. It is the only real radial point on the earth's surface. Land, ocean, coast, all productive areas, are naturally tributary and accessible to it. If the ocean carrier comes hundreds of miles inland, it passes rich areas, full of business. If it brings freight for this coast area, that freight must have a double needless journey after delivery inland. The coasting vessels must come far inland to exchange freights with the ocean carrier. The railways must carry back nearer to the coast what has already passed the coast inbound.

In support of the seaport far inland, and expensive projects for bringing vessels to it, existing ports are quoted. They are the main apparent argument. It is said that vessels must penetrate to the consumer, and the mart of trade be there. The consumers passed and ignored on the inland journey are not counted as of importance. A few figures in Table No. 6 will indicate what rich productive areas would be run through if a vessel could and did come far inland past the territory of each road named.

TABLE No. 6.

Road.	Through tonnage.	Local tonnage.
New York Central	2 023 133	14 598 064
Pennsylvania	2 189 804	35 809 588
Southern Pacific, Pacific System	738 914	4 949 142

Such are some of the rich tonnage-producing territories which the new Lake vessels would ignore and run past, in order to reach the

inland port and get "nearer the producer and consumer." The vastly greater number of buyers, greater number of profitable whole-cargo lots, greater radial possibilities of distribution to all the country, are ignored by the vessel whose only aim is to go burrowing far into the land, in place of sending its load, promptly on reaching land, to every possible direction. Mr. Schenck.

The actual ports cited as favoring the vessel's search inland, are numerous. None of them is really far inland, many wish they were less so. Some of the "inland ports," as contrasted with the ocean or coast ports, are the following:

"Inland ports"—Hamburg, Bremen, London, Glasgow, Gloucester, Montreal, Philadelphia, Baltimore and New Orleans.

Some of the coast ports are Liverpool, Southampton, Boston, New York and San Francisco.

These inland ports were located before railways reached readily to the coast. They attained importance when slow water transportation was the only form of transportation. They were in areas which had developed close to the rivers (then the only transportation routes) as towns now spring up along railways. The draft of vessels was small. Time was not an object in travel, in making sales, in turning over capital. In the present days time governs in all these, as essential; speed in transactions multiplies capital. Quick fluctuations compel quick deliveries by ocean carriers. Protection is afforded to commercial cities at the coast. Fresh water does not have to be sought far inland.

Hamburg was founded in 808 A. D., became a free city, and secured valuable commercial privileges which more than compensated vessels for going a short distance inland. It is in a fine radial location of water transportation, having tributary to it 2 000 km. of waterways. It has expended immense sums for facilities, having 300 acres of basins, many miles of wharves, 30 000 sq. ft. of storage. Its facilities are worth $50 000 000 per annum to commerce. Railway transportation came too late to change its location.

Bremen is not a port, having only 8 ft. of water.

London was located under early conditions. Although it often in early days congratulated itself on being inland a short distance, because of protection from hostile fleets, it has regretted it for commercial reasons, and made every effort to overcome its disadvantages. Since the increase of railway facilities, its annual tonnage has become about 17 000 000, against 20 000 000 for Liverpool and much for Southampton.

Glasgow is in a great coal and iron producing country. It is not a commercial but a manufacturing city, and attained a population of 300 000 before railways came.

The American ports named as inland were also located and grew to

Mr. Schenck. large size, as a result of water facilities, before railways came. New York, a coast port, once much smaller than Philadelphia, is now many times larger. Boston's foreign commerce exceeds Philadelphia's many times. The inland port of Albany and the inland ocean waterway, the Hudson River, were deserted by ocean vessels through railway competition which placed the freight down at the ocean for the ocean carriers, and through competition of the great tows on the Hudson. No better practical illustration of how needless is an inland waterway for ocean vessels can be found than this same Hudson River, on which this inland attempt is desired to be renewed.

Philadelphia made every possible effort to draw ocean vessels generally to her wharves. The immense capital and tonnage of the Pennsylvania Railroad were backing the attempt. Great elevators were erected. The river was deepened and range-lighted by Government. The ocean business failed to come, except in years when coast ports and steamers of all drafts were overtaxed. Much of the elevator machinery went to the junk shop. President Roberts read the burial service as follows before the directors:

"Some years ago you established a steamship company, the American Steamship Company, fostered by your corporation, in which you took four-sevenths of the original stock, afterward increasing that from time to time until you had nearly $1 200 000 invested in the capital stock of that steamship company.

"That $1 200 000 has been totally sunk. In addition to that, this company has faithfully paid the obligations, all these years, at the date of their maturity, so that they have paid in the neighborhood of $2 500 000, more as a contribution on the part of your corporation to endeavor to build up the commerce of this port; not an illiberal contribution, and one which has given the managers of your corporation some anxiety as to whether it was right or proper; but right or not, it is all gone."

Similar bad results are becoming observable at Montreal, New Orleans and Portland, Ore., while Boston, New York and San Francisco, well to seaward, are wearing no funeral colors, and plucky Galveston, seaward and even in the sea, is reviving, and shipping its greatest cargoes to-day. There may be another Philadelphia experience on a scale of hundreds of millions instead of millions. This time a rich nation, instead of a rich corporation, will be asked to take the risk and bear the loss.

Mr. Clarke. THOMAS CURTIS CLARKE, Past-President, Am. Soc. C. E.—All are agreed upon the necessity of some water communication, between Lake Erie and New York, better than that which now exists, as it has shown its incapacity.

Some advocate a 12-ft. canal to pass barges which they hope could navigate the Lakes and avoid transhipment. The speaker formerly favored this plan, as he then believed that the cost of a deeper canal would be prohibitory.

Mr. Clarke.

Mr. Wisner has shown that the cost of a canal 21 ft. deep, from Lake Erie to New York, through the Mohawk Valley, would be $200 000 000, and by way of the St. Lawrence and Lake Champlain $15 000 000 less.

He considers that if the canal were made deeper, the greater interest on its cost and its depreciation would exceed the saving in transportation.

He shows that a 21-ft. canal gives economy of transportation even after adding the charges for interest and depreciation. This is the first time that this statement has been positively made.

The writer agrees with Mr. Wisner in believing that the sum of $200 000 000 is not prohibitory—that a canal for the largest ocean vessels will never be made on account of its enormous cost—and that his figures of cost of transportation are substantially correct.

His reasons are briefly these: Experience has shown that a canal, requiring transhipment of freight at each end, cannot compete with all-rail, or rail and lake navigation.

A canal 12 ft. deep would probably pass tows of barges large enough to navigate the Lakes. It would, however, require transhipment at sea ports, and could not compete in economy with a canal large enough to pass Lake and ocean vessels, if the cost of such a canal did not stand in the way.

In the speaker's opinion, Mr. Wisner has proved that its cost would not be great enough to overbalance its economy of transportation, if made 21 ft. deep.

Can steamers, fit to navigate the Lakes and pass this canal, navigate the ocean economically?

The answer is, that such vessels, or even smaller ones, have done so for many years. What is to prevent a tow of large schooners with five or six masts and having auxiliary power, from passing through such a canal?

It is perfectly true that larger vessels, such as those running between New York and European ports, would show greater economy on the ocean, but not in artificial waterways.

It is believed that the Champlain Route would be less opposed by vested interests than the Mohawk Route, as the destruction of property would be much less.

The estimates of damages are:

By Mohawk Route	$14 039 000
" Champlain Route	6 259 000

Private capital probably would not invest in either undertaking; nor would the State of New York do so without an entire change of its present policy, which seems unlikely.

The United States—the richest power in the world—can easily afford to construct this line of communication. It is necessary to first

Mr. Clarke. get a congressional majority in its favor. Some years ago that would have been impossible. Now, it seems to be possible.

The City of Chicago, after having expended $34 000 000 on her drainage canal, 24 ft. deep and large enough for Lake vessels, has offered it to the General Government on condition that the same scale of navigation shall be continued to the Mississippi River.

L. E. Cooley, M. Am. Soc. C. E., in a paper presented to The Western Society of Engineers, estimates the cost of a 20-ft. waterway from Chicago to St. Louis, 360 miles long, at not over $140 000 000.

The problem of extending the same depth from St. Louis to the mouth of the Mississippi, some 1 100 to 1 200 miles, is as yet unsolved.

There is much to be said in favor of the plan advanced by James A. Seddon, M. Am. Soc. C. E. (described in a paper presented to The Western Society of Engineers), for preventing floods and maintaining depth of water in the Lower Mississippi by the construction of a reservoir system formed in the bottom lands between the river and the bluffs. These reservoirs would retain flood waters, and let them out during seasons of low water.

Thorough surveys should be made by the United States Government to ascertain the facts. Mr. Seddon's estimate—$50 000 000—seems to be low.

The interesting fact is, that a union of interests between the western, southwestern and eastern States can now possibly be brought about to construct this great system of internal navigation from New York to Chicago and thence *via* New Orleans to the Gulf of Mexico.

The proposed Isthmian Canal is a vital part of this scheme. If that canal alone were built it would give only the Atlantic cities and New Orleans a short line to the Pacific, and the Pacific cities one to the Atlantic. If the entire scheme were carried out, it would take in the whole central part of the United States.

The total cost, including the Isthmian Canal, would probably not exceed $600 000 000. Some years ago that sum would have shocked careful financiers. But now that United States bonds can be placed bearing 2%, it means a yearly tax of $12 000 000, or about 16 cents per head of the population.

The United States has now become a World power, exporting its agricultural and manufactured products and coal, iron and copper to all countries. Its imports and exports have increased nearly 50% during the last six years, or from $1 710 000 000 to $2 427 000 000. The exports alone have risen from $921 000 000 to nearly $1 500 000 000. Such an increase has never before been seen in the history of the world.

If we wish to preserve this enormous trade, in the face of the sharper competition which must inevitably soon take place, we must give it the least expensive routes of transportation. The railways will take

care of it during the season of closed navigation; during the open months, such a scheme as has been outlined is necessary to enable all of our States to reach countries on both the Atlantic and Pacific Oceans by the shortest and least expensive routes. Mr. Clarke.

Nature has already done much to create this vast internal navigation, and we would be lacking in energy if we did not apply from our surplus revenues the sum necessary to complete what Nature has begun.

When this is done, the seat of the world's commercial empire will be transferred from Europe to that part of the United States where grain, coal, iron and copper lie side by side, controlled by the most energetic and inventive people that the world has ever seen.

EDWARD P. NORTH, M. Am. Soc. C. E.—It is a matter for congratulation that members of this Society turn from considering expert measurements of quantities to questions involving large additions to the service and convenience of mankind, and essay to direct those large sources of power in Nature—public opinion and the service of capital—to increasing the facilities for transportation and decreasing the cost of distribution over large areas of this country, as in the two admirable papers before us. Mr. North.

It is also pleasing to notice that both authors, though proceeding by different routes, reach substantially the same conclusions, viz., that with near-by railroad trains carrying 2 400 tons and a 14-ft. canal parallel to it, a 10-ft. canal may influence freight charges, but can neither control such charges, direct the course of commerce, nor prevent the present system of discriminating charges against production and consumption in this country generally and the commerce and industries of the City of New York in particular. And any money invested in procuring such inadequate transportation facilities must fail to return a normal reward.

In view, however, of the rapidly expanding trade of the Great Lakes, the determination to make their governing depth 24 ft. and the probability of a waterway of large capacity connecting the Lakes with the Gulf of Mexico, doubts may be felt as to the advisability of limiting to 21 ft. the depth of what should be one of the largest channels of distribution, or as to imposing tolls on a traffic which it is wished to develop. The toll proposed is not large, either absolutely or in relation to the probable value of the goods to be transported, 2 to 2½% on the value of the goods passing The Soo, and will not operate as does the yearly abstraction of $15 000 000 or $16 000 000 from the resources of those trading through the Suez Canal; a most onerous impost that must in part account for a tonnage of 8 000 000 or 9 000 000 sufficing for the wants, or capacity, of 1 100 000 000 people, while the 40 000 000 served by our lake carriers use 40 000 000 tons.

But to some extent any toll decreases traffic, principally by preventing or hindering the production and marketing of low-valued commodities, also by diverting travel or freights to other routes. The

Mr. North. marked differences in the development of freight and passenger traffics in this country on the one hand, and in European countries on the other, seem to show the effect of high and low charges on the volume of traffic. Poor's Manual gives very full statistics of the traffic on all our railroads, commencing with 1882. From these data Table No. 7 has been made, excepting in the columns as to population, which for 1899 has been taken as 75 500 000.

Here, the average passenger rate is persistently high; higher, it is thought, than in other countries. The freight rate, which in 1882 was lower than in any other country and about half the passenger-mile rate, is now, according to Mulhall, less than half the freight rates in any European country, and 36.3% of the passenger rate. As a consequence, the passenger mileage has increased 93.3%, while the ton mileage has increased 223.1 per cent. In European countries, on the other hand, there has been conservatism in the matter of freight rates; but, through zone tariffs, improvements in the comforts of third-class carriages, etc., average passenger rates have been materially reduced, and though there is no knowledge of such full statistics as those for American roads it is thought safe to say that the passenger traffic has had a growth approximating to that of American freights, while the freight traffic has developed on lines somewhat parallel with the development of passenger movement on American railroads.

TABLE No. 7.

Year.	Miles operated.	Rate per Mile, in Cents.		Miles transported. 1 = 1 000 000.		Gross Receipts. 1 = 1 000.		Transported per Mile Operated.		Transported per Inhabitant. Miles.	
		Passengers.	Freight.	Passengers.	Freight.	Passengers.	Freight.	Passengers.	Freight.	Passengers.	Freight.
1882....	95 725	2 447	1 236	7 688	39 302	188 137	485 778	80 313	410 572	146	748
1899....	186 280	2 002	0 726	14 860	126 992	397 560	922 436	79 770	681 725	197	1 682
Inc. or Dec..	+94.6	−18.2	−41.2	+93.3	+223.1	+101.7	+89.9	−0.007	+66.0	+34.9	+124.9

It may be claimed that the figures given in Table No. 7 do not fairly present the relations existing between our freight and passenger traffics, as no account is taken of the influence of electric railways paralleling our railroads, and the decrease in passenger traffic since 1894. But the effort of continued conservatism in upholding passenger rates, after a full recognition of the fact that a reduction would develop

such large accessions to the traffic as to require additional accommodations, is exactly the point for which attention is solicited. By this general conservatism, which is well typified by the story of the New England directorate which refused its commuters' request for an additional train with the assertion that in a short time they would crowd it as uncomfortably as the trains they then had, the railroads of this country possessing a right of way with much if not all of the necessary grading and track laying done, and its passenger department organized, have forced their patrons into the expense of new organizations, new rights of way, grading and track, and experiments with an undeveloped motive power, that the circulation of passengers on lines parallel to their tracks might not be unduly curtailed by high rates of passage. This does not, it is submitted, vitiate the deductions apparent from the table. Nor does the fact that the electric roads are likely to add materially to the convenience and wealth of the country throw doubt on the inference that if passenger fares had been reduced as freight rates have, the passenger mileage would have kept pace with the freight mileage. Mr. North.

The comprehensive and valuable system of statistics inaugurated at The Soo in 1887 and continued by the United States Engineers since then, statistics which have done so much to strip the value from those conservative predictions which vested interests will always call expert and authoritative, show that, for the five years ending with 1891, 37 352 066 tons of freight were carried for $45 913 715, or $1.23 per ton; and for the five years ending with 1899, 96 774 880 tons of freight were carried for $77 065 077, or $0.796 per ton. Here a fall of one-third, in the cost of transportation, was paralleled by an increase of nearly two and two-thirds times, in the freight transported. Or, to use another standard of measurement: During the first period mentioned, the freight charges on cargoes valued at $12.73 per ton were 9.66% of such values; and during the last on cargoes valued at $11.23 per ton, the freight charges were only 7.09 per cent. The average hauls were, for the first period 806 miles, and for the second 835 miles.

A case somewhat analogous to the foregoing was mentioned by Mr. Vreeland, President of the Metropolitan Surface Railway of New York City: On the Madison Avenue line the time of a trip with horses was about 1½ hours, and the road carried 17 000 000 passengers. On the introduction of electricity the time was shortened to about 1 hour and the road carried 50 000 000 passengers. In the first case a reduction of one-third in the cost of service was accompanied by an increase of freight carried by two and two-thirds times. In the other, a reduction of one-third in time without reduction in cost of passage was accompanied by an equal increase in the number of passengers carried.

Unless the above-quoted figures are inappropriate, they show that if we are to have the full encouragement to production and consump-

Mr. North. tion due to a waterway convenient for the exchanges between the 40 000 000 tons of traffic on the Great Lakes and the 79 000 000 of entrances and clearances at the Port of New York, as now estimated, the traffic should pass through ample channels unvexed by toll gates as well as transfer charges. But if the prospect of tolls is necessary to induce the construction of the canal, they should be imposed, as there would still be left a large margin of profit in comparison with present rates.

It may be well at this place to recapitulate the work done and the projects brought forward for cheapening transportation between the Great Lakes and the harbor of New York. Commenced in 1817 and completed in 1825, with a cross-section of 136 sq. ft., the Erie Canal added so much to the wealth of the State that it was determined in 1835 to enlarge its cross-section to 429 ft., an increase of 215 per cent. This enlargement was not legally completed until 1862, although the increased cross-section was available for traffic some years before that date. The law authorizing this enlargement prescribed a nearly rigorous adherence to the original location. In 1897, work was commenced on a further enlargement to 562 ft., an increase of 23 per cent. This was to be accomplished on the plan advised by Horatio Seymour, M. Am. Soc. C. E., then State Engineer, by deepening the prism from 7 to 9 ft. An appropriation of $9 000 000 was made for this purpose, no improvement on the original location being contemplated. In the summer of 1898, the appropriation being exhausted, work was suspended, the State Engineer estimating that it would cost $4 800 000 to finish the improvement of the Erie, Oswego and Champlain Canals. Work has not been resumed.

Tolls, which in their aggregate are greater than all the expenditures on the canals of the State up to the present time, excepting for interest, were collected on local and through freights until 1882.

In 1884, Elnathan Sweet, M. Am. Soc. C. E., State Engineer of New York, read a paper before this Society advocating the enlargement and relocation of the Erie Canal so that it should pass the largest lake vessels and be fed for its entire length from Lake Erie.

In conformity with a law of March 2d, 1895, a mixed commission, consisting of three citizens of the United States and three of Canada, reported in January, 1897, recommending, as was thought probable, surveys of two routes, both through Lake Ontario, one *via* the St. Lawrence and Lake Champlain, the other through Oneida Lake and the Mohawk Valley.

An appropriation bill of 1897 provided for the appointment of a board of three engineers known as the Board of Engineers on Deep Waterways, to make surveys, examinations and estimates for a deep waterway between the Great Lakes and the Atlantic tide-waters. This board was confined in its power to making surveys, etc., on the

routes recommended by the Deep Waterways Commission, which reported in January of that year. This report has not yet been given to the public, but some of its conclusions are given in the paper by Mr. Wisner, who was a member of this board. Mr. North.

In 1897, under requirement of the River and Harbor Act of 1896, directing the Secretary of War "to cause to be made accurate examinations and estimates of cost of construction of a ship canal by the most practicable route, wholly within the United States, from the Great Lakes to the navigable waters of the Hudson River, of sufficient capacity to transport the tonnage of the Lakes to the sea," Major Thomas W. Symons, M. Am. Soc. C. E., reported on what he calls "the broader view of the subject," recommending a ship canal through Lake Ontario, water which is owned in common by this country and Great Britain. That on completion of the Seymour plan, and the removal of restrictions as to capital of companies operating on the canal, the Erie Canal would "give commercial advantages practically equal to the commercial advantages which would be given by a ship canal." That a barge canal, with a cross-section of 1 208 sq. ft., "will enable freight to be transported between the East and West at a lower rate than could a ship canal (cross-section 4 176 ft.) navigated by large lake or ocean vessels."

Actuated, possibly, by the political uses made of the failure of a popular estimate of cost to complete the enlargement provided for in "the $9 000 000 act," the Governor of this State asked five eminent engineers and business men in connection with the State Engineer and State Superintendent of Public Works to form themselves into a "Committee on the Canals of New York State." This committee reported, in January, 1900: That the canals should not be abandoned:

"That the project of a ship canal to enable vessels to pass from the Upper Lakes to New York City (or beyond) without breaking bulk is a proper consideration for the Federal Government, but not by the State of New York."

They also recommended a modification of Major Symons' barge canal, cutting down the maximum size of the boat usable from 200 × 30 × 10 ft. to 150 × 25 × 10 ft., thus reducing the capacity from 1 500 to 1 000 tons. This, by some unexplained mistake, is characterized as enlarging the waterway of the State of New York to its "utmost limit."

In the spirit of this report, a law appropriating $200 000 for a survey for a canal of the recommended size was passed in the spring of 1900, limiting the survey to "the present route of said canal," excepting some minor changes between Buffalo and Syracuse. This estimate is expected in January, 1901.

Lastly, "The New York Commerce Commission," consisting of five eminent citizens, appointed in 1898, made a report in January, 1900, setting forth, among other conclusions:

Mr. North. "The decline in New York's commerce has been steady and continuous for many years; it has been more pronounced during recent years, and has now reached serious proportions in actual loss of exports."

"The loss to New York is due in great measure to a discrimination against New York in railroad rates imposed by an agreement, known as the differential agreement, between the trunk-line railroads of the American Atlantic sea ports, including the New York railroads."

"The differential rate applies, not only on products destined for export, but also destined for local consumption by the people of New York City."

"The State has it within its power, through an adequate improvement of the Erie Canal, not only to apply the remedy that will secure it against further loss of its commerce, but will secure to it, as well, the restoration of that which has already been diverted."

The commission recommended the completion of the current enlargement, with a cross-section of 562 sq. ft., as adequate, acquiring additional terminal grounds and piers, and that certain laws be amended, particularly as to elevating grain.

This arrangement between the trunk lines reaching the Atlantic seaboard, giving differential rates on grain, both for domestic consumption and for export, is, in the aggregate, a serious injury to our farmers. The cheapest route for export grain *via* Atlantic ports is through New York, but, in favor of other roads and other cities, grain for New York has been for years penalized with a differential that amounted to 1.8 cents per bushel on wheat and 1.68 cents on corn; this has been lately reduced to 0.9 cent. During the five years ending with 1899, the average of the average rates charged on wheat, Chicago to Buffalo, as returned by the Merchants' Exchange of Buffalo, has been 1.82 cents.

Of course, it is perfectly competent for a committee of the New York Chamber of Commerce "to look with complacency, if not with actual pleasure" on the consequent diversion of grain exports to other ports. But is the man who grows the grain, which is burdened in any case with a relatively high cost of transportation, to look with any approximation to pleasure on a practice which puts an additional charge on his produce? As the committee says: "An adequate improvement of the Erie Canal," will apply the remedy. Their opinion, however, as to what is adequate, is even more restricted than that of the Committee on Canals of New York State.

It will be noticed that though sixteen sessions of the State Legislature have been held since Mr. Sweet read his able paper before this Society, advocating a radical enlargement of the Erie Canal, and two commissions have been appointed by as many presidents, some influence has seen to it that no survey has been made for a ship canal which would not be of greater use to Canada than to the United States. And it is possible to assert that a route through the richest and at one time the most prosperous section of this State "is not available for a

ship canal," although no one can show any reason that on a judicious location a canal of large cross-section should cost more per mile between Buffalo and Oneida Lake than between that lake and the Hudson River. Mr. North.

Agreeing fully with Mr. Wisner that: "The dimensions which should be given to a waterway between the Lakes and the Atlantic depends upon the economic depth which can be obtained and maintained in the water routes to be connected," and accepting his estimate that the interest and fixed charges for an increased depth in the Lakes from 21 to 30 ft. would amount to an annual charge of $2 205 000, or 5.51 cents per ton on the traffic now passing Detroit, but taking exception to his declaration: "The actual saving in the cost of transportation on the Lakes by using deeper draft vessels is only about one-half the above estimated fixed charges," and differing from the deduction, "the interest and maintenance account will exceed any expected returns from lower transportation rates;" reference is made to the previously mentioned valuable statistics of the traffic at The Soo.

Although, as set forth in the Report of the Committee on Canals of New York State, previously referred to, the railroads were able to partially starve the Lake marine when there was a depth of only 11½ ft. on the miter sills of the locks at The Soo; the traffic through that waterway since the opening of the Weitzel, or 16-ft., lock in 1881, has been phenomenal. Opened in 1855 to a traffic of 106 296 tons, its traffic for 1881 was only 2 092 757 tons, but in 1895, the year the Canadian, or 20-ft., lock was opened, it had increased to 16 806 781 tons. In 1899, 21 956 347 net registered tons of shipping passed through the locks at The Soo, carrying 25 255 816 net tons of freight valued at $281 364 750. This freight was carried an average distance of 827 miles for $21 959 707, or at the rate of 1.05 mills per ton-mile, which is about 13% higher than the average cost for the three preceding years.

During the thirteen years for which we have returns covering the tons and value of freight, the distance carried and the cost of carriage, as well as the registered tonnage, the cost per ton-mile has decreased from 2.30 mills in 1887, when 5 494 649 tons were carried, to 1.05 mills in 1899, when 25 255 810 tons were carried. This is a decrease concurrent with the deepening of the Lake channels, and largely dependent on such deepening, and justifies the prediction the president of the Great Northern Railroad made while discussing the project of General O. M. Poe, M. Am. Soc. C. E., for a 21-ft. channel: That it would cut the Lake freight rates in two.

Mr. Wisner estimates the annual charge to the 40 000 000 tons passing through the Detroit River, due to deepening the Lake channels and harbors to 30 ft. as 5.51 cents per ton. If the length of haul by Detroit is equal to that through The Soo, 827 miles, the cost of deepen-

Mr. North. ing the Lake channels and harbors will be 0.076 mill per ton-mile. This would set the cost of transportation back to about the average of 1892 and 1893, if there was neither reduction in freight charges due to the larger channels nor increased production of freight to be transported. Neither of these suppositions is possible. The increased charge of 5.51 cents per ton would be divided by 80 000 000 within a few years after the deepening was effected, and while the larger carriers might not again cut freight rates in two, they could be expected to cut off about one-third of the present rates, leaving an average freight rate of 0.6 mill per ton-mile. This does not seem an unprofitable rate, in view of the volume and steadiness of the traffic and the average sea rates.

The assumption, that the traffic passing Detroit soon after, if not with, the earliest possible completion of 30-ft. channels and harbors will be 80 000 000 tons, is not thought too large for realization, as the traffic for some time, as shown by Mr. Mayer, has been doubling every six years, and the Chief of Engineers is reporting to Congress that prompt action in doubling the capacity of the St. Clair Flats Canal is necessary and important to the safety and convenience of the vast commerce of the Great Lakes. And deepening the channel to 23 ft. has already been commenced in the St. Mary's River.

Proposals have been made that the Erie Canal should be given to some railroad as a right of way, and numerous other schemes have been proposed like "the $9 000 000 plan," to impose a general hardship to the financial advantage of a few transhippers at Buffalo and New York. And lastly, we have the project to be submitted to the next Legislature of New York State to abrogate the necessity of transhipment at Buffalo by imposing a high rate of freight on the Lakes. The possibility of this perspective, however, has doubt cast upon it by the vociferous demands of "the friends of the canal," that large expenditures shall be made at both the Lake and Atlantic termini, that freight may be transferred more conveniently. All of these plans oppose the interests of both producers and consumers.

It is to the advantage of the City of New York and the New Jersey cities on the bay that this improvement should be so ample and commodious that no better or cheaper route between the interior of the country and the seaboard can be reasonably expected.

It is to the advantage of the State of New York that the canal should pass through the State and should be so large that the factory sites along its banks may have all the facilities for manufacturing offered by low freight rates, without transhipment, that the cities on the Lake front now have.

It is to the advantage of those living in the country to the west of this canal that it should be so located as to build up centers of production and consumption as near to them as practicable, and that its

capacity for speedy and economic traffic should give the greatest possible encouragement to marketing those low-cost products which cannot stand railroad charges; as well as reducing the cost of all freight transportation, that the producer and consumer may have more to divide between them. Mr. North.

It is also to the advantage of the whole country that many waterways should be constructed or developed, that their control over railroad freight rates, as described twenty years ago by the late Albert Fink, Past-President of this Society, may be maintained and increased.

It is fortunate that the Hudson River carries a depth of 30 ft., excepting at a few unimportant points, nearly to Coxsackie.* This leaves only about 375 miles, out of approximately 1 500, to be improved. With an economic depth of 30 ft. obtained in the Great Lakes, there seems to be no doubt that the channels connecting their commerce with our cities on the Atlantic coast and with the water front of the whole world should have an equal depth.

As no survey has been made for a ship canal between Lockport and Syracuse, an approximate estimate of cost may be made by adding 50% to the estimate of the Board of Engineers on Deep Waterways for the low-level, Oswego-Mohawk 30-ft. plan from Duluth to New York, given in Table No. 2, viz., $317 284 348, as covering the increased cost of building the canal wholly within the United States, and a further addition of $36 000 000, the estimated cost of improving the Lake harbors. The aggregate, $512 000 000, at 3% interest, and 0.5% maintenance, would give an annual charge of $17 920 000. This sum would be directly repaid to the country by a saving of 0.27 mill per ton-mile on 80 000 000 tons carried an average distance of 827 miles.

The sum of 0.27 mill per ton-mile necessary to be saved on the estimated Lake traffic, when the channels from Duluth to New York are deepened to 30 ft., to directly recoup the country for the total expenditure would be reduced by the additional haul of 500 miles on all freight either destined to or originating in the Port of New York or beyond it. There would also be a further reduction due to eliminating the cost of transhipment at Buffalo, which Major Symons puts in his report as 27.1 cents per ton for package freight. These direct savings can be partly estimated, but the indirect saving due to the absence of delay and damage inseparable from transhipment cannot be computed with accuracy.

In estimating the amount of traffic attracted to a 30-ft. canal of sufficient cross-section, notice should be taken of the facts that there is no other people able to employ a ton-mileage equal to ours. The traffic passing Detroit is larger than any other water-borne traffic through one channel and is growing rapidly. The entrances and clearances at the Port of New York are more than twice those of any other seaboard port.

* From there to Troy the State and General Governments have increased the depth from 4 to nominally 12 ft.

Mr. North. The traffic of this port has for some time been estimated at 70 000 000 tons. E. L. Corthell, M. Am. Soc. C. E., after careful research, preparatory to presenting a paper, "The Harbors of the World," to the VIIIth Congress on Navigation, held at Paris in 1900, states that the total entrances and clearances at this port are 79 544 653 tons. This tonnage he divides as follows:

In the interior trade	25 093 000	tons.
" coastwise	39 250 000	"
" foreign*	15 201 653	"

As a matter of comparison, the entrances and clearances are:

For Liverpool	25 093 000	tons.
" London	30 215 000	"

It will be noticed that less than 20% of the tonnage of this port is engaged in foreign trade, and the aggregate tonnage of Liverpool and London, foreign and domestic, is 16% less than the domestic tonnage of this port.

Some nine years ago a board of engineer officers on the improvement of the Hudson River reported that the traffic of this stream was 18 500 000 tons, 3 500 000 of which were contributed by the State canals. Although it may seem incredible, it is a fact that many of the best thinkers to be found in the City of New York oppose materially increasing this contribution of the canals to the tonnage of their port, on the plea that the grain-handlers here and in Buffalo will be deprived of their opportunity to penalize production in the Northwest by exorbitant transfer charges on grain for export.

The population and wealth of the Port of New York is not dependent on its exports. Its unique natural position, making it the largest "port of call" in the world, is due to its having two large waterways back of it, viz., the Hudson and East Rivers. Adding to its natural advantages, the construction of the Erie Canal forced all railroad terminals to concentrate here, making its position the best in the world for manufacturing; for here raw products for manufacture can be assembled, and, the finished materials distributed to consumers at less cost than elsewhere, and, both in value of merchandise produced and in amount of wages paid, it is peerless. The present census will probably show that the wages paid in manufacturing industries in the cities and towns on the Bay of New York are nearly equal to the value of exports from the Port, and provision for traffic through the proposed canal that neglects the demand of New York and the cities ancillary to it on the Atlantic coast will fail in economic value, not only to the Atlantic front of our country, but to the interior.

No estimate of the freight entering and leaving the canal should

* A writer in the *Forum*, December, 1900, states that the total entrances and clearances at the ports of the United Kingdom for 1899 were 209 348 081 tons. Of this, 97 782 887 tons were entered and cleared in the foreign trade.

neglect the manufacturing potentialities presented by the banks of the proposed waterway, which for nearly its whole length would offer factory sites with water and rail communication. The present status of manufacturing enterprises in the canal towns and cities is pathetic. From the inception of work on the Erie Canal until forty or fifty years ago this region offered higher rewards to ambition than any other locality, enjoying the cheapest and most convenient transportation facilities. Since the Erie Canal lost its pre-eminence as a carrier, through the development of railroads, while it remained unimproved, and rates in discrimination against short hauls have been imposed, the advantages offered by cheap transportation to other localities have worked injury to the interests of the center of the State. Mr. North.

This fact has been recognized as to agricultural lands for some time. The twelfth census shows that in seven cities of 25 000 or more inhabitants in the center of the State, viz., Troy, Albany, Schenectady, Utica, Syracuse, Rochester and Auburn, all the cities of that class in the interior of the State, the rate of increase since 1880 has not been quite 52%, one city showing an absolute loss. The City of New York, with the dependent cities in this class, Jersey City, Newark, etc., show a gain of 81%; while the twelve cities on the Great Lakes, which have over 25 000 each, show a gain for the twenty years of 168.5 per cent. So low have our canal towns fallen in the estimation of some that they regard any attempt to relight the fires in their abandoned factories, or rebuild their ruined smelting furnaces as an attack on some inexorable law.

It is not proposed at this time to labor with the education of those who have been taught that production at one locality is always at the expense of another, and that the relative, to absolute, decline in value and population of the country along the line of the Erie Canal has aided the development of either the City of New York or the cities on the Great Lakes, but simply to assert the conviction that if the center of this State had continued the rate of development attained in the early canal period, both the lake front cities and New York would offer even higher prizes to ambition than under present conditions.

About two-thirds of the coal produced in Pennsylvania is mined east of the line of equal distances between Cleveland and the head of Seneca Lake. This should give shipping ports at the heads of Cayuga and Seneca Lakes the control of the anthracite trade of the upper Lakes, the control of all coal shipments to Lake Ontario and the upper St. Lawrence, and allow these ports to mediate effectively in the coal trade of the Atlantic coast.

The important influence of this proximity of coal on the manufacturing potentiality of this State, when combined with cheap and ample transportation, and particularly to that portion of the State

Mr. North. which has been so carefully excluded from all plans for a ship canal cannot easily be overestimated. Over 50% of the tonnage on the canal at present is local, and under improved conditions it is possible, if not probable, that the same proportion of any traffic will be stopped either for consumption or to be reshipped with advanced value from manufacture.

At a deep waterways convention it was asserted by the President of the Lake Carriers' Association that two-thirds of the freight passing Detroit was on its way to be advanced in value by manufacture. If we can expect 80 000 000 tons to pass Detroit by the time Lake channels and the Erie Canal could be deepened to 30 ft., and that the tendency to advanced manufacture on the Lakes is offset by the increased production of low-value goods due to cheaper freight rates, the average value per ton remaining as in the past, $11.14, the aggregate value transported will be $891 000 000. So moderate an assumption as that the two-thirds to be manufactured would be doubled in value by that process gives an ultimate value of $1 188 000 000 for the portion under consideration. If 65% of the increased value went to labor, it would employ 772 000 men at $500 a year; this would, as usually computed, support a population of over 3 500 000.

If one-third of the advance in value to Lake freightage by manufacture was given along the canal and in the City of New York, an addition of 1 200 000, or 17% to the population of the State would ensue, and to that extent would increase the local markets of our farmers. As the average capital of the inhabitants of this State will probably be shown by the present census to equal $1 250, such an addition to our population would increase the wealth of the State by $1 500 000 000. If the foregoing figures are approximately correct, the opinion that a ship canal should not "receive any serious consideration from the State of New York" might be sufficiently revised to permit further examination of the subject.

It is difficult to make an estimate that will be satisfactory to all, covering the proportion of Lake commerce which will enter a canal of sufficient cross-section: to be retained for consumption or manufacture in the State of New York, to be distributed to points in Pennsylvania and Lower Canada, or along the Atlantic coast, and to be exported. The City of New York and its environs could easily furnish capital and labor to absorb any possible increase of raw material offered for manufacture. The superior advantages presented by both banks of the enlarged canal from Buffalo to Albany would possibly give it a larger percentage of the total traffic than the present canal does, viz., 53 per cent. And the fact that 80% of the entrances and clearances at the port of New York are in the domestic trade, may justify an expectation that about half of the Lake freightage would enter or pass through the canal.

Mr. North.

Although all sorts of statements have been made asserting the unsuitableness of vessels with a draft of 8 ft. and more, even in deepened lake channels, no one is known to have had the temerity to put a limit on the growth of the Lake traffic, and it would evidently be unwise to construct a canal too small to carry a traffic of less than 40 000 000 tons with the economy dependent on speed and convenience. An approximate estimate of the cost of such a canal, without reference to deepening the Lake channels, may be made by subtracting Mr. Wisner's $27 000 000, the cost of deepening the Lake channels, from the before-used $317 284 348, and adding 50% for building the canal through the United States. At 3% interest and 0.5% maintenance, the annual charge would be, say, $15 250 000.

If a probable ton-mile rate of 0.8 mill is accepted as fair for Lake freights—and for 1898 the average rate was 0.79 mill—and a retardation of 50% for the entire distance of 500 miles between Buffalo and New York is assumed, the freight charge should be 60 cents per ton.

If, on the other hand, a railroad rate of 3 mills per ton-mile for the 440 miles between Buffalo and New York is taken, the cost will be $1.32, making a difference of 72 cents per ton. At this rate, the transit of less than 22 000 000 tons would be required to directly pay the country for the expenditure. This amount is less than the average freight through the Soo for the three years ending with 1899. Obviously, to contend that, neglecting entirely all incidental benefits, there could be any loss to the country by constructing a 30-ft. canal through the State, it is necessary to show that either the estimate of cost is too low or an insufficient allowance for retardation has been made.

As the decisive element in this discussion hinges on the relation between the possible cost of a deep waterway from Chicago and Duluth to the Port of New York, and the probable saving in the cost of transporting the immense volume of freightage to be developed by the improved waterway, indulgence is asked for the presentation of another postulate: If it is thought that the previous estimate should be doubled to cover all contingencies, instead of having 50% added as before, the annual charge would become $22 190 000. Assuming that the expected 80 000 000 tons on the Lakes were carried the present average distance at a saving of 0.25 mill per ton-mile, it would leave $5 650 000 to be earned on the canal. If there was no larger saving per ton carried in the canal than the cost of transferring package freight at Buffalo, viz., 27.1 cents per ton, as mentioned by Major Symons, it would require only 21 000 000 tons to pass through the improved canal to repay the country the interest and maintenance on the entire outlay.

The above-estimated cost is supposed to be greatly in excess of any necessary expenditure, and, in view of the fact that the Government is borrowing money at 2%, the allowance of 3.5% for interest and

Mr. North. maintenance will be thought sufficient. Deepening the Lake channels from 16 to 21 ft., and enlarging the locks, has resulted in reducing freight rates from 2 to 1 mill, and it is not probable that anyone will contend that a further deepening from 21 to 30 ft. would not force a greater reduction in rates than ½ mill. The only ground for attacking the estimate appears to require a prediction that our producing, manufacturing and consuming capacity is about to be paralyzed.

Although a very large sum, compared with the cost of preceding public or corporate improvements, will be required for this work, it is suggested that there is no necessity for a partial improvement. The United States has not only abundant credit, but the necessary capital in hand to carry the enterprise to a prompt completion. And it is confidently predicted that in no year will the expenditures on this improved waterway exceed our loans of the current year to foreign countries,—loans that aid them in industrial competition.

In a paper on the Suez Canal, lately read before the Institution of Civil Engineers, London, by Sir Charles Hartley, it is stated that questions addressed to many captains and pilots navigating that canal showed a consensus of opinion in favor of a canal that could pass vessels at a speed of 8 knots—9.22 miles—per hour. For this, 3 to 3½ ft. under the bottom of the vessel, with 1½ clear beam widths between vessels when passing, and 40 ft. between the vessel and the bank was desired, with an ultimate depth of 32 ft. 9 ins.

The present scheme for enlargement provides for a depth of 29 ft. 6 ins., a bottom width of 200 ft., or a cross-section of 8 075 sq. ft. Mr. Lionel B. Wells wrote, in discussion of that paper, that with a vessel said to have a speed of 11 knots in the open sea, he made 8 knots per hour in the canal. The cross-section of the vessel was 819 sq. ft., and the probable cross-section of the canal 5 384 sq. ft., or about 6½ times the cross-section of the vessel.

The Bureau of Statistics, in its "Miscellaneous Tables," gives the mean duration of passage of vessels navigating the Suez Canal by night and day as 17 hours 16 minutes, or an average of 5.8 miles per hour, including the necessary delays at "gares," as often as vessels meet.

It is noticeable that a depth of 29 ft. 6 ins. is being obtained, and a depth of 32 ft. 6 ins. is expected in a canal that for some time has not had half the traffic of The Soo, and probably will never pass one-fourth of the traffic of the Detroit River. The paper of Sir Charles Hartley, with the discussion, strengthens the general conviction that the speed of boats in contracted channels increases faster than the ratio of the cross-sections of the channels to that of the boats; and any desired speed can be maintained in a canal of sufficient size. The remarkable absence of accidents delaying traffic in the Suez Canal gives strength to the theory that accidents are less to be dreaded in

canals than in comparatively short passages of restricted width in the course of a free navigation. And it should be remembered that while a small canal is impracticable for a large vessel, the small vessel moves with less expense than in a more contracted channel. Mr. North.

It seems that with a channel of sufficient cross-section to conveniently accommodate the tonnage which may be fairly expected, the saving in the cost of the freight carried will more than pay the assumed rates of interest and maintenance. And that, without regard to indirect and collateral benefits, the investment will afford a profit to the public after paying interest and maintenance charges.

With these considerations, it does not seem entirely proper to imagine a canal of inadequate capacity and predicate on that conception the necessity of high freight rates because of a speed of "only 5 miles per hour."

The very full and complete statistics kept by the engineer corps at The Soo, which cover the last thirteen years, show that concurrently with deepening and enlarging the Lake channels, and, it is confidently thought, in consequence of such deepening, the tons carried in 1887, 5 494 649, increased to 25 255 810 in 1899, while the cost for such carriage increased from $10 075 153 to $21 959 707. A very important relation of this traffic to the production and wealth of the country is shown by the fact that, while the value per ton was $13.83 in 1887, reaching its minimum, $10.59, in 1895, and averaging $11.14 for the thirteen years, the cost of transportation, compared with the value of the freight, has decreased from an average of 10.88% for the three years ending with 1889, to 6.34% for the three years ending with 1898, having averaged for the thirteen years 7.84% of the value of the goods transported an average distance of 827 miles.

It is unnecessary to explain the effect of this low freight rate on the production and wealth of this country, much of the produce having been sold on Lake Erie docks for less than the railroad charges between the points of production and sale, according to E. S. Wheeler, M. Am. Soc. C. E., for some time superintendent of the locks at The Soo. The cost of transportation by railroads would probably have prevented the production of fully half the freight carried, as they would have left only about 45% of the value to the producers. And the low freight rates on the Lakes have been an important if not governing factor in our ability in the nine months ending with September 30th, 1900, to sell manufactured goods valued at over $338 000 000 to foreigners. During the period under consideration the total value of the 169 333 701 tons passing through The Soo was $1 986 374 573, and $155 798 436 was received for its carriage on the Lakes. If this ton-mileage had paid railroad rates, it would have cost over $1 000 000 more. This sum applied to the total expenditures of the country for rivers and harbors—$320 000 000—gives a very satisfactory dividend.

Mr. North. Comparing the railroad rates of 1899 with the water rates, there is a saving of $129 715 811, or a dividend of 40.5%, on the above-mentioned expenditure.

It is recognized that objections are made to taking the average of all railroad freights in comparison with water rates on lower-classed goods, but as a rough measure the method may be inadequate, in view of the facts that the tonnage through The Soo is about half of the traffic on the Great Lakes, and our average freight rates, which are less than half those of any other country, are declining rapidly. This is due more to the competition developed by our ample Lake channels than to any other cause. If the entire cost of transportation must be taken in some proportion from the resources of both producer and consumer, it is probable that the direct gain to the people served by the enlargement of the locks and channels during the last thirteen years is greater than $2 000 000 000, rather than less than $1 000 000 000. This is in addition to indirect gains from the employment given in producing such part of the $1 986 374 570 as was made possible by low freights and the consumption consequent on such production.

History takes no cognizance of an occasion when a proposition to enlarge channels of distribution and cheapen transportation has been submitted to the public without decided opposition to the project from those having an interest in existing transportation facilities. Most of us remember the Lake Carriers opposition to General Poe's plan for a 20-ft. navigation, and Thomas C. Keefer, Past President of this Society, in a paper—"The Canals of Canada"—read before the Royal Society of Canada in 1893, has preserved a record of some of the influence brought to bear on a Commission appointed by the Dominion Government in 1870 to decide on the depth of the Canadian Canal system.

A ship owner, of Oswego, thought vessels 200 ft. long with a burden of 750 tons "ample for the internal commerce of the Lakes, the Lower Provinces and New England; longer locks would cause expense, a strong current and delay, not warrantable in order to provide for a few and rare cases where large vessels would desire to pass to and from the ocean."

Another, "whose life had been spent in the commerce of the Lakes" said:

"It had been clearly established that vessels of over 700 or 800 tons were not so profitable on the Lakes as vessels of a smaller size. Nature has placed barriers in front of most of our harbors, also wide flats across some of our greatest thoroughfares, that will, in spite of art, for ages to come, make it necessary to build lighter-draft vessels. Sail vessels of over 800 tons could not safely navigate the lakes even if harbors were deep enough."

Mr. Keefer does not make it entirely clear whether this testimony is preserved to palliate the insufficient size of the Canadian canals, or

to show the exuberant imagination developed by the possession of some small wooden schooners, in the minds of otherwise reliable citizens. The conclusions and opinions reproduced bear a striking resemblance to some published thirty years later by the Committee on Canals of New York State. Mr. North.

Notices of large-sized steamships which have been built lately, for as certain an ocean trade as that on the Great Lakes, seem to be responsible for a theory that all over-sea commerce is carried in ocean leviathans, and that if we cannot have ocean leviathans we cannot trade. But the sea-borne exports from the United States are carried in vessels that average 818 tons net register; 32 108 vessels with an aggregate of 26 265 976 net registered tons cleared from our ports in the foreign trade during the fiscal year 1899. The average register of sailing vessels was 371 tons, and of steamships 1 064 tons. As a matter of comparison, the American sail vessels passing through the locks at The Soo during the calendar year 1899 averaged 944 net registered tons, and the American steamers 1 162 tons, or nearly 9½% greater tonnage than those engaged in our foreign trade. And the vessels, sail and steam, carrying our exports to foreign countries averaged only three-quarters of the tonnage of our vessels carrying between Lake Superior and the lower lakes.

The steamships out of the Port of New York, in our export trade, averaged 2 324 tons; those to English and European ports north of Gibraltar, 3 055 tons. To the Mediterranean, the average was 2 104 tons; to South American ports, 1 517 tons, and to West Indian ports, 1 453 tons.

The greater average tonnage to ports north of Gibraltar is due in part to their larger consumption, and in part to our necessity of shipping *via* those countries whose persistent subsidies to steamer lines has enabled them to become distributors of the goods of less alert nations to those small but more profitable markets which are not reached by direct lines from this country.

Although there is no doubt that the large vehicle with full loads always carries freight at a lower cost than the smaller one, the small carrier is necessary to commercial development and occupies the same position to ports of smaller production that our cheaply constructed railroads of the '40's and '50's held in the development of this country. A large commerce is as impossible without development by the small carrier as the four-tracked railroad would be without its forerunner, the cheap road.

Much has been said about barge and other towing on the Lakes, the Atlantic and on rivers. Towing on the Lakes, though theoretically economical, has not proved sufficiently profitable in practice to receive much development. The peculiar fleet-towing on the Ohio and Mississippi, viz., the towboat pushing the fleet or holding back in

Mr. North. bends, gives probably the cheapest transportation known, but that method would be impossible on the Lakes, of doubtful practicability on any canal, impracticable in a stream like the Hudson, where the tide ebbs and flows from New York to Troy, and would be dangerous at times in the Tappan Zee with the style of boats used on the Ohio. String and fleet-towing are both practicable on nearly all rivers, and there seem to be no theoretical objections to its economy; but, excepting fleet-towing on the Hudson, it has not been much developed in this country.

String-towing has for some time been practiced both on the Lakes and ocean, particularly in the coasting trade. At one time the large coal railroads moved substantially all of their coal to eastern ports by towboats and barges. In the dark days between 1893 and 1897 it was thought the coasting schooner was doomed. It is asserted that for one year only one schooner was launched in eastern ship yards. That is now ancient history. For some time there has not been a vacant berth in an eastern yard. A list of this year's launchings shows eighteen 4-masted to 6-masted schooners, averaging over 1 700 tons gross register each. The coal companies will doubtless continue towing barges along the Atlantic coast, but it is doubtful if that method of transportation will again attain the relative importance of four years ago; it, however, does not seem likely to be discontinued, and will probably always present a means of cheap transportation.

Fear is felt by some that foreign governments would subsidize steamer lines to our Lake ports and not only capture the trade of the Lakes, but destroy the ship-building industry established there. Such fear does not seem well founded. No vessel under a foreign flag could do a way business between any ports in the United States, and, though a few foreign steamers would discharge and load cargoes at Lake ports, their entire participation in that traffic would probably not exceed the 2 to 4% of British tonnage now passing The Soo. But it can be confidently predicted that, with a practicable channel to the sea, there are few vessels which would not add to their cost the 8 to 10% necessary to put in condensers and enable them to compete for the very high ocean freight rates now exacted, which are substantially paid by our producers.

This theory, that either American carrying or American ship building would be injured by an ample waterway connecting the Lakes and ocean together, has great doubt thrown upon it by the attitude of the agents of foreign steamships sailing from our ports and that of their employees and other servants. These lines are generally subsidized by the governments whose flag they fly, governments that know what they are paying their money for, and their agents in this country must uphold the interests of such governments, as well as the interests

of the stockholders and those of the closely affiliated railroads in this country, or make way for some one better fitted for the position. It is doubtful if, in the whole category, a single advocate for anything larger than a barge canal can be found. And the fear that a ship canal of ample cross-section would enable these foreign shipowners to increase or even maintain their present high rates for freight and passengers seems to have no more foundation than a fear that the same canal would allow the New York Central to maintain either its general freight rates or its discrimination against the profits of our wheat growers. Mr. North.

The importance, to our national prosperity, of our present low freight rates and their further development in this direction cannot be overestimated. Any history treating of national power, from the dim days when its seat lay on the Euphrates route between India and the then West, to the present, shows, probably without assertion, that possession of the cheapest and most efficient transportation has been necessary for the greatest development of production and wealth.

Forty years ago Great Britain possessed the cheapest internal transportation, and, while seeing the destruction of our merchant marine, was building up its own with subsidies of nearly £1 000 000 per annum. In the aggregate rewards of its industrial energy it was apparently unapproachable. But the principle of conserving vested rights has enabled the managers of her railways to maintain freight rates without sensible diminution during this period, and prevented competition from enlarged canals. Other nations, however, have enlarged their waterways and reduced freight rates. Then, England stood easily first among the nations and this country third. Now, the United States, having the lowest freight rates, stands first, and there is doubt whether England, whose manufacturers and consumers pay more per ton-mile for transporting their freight than any other people, stands second or third in rank.

The managers of our railroads, unable to establish legislative principles analogous to those of England, except possibly along our northern seaboard, have adopted the expedient known as "community in stock ownership" with the apparent, if not avowed, intention of preventing healthy, or other, competition in the future. Success in this plan will probably result in no further diminution of freight rates in this country, unless there is more effective competition from water routes. Few doubt that the opening of the combined Lake and canal route in the spring still influences, and to some extent controls, all freight rates east of the Mississippi, as they did to a greater extent when Mr. Fink gave his testimony. None can doubt that a larger waterway than the present would have more influence; nor can it be denied that the larger the waterway, the greater the influence exerted

Mr. North. by it. The only tenable question is as to the economical size of the canal.

Approaching the discussion of these papers with a predilection in favor of large channels, that some might call a prejudice, a collation of the figures, presented from year to year by the members of the United States Engineer Corps in charge of the locks at The Soo, showing the important items of the traffic, has resulted in astonishment at the combined growth of tonnage and decrease in cost of transportation, both absolutely and in terms of value carried. They show a traffic that is unprecedented for volume per inhabitant served, and is also thought to be unprecedented in both the small value per unit of commodities carried and in the low cost of the service. This is from a lake, the shores of which were visited by the explorers of sixty years ago.

When the first locks at The Soo were opened to traffic we were a poor nation, and capital for small enterprises was secured with difficulty, mostly from foreigners at money-lender's bargains. To-day we seem to be the world's favorite gathering ground for national loans.

A consideration of the curves of increase and decrease, shown by these returns, and their extensions, have appeared to justify the foregoing assumed increases in tonnage and decreases in cost of service. They also are thought to warrant the prediction that no channel from the heads of the Great Lakes to the Port of New York in the service of distribution between the area which seeks outlet on that channel and the Atlantic coast of our country, will be so large as not to be inconveniently crowded by the passing traffic. The direct returns from the lower freight rates, on an ample and commodious channel, will undoubtedly exceed the annual expense to the nation for interest and maintenance.

As the statistics referred to may not be easily accessible to all members of the Society, they are given in Tables Nos. 8 and 9, with the addition of the railroad rate per ton-mile, as given in Poor's Manual, and some calculated deductions.

There is much reiteration by Mr. Wisner of the statement that 21 ft. limits the economical depth for navigation on the Lakes, but probably not more than was used by vessel owners against the first deepening of the waterways, nor more than was urged against General O. M. Poe's splendid project for the 21-ft. channels. The economic size of the carrier is thought to be governed not so much by the length of haul as by the amount of freight offered, which is probably the most important factor in determining the size of the vessel used. That the present limitation of the draft of vessels on the Lakes is not satisfactory to the business interests of Chicago, may be seen from the 24-ft. channel of their Main Drainage Canal, which was commenced in Sep-

tember, 1892, and has been pushed through rock and earth to a navi- Mr. North.
gable conclusion. This depth, however, apparently does not measure the ultimate ambition of Chicago for cheap transportation, as it seems the Drainage Trustees are proposing to deepen the Chicago River to 30 ft.

TABLE No. 8.—NET REGISTERED TONNAGE PASSING THE LOCKS AT SAULT STE. MARIE (THE SOO).

Year.	Tonnage.	Year.	Tonnage.	Year.	Tonnage.	Year.	Tonnage.	Year.	Tonnage.
1855	106 296	1864	571 438	1873	1 204 446	1882	2 468 088	1891	8 400 685
1856	101 458	1865	409 062	1874	1 070 867	1883	2 042 259	1892	10 647 203
1857	180 820	1866	458 530	1875	1 259 534	1884	2 997 837	1893	8 949 754
1858	219 819	1867	556 899	1876	1 541 676	1885	3 035 937	1894	13 110 366
1859	352 642	1868	432 563	1877	1 439 216	1886	4 219 397	1895	16 806 781
1860	403 657	1869	524 885	1878	1 667 136	1887	4 897 598	1896	17 249 418
1861	276 639	1870	690 826	1879	1 677 071	1888	5 130 659	1897	17 619 933
1862	359 612	1871	752 101	1880	1 734 890	1889	7 221 935	1898	18 622 754
1863	507 434	1872	914 735	1881	2 092 757	1890	8 454 435	1899	21 958 347

Lock.	Dimensions.	Date of opening.
First lock; double lift.....	350 × 70 × 11½ ft.	June 18th, 1855.
Weitzel lock.....	515 × 80 × 17 ft.	Sept. 1st, 1881.
Canadian lock.....	900 × 60 × 20 ft.	Sept. 9th, 1895.
Poe lock.....	800 × 100 × 21 ft.	Aug. 3d, 1896.

NOTE.—Deepening the lake channels from 16 ft. to 21 ft., which was commenced in the spring of 1893, is not yet fully completed.

No other demonstration, of the results which will accrue from larger channels, is possible than the increase of tonnage and decrease of freight rates which have followed increasing the depths in The Soo locks from 11½ to 21 ft. Nor should any other be required. Anyone who, 20 years ago, had predicted that the 2 000 000 tons then passing The Soo would expand in the year 1900 into 25 000 000 tons would have been considered of unsound mind. Then, Minnesota, the two Dakotas, Wyoming and Montana, with an area slightly less than 500 000 sq. miles, had a population of 2 per square mile; now, the population is 6. And, though we may not predict another increase of 200% in the next twenty years, a much larger absolute growth is certain if a continued decrease in freight rates is maintained. The same statement seems authorized as to the traffic. The traffic and population being interdependent, and both dependent on the freight rate, their development will be governed by the size of the Lake channels.

The minimum depth on the miter-sills of the next lock built will be 24 ft., and it is possible that the depth will be 30 ft.

It may be doubted if the expected and desirable change in the character and value of materials exported will result in a decreased

TABLE No. 9.—TRAFFIC THROUGH THE LOCKS AT ST. MARY'S FALLS, OTHERWISE CALLED "THE SOO."

Year.	Tons carried.	Ton-miles.	Average distance.	Lake cost of carriage.	Cost per ton-mile.	Average railroad freight rates per ton-mile.	Railroad cost for equal ton-miles.	Total value of freight carried.	Value per ton.	Percentage of value paid for transportation.
(1)	(2)	(3)	(4)	(5)	(6) Mills.	(7) Mills.	(8)	(9)	(10)	(11)
1887.....	5 494 649	4 458 544 804	811	$10 075 153	2.3	10.34	$46 101 353	$79 031 757	$13.83	12.75
1888.....	6 411 423	5 173 132 972	806	7 883 077	1.5	9.77	50 541 509	82 153 019	12.81	9.59
1889.....	7 516 022	5 940 646 352	790	8 634 247	1.5	9.70	57 624 270	83 732 527	11.14	10.31
1890.....	9 041 213	7 207 299 415	797	9 472 215	1.3	9.27	66 811 666	102 214 948	11.30	9.26
1891.....	8 888 759	7 202 462 269	820	9 849 023	1.35	9.29	67 746 974	128 178 208	14.42	7.68
1892.....	11 214 333	9 223 773 938	822	12 072 851	1.31	9.41	86 786 303	135 117 267	12.49	8.93
1893.....	10 796 572	8 980 310 240	832	9 957 483	1.10	8.93	80 194 170	145 436 957	13.47	6.84
1894.....	13 195 860	10 927 871 324	821	10 798 310	0.99	8.64	94 416 808	143 114 562	10.80	7.54
1895.....	15 062 580	12 502 548 892	830	14 238 758	1.14	8.29	104 896 385	159 575 129	10.59	8.92
1896.....	16 239 061	13 582 641 886	836	13 511 616	0.99	8.21	111 513 490	195 146 842	12.02	6.87
1897.....	18 982 755	15 969 393 576	841	13 220 100	0.83	7.97	127 276 067	218 235 927	11.49	6.06
1898.....	21 234 664	17 891 597 030	843	14 125 896	0.79	7.58	135 618 305	233 069 740	10.98	6.10
1899.....	25 255 810	20 891 944 628	827	21 959 707	1.05	7.26	151 675 518	281 364 750	11.14	7.80
Totals and averages.	169 333 701	140 041 167 326	827	$155 798 436	1.11	8.43	$1 181 202 818	$1 986 374 573	$11.14	7.84

Mr. North.

aggregate tonnage of exports. For thirty years we have been steadily bringing the factory nearer to the producer of raw materials, with a concurrent phenomenal increase in freight tonnage, although, theoretically, the process should diminish the ton-mileage of the country. Mr. North.

Undoubtedly, "a waterway from the Lakes to the Atlantic should have dimensions which will permit the passage, at economical speeds, of ships best adapted for the traffic of the water routes connected;" and, "it should be the one best adapted for the distribution of products and manufactures for domestic uses." The first of these assertions calls for a waterway that will connect the immense power of production in the area tributary to the Lakes, not only with our Atlantic and Gulf Coast ports, but with all ports fronting on the ocean; and it will not be denied that the larger the waterway the less the cost of transportation through it. The second will lead to the location of the waterway near the line of the present canal, where it will feed and develop our central aggregation of population and capital.

Attention is once more asked to Mr. Corthell's figures for the tonnage of the Port of New York. He found the entrances and clearances in the foreign trade to be 15 201 653 tons, and in the coastwise trade 25 093 000 tons; that is to say, the volume of the foreign trade of New York is less than 61% of the coasting trade. But to both of these trades the canals of the State of New York contribute only about 3 000 000 tons. We have no standard which enables us to correctly predicate the tonnage of the harbor when the Erie Canal contributes 30 000 000 tons instead of 3 000 000.

The statement that 40 000 000 tons passed through the Detroit River was, in fact, taken from Mr. Mayer's paper, and erroneously credited to Mr. Wisner.

Thomas Monro, M. Inst. C. E. (by letter).—Mr. Wisner makes out a fair case for a 21-ft. channel, but the writer must take exception to the way in which he has bracketed the "decline of traffic on the Erie Canal since 1880," with the alleged "failure of the 14-ft. Canadian canals to divert commerce from the lake and railroad lines." It is true that the traffic of the Erie Canal, at its present dimensions, is constantly declining—and for obvious reasons—but the 14-ft. channel between Lake Ontario and the sea has not yet been tried, and, therefore, cannot correctly be said to have failed. As is well known, the Canadian canals were only fully opened to the new dimensions this spring, and there are as yet but few vessels of full "Canadian canal size" in existence. The fleet that can navigate the St. Lawrence route to the greatest advantage has yet to be built, but the writer believes that a number of suitable craft are now in hand on the Upper Lakes. As a matter of fact, only one vessel drawing 13½ ft. passed through to Montreal, so far, this season. The writer is inclined to believe, however, that the $65 000 000 spent by Canada on a line of navigation Mr. Monro.

Mr. Monro. which will enable vessels drawing 14 ft., and carrying from 2 000 to 2 200 tons, to pass directly from Duluth, Fort William or Chicago to Montreal without breaking bulk, has not been entirely thrown away—but, on the contrary, this magnificent route, when fully developed, as it will be ere long, must prove a valuable means of transporting grain, ore, steel, etc., for export to Europe *via* Montreal.

If the arguments set forth in Major Symons report of 1897 are correct (and the writer believes this to be the ablest and most practical paper which has so far been written on the subject), and his conclusion sound, that the best thing New York can do is to construct a 10-ft. barge canal across the State 350 miles from Buffalo to Albany, then there appears to be a chance for Canada to do some business, on a 14-ft. basis, with boats of 2 000 tons, and only about 70 miles of canals between the Great Lakes and the seaboard. Of course, it is a fact that the domestic commerce of the United States, between the centers of population in the East and the producing regions of the West, constitutes the vast bulk of the present traffic; and the Canadian route does not lead directly either to the New England States or New York; but the development of Canada's own vast areas of production in the Northwest must soon give plenty of work to do. There will be enough to crowd all the avenues of transportation both by rail and water. The railways are making such rapid strides in cheapening freights that the water routes will have to look to their laurels. Brindley said that rivers were made to feed canals; but one would not now be surprised to hear of railway men proposing to fill them both up so as to form roadbeds for their ever-multiplying tracks. If the water routes fail entirely, as has been so confidently predicted, the Canadian route cannot prove the exception hoped for. At all events, Canada is past the theorizing stage. Its main canal system is now completed, such as it is, and, if it fail, all that can then be said for Canadians is that they deserve more credit for pluck than for foresight. However, the end is not yet, and the writer, for one, fully believes that Canada will, in due time, reap the full benefits of her large expenditure on her national route.

Mr. Hinds. FRANK A. HINDS, M. Am. Soc. C. E. (by letter).—Mr. Wisner's paper is valuable in that it shows the necessary dimensions a canal must have in order to do a successful business between the Great Lakes and New York, and Mr. Mayer's paper throws much light on the question of who should build it.

That the enterprise belongs to the United States and not to the State of New York alone, becomes more evident the more it is considered, although New York State will always be much interested in it, on account of its necessary location through her territory.

Nature made a great valley through the middle of New York State, cutting the Appalachian system of mountains below the level of Lake

Mr. Hinds.

Erie and making it possible to construct a self-supplying waterway from this lake directly to tide water at Troy on the Hudson. In the report of the Deep Waterways Commission of 1897, this route, although mentioned, was not reported with favor, but, instead, a route was mentioned from Lake Erie to Lockport and to Lake Ontario at Olcott Harbor, and thence by way of Lake Ontario to Oswego and up again through the Mohawk Valley and finally down to Troy. The reasons given in favor of the Lake Ontario route were: Less standard canal to build; the development of new business on Lake Ontario, and a supposed saving in total length.

It would seem now, however, in view of the comparative speed which a vessel may be able to make in the canal, as is shown in the two papers under discussion, and also in view of the expense and difficulty of providing an adequate and reliable water supply for the summit of the Mohawk Valley Canal, that the subject of a self-supplying, continuously-descending-grade canal from Lake Erie to the Hudson should have further consideration.

The level of Lake Erie is 573 ft. above tide; that of Lake Ontario 247 ft. The long Rome Level of the present Erie Canal is 430 ft. above tide, or 183 ft. higher than Lake Ontario. Of all the business to be done on a waterway, probably more than seven-eighths would come from Lake Erie or points farther west.

Now, if, as recommended by the Deep Waterways Commission in 1897, the route is from Lake Erie through Lockport and Olcott to Lake Ontario, and thence by Oswego and Oneida Lake to Rome, all the through tonnage must be lowered to the level of Lake Ontario and then raised again to the Rome Level, making, as compared with a continuous-down-grade canal, twice the difference in level of 183 ft. from Lake Ontario to Rome, or 366 ft. of unnecessary lockage.

A new canal would, of course, be made with fewer locks and greater lifts than was the old practice, but the delays consequent upon 366 ft. of lockage would offset any advantage of greater speed to be made by vessels while in the open lake between Olcott and Oswego. The time required for this 120 miles of lake navigation at 12 miles per hour would be 10 hours, while in the standard canal, at 8 miles per hour, it would take 15 hours, or 5 hours more than on the open lake; but this would be a short time in which to accomplish the 366 ft. of unnecessary lockage. A comparison of the total distance over each of the two routes, although it cannot be made with certainty until each is carefully surveyed, would probably not disclose a great difference between them; but the probabilities are, judging from a study of the topographical maps of the United States Geological Survey, etc., that the shorter line will be found to be by way of the continuous canal.

With the continuous down-grade, the water from Lake Erie would practically supply the canal through to the Hudson, and avoid the

Mr. Hinds. very difficult problem of providing an entire water supply at the Rome Summit.

It is to be remembered that, where a canal passes over a summit, the water must be supplied at the top, and each boat in passing over requires a supply of water equal to the horizontal size of the lock multiplied by the total height of all the locks ascended or descended.

A ship canal passing over the Rome Summit Level, from Lake Ontario to the Hudson, requires a total lift of $183 + 430 = 613$ ft., and all the water for operating the locks in either direction must be supplied at the summit.

It was proposed, in the Deep Waterways report, to make a moderate development of the canal through western New York, with a high level from Newark to Syracuse, as a means of water supply for the Mohawk Valley Summit.

Now, if it is practicable to make a feeder along this line, then it cannot be very impracticable to make the feeder into a canal, and the cost of the first construction, and the maintenance of this feeder afterward, can be applied in the first cost of the continuous canal, which will be self-feeding. Lake Ontario may be fully accommodated and its interests as thoroughly developed by a side cut, probably through Oswego and the Oswego River, as it would if all the business from the Upper Lakes was compelled to pass down to its level and then be raised again over the summit.

With the necessary water supply reduced to a minimum, and it takes much less water for the through business if the unnecessary lockage into Lake Ontario is avoided, and with the supply of water coming from a source as constant and secure, and as well situated, as that of Lake Erie, it is plain that this route deserves more attention and consideration than it seems to have received.

With the work in the hands of the United States Government, it can certainly afford, if it builds a canal, to build the one that is best and most desirable for the whole country, and cheapest and simplest to operate.

Mr. Sweet. ELNATHAN SWEET, M. Am. Soc. C. E. (by letter).—These papers, relating to the same propositions, and reaching similar conclusions, deepen the conviction the writer has held, ever since bringing this subject to the attention of the Society, in a paper read at the Buffalo Convention in 1884,* that no waterway built to connect these waters can hope to secure great commercial importance unless it can accommodate the largest vessels navigating the Great Lakes.

In a communication to the *Times*, opposing the endorsement of the proposition then before the people of New York to authorize the expenditure of $9 000 000 for slightly enlarging the State canals, the writer urged that "the expenditure of great sums of money for so-

* *Transactions*, Am. Soc. C. E., Vol. xix, p. 37.

called improvements of the present canal were to be discouraged as wasteful, and because they tended to postpone the wise and statesmanlike undertaking of a deep-water outlet for Lake commerce." Mr. Sweet.

There are three fundamental and controlling elements of advantage in the deep waterway:

First.—The elementary physical law, that the resistance to motion in vessels of like model varies directly as their immersed surfaces, while their tonnage varies as the cubic contents of their immersed section, ensures enormous economy in large boats.

Second.—The obvious and controlling advantage of passing from terminal to terminal without transfer of cargo.

Third.—A large fleet adapted to the navigation of the deep waterway already exists, while, to put the smaller canal into operation, a new marine equipment must be created, for which there could be little use elsewhere.

Mr. Wisner makes a very strong argument, showing the disproportion of benefit to cost in the deepening of Lake channels and harbors beyond the depth of 21 ft., now in process of attainment by the Government engineers. This seems to the writer to afford definite authority for fixing the depth of the canal, and, in the light of the extended experiments made since 1890 in France and Germany on the resistance to traction in narrow channels, the writer thinks that the making of the wet cross-section more than five times as great as the midship section of the largest vessel it is intended to accommodate, will insure about two-thirds of the speed secured in open water with the same expenditure of energy; but for economy of movement with 19 ft. draft he thinks that the proposed form of section should be modified by increasing the depth 2 ft. and narrowing the width sufficiently to secure the proposed sectional area of channels. He cannot share the sanguine expectation expressed by Mr. Mayer as to the volume of tonnage likely to soon seek transportation on such a canal. Its chief function will be the transportation of freight destined for Atlantic ports and for export. The ore and coal freights which form so large a fraction of lake tonnage cannot, under present conditions, contribute business to this canal.

It must be remembered that the railroads are the natural and necessary distributors of even the coarsest and heaviest freight, as their side tracks reach the yards of all large manufacturers and consumers throughout the country. A large part of the tonnage received by lake at Buffalo and other Lake Erie ports is destined for distribution in the interior of the Eastern-Middle and New England States, and this freight will all probably continue to be handled from Lake ports by the railroads, for the reason that the branch roads making this distribution are controlled by railroads reaching these Lake Erie ports, and their control of local rail rates will enable them to route this busi-

Mr. Sweet. ness through from these ports instead of from points farther east on the canal.

There is a question of public policy to be considered as to the effect of withdrawing so large a part of the business of the railroads, in which so large a part of the capital of the country is invested, upon their earning capacity and the rates it would be necessary for them to impose on their remaining traffic.

Account should also be taken of the question whether all freights may not ultimately be handled more cheaply by rail, in favor of which there are both practical results and theoretical considerations.

The great and uninterrupted decrease in unit-cost of railroad freight transportation during the entire period since its introduction may be cited as the practical result, and the fact that the resistance to the movement of rail freight varies directly as the velocity and on account of the labor item is (within limits of safety to track and equipment) cheaper at high than at low speeds, while the resistance of vessels increases at a rate greater than the square of the rate of speed, and is absolutely greater than the rail resistance except at a moderate rate, may be urged as the principal theoretical consideration.

The fact that our resources are growing so rapidly, that the Lake Region and the line of this canal furnish the most suitable field for a great export business in heavy manufactures, and that the export freight business is necessarily carried on by water, make it probable that both the railroads and the canals will be needed in a future not remote, and both be profitable.

Mr. Hunter. W. HENRY HUNTER,* M. Inst. C. E. (by letter).—In responding to the invitation to take part in the discussion of a subject of such world-wide importance, the writer can but express his regret that lack of time and pressure of official engagements have rendered it impossible for him to find opportunity for the close and detailed study of the question which is rendered so necessary by the great issues involved in it, or to grasp anything more than the outlines of the different schemes which have been propounded; though in this particular the papers by Messrs. Mayer and Wisner have been of the greatest possible assistance.

It appears very clear, to many outside the boundaries of the United States, that the time has arrived in which it is essential, even in the interests of their own country, that the economists and engineers of the United States should take more practical cognizance of the fact that their great Republic has an even more important part to play as an integral portion of the human family, than as a more or less self-contained community (or concourse of communities) dwelling alone, devoting itself to the development of its own resources, and the advancement of its own interests, whether material or moral.

* Chief Engineer to the Manchester Ship Canal; President of the Manchester Association of Students of the Institution of Civil Engineers.

Mr. Hunter.

The date of the accomplishment of the poet's dream of "The parliament of man, the federation of the world," is as far off as ever; yet, of nations as of individuals, it is true, and is becoming more and more apparent, that "No one . . . liveth to himself, and no one dieth to himself," for the interdependence of humanity is growing more and more pronounced as population increases, and as the reserves, in the way of uncultivated prairies and unwatered steppes, are being brought into use for the benefit of man.

Interdependence involves intercommunication: The one is a question for the economist, the other a matter for the engineer; the present danger is that in each case too narrow a view may be taken, either of the responsibilities of the position, or of its possibilities.

The problem of the future, the not-far-distant future, will resolve itself into the broad question of the supply of food to the rapidly increasing masses of mankind. Civilization is becoming effective at last, and the result is that population is growing at an unprecedented rate, so much so that the academical question—the question of the theorists—of former generations, as to the sufficiency of the earth for the support of her children, will soon loom largely on the horizon of practical politics, and men will have to rouse themselves to deal with it.

If this were admitted and its significance realized, proposals for constructing barge canals, or even ship canals of insignificant depth, between the Atlantic seaboard and the Lakes which form the natural center of one of the greatest food-producing districts on the earth, would drop into the limbo of "lost causes"; the insufficiency of either one or the other would become so manifest.

The first and greatest of the means of intercommunication on our planet is of Nature's providing, is the ocean; whether in the East or in the West, the sea is the trunk highway for mankind; and the primary object to be sought in the projection of a waterway originating on the seaboard, and consisting, as is generally the case, (*a*) of the improvement of a river, (*b*) of the formation of a canal, should be the carrying of the sea as far into the interior as is reasonably possible; for the farther the sea is carried into the interior, the greater will be the benefit to the district affected, and the more marked will be the ultimate success of the waterway.

The Manchester Ship Canal furnishes an example of a waterway of this class. The terminal docks are 50 miles inland from the coast line, a distance which is covered by 15 miles of estuarial and river improvement, and 35 miles of canal construction, construction which was carried out in the midst of a dense population, in the teeth of bitter and prolonged opposition, and in despite of a crowd of ancient rights and vested interests, which are always short-sighted, inert, and desperately opposed to progress, but which, in a country like Great

Mr. Hunter. Britain, must be reckoned with, must in some cases be conciliated, in others coerced. Either process costs money, hence the Manchester Canal cost more than it should have cost, to which cause, added to that presented by the obstructions thrown in the way of its working by the same vested interests, is due the fact that up to the present time it has returned no reward in the shape of dividend to its original shareholders.

This fact had led pessimistic persons to the conclusion that the canal has failed in the accomplishment of the purpose for which it was promoted and constructed. There never was a greater fallacy, a greater error. A prominent public man in Manchester,* the head of one of the largest manufacturing firms in England, has declared again and again, and has met with no one bold enough to deny his proposition, that the citizens of Manchester are receiving more than 20% per annum for the $25 000 000 which the city invested in the canal, while directly and indirectly the canal has influenced for good a district in which the population is more dense than in any other on the face of the globe, and has sensibly advanced the welfare of some 10 000 000 human beings. At the same time trade is growing, developments are accruing, terminal facilities are being increased and extended, and the cost of handling traffic diminished to such a degree that there is every prospect of a return being made ultimately, even to the patriotic shareholders who put their money into the undertaking.

The depth to which the Manchester Ship Canal was constructed was 26 ft., but the sills of all the locks were put down to 28 ft. Steamers of a dead-weight capacity of 10 000 tons are now traversing the canal regularly, and others, still larger, are projected. Thus it is clear that the deepening of the waterway to 28 ft. will have to be taken in hand at no distant date, for the course of trade is now such that large steamers can run and prove themselves profitable to their owners, where small or even moderate-sized vessels cannot live.

Applying this experience to the case under discussion, which is really not one of carrying the sea inland, so much as it is one of joining two seas, or, in other words, of severing an isthmus, it would appear that a dead-weight capacity of 8 600 tons for the ocean-going steamers proposed to be sent on, *via* the new waterway, to Chicago, would be insufficient. Such steamers would hardly compete now with the large and improved boats which are continually being launched, and in twenty years' time they would be altogether out of the running. If the canal were formed with a depth of 24 ft. in the first instance, and all lock sills, syphons, and other works which would limit the possibility of future deepening, were put down to 30 ft., it is probable that the waterway would meet the world's needs for the next half century, which is all that can be required.

* Mr. Reuben Spencer, the head of the firm of John Rylands & Co.

Mr. Hunter.

The regulation speed of vessels on the Manchester Canal is, as stated by Mr. Wisner, 6 miles per hour, but licenses are issued to steamers permitting them to run at the rate of 8 miles, as experience has proved that at this speed no sensible amount of injury is caused to the banks by wash; the licenses are, therefore, granted as a matter of course, there having been only one instance in which the writer has had to refuse his consent to the issue, and that was in the case of a large paddle steamer (an undesirable craft on a canal), which was already really running at the rate of over 8 and wanted to run 10 miles per hour. There need, therefore, be no apprehension as to the safety of the estimate of 8 miles per hour as the average speed of large steamers on the proposed Lakes Canal.

The writer would like to add one observation in respect of the argument that if the Lakes Canal were really required "the play of economic forces" would be sufficient to bring about its construction. The economic theories—or principles, if the word is preferred—of one age become the fallacies of another, as life becomes more complex, and the interdependence, to which reference has already been made, grows more complete. From this proposition, if sound, it follows that strict adherence to theories as to demand and supply, as if these theories were natural laws which could not be broken with impunity, would inevitably arrest the advance of civilization and the progress of the human race.

The ordinary investor, as such, is neither a philanthropist nor a patriot—he does not think—nor indeed is it possible for him to think, except in a secondary way, of the good of mankind, of which he is a part, or of the welfare of the State, of which he is a subject. On the contrary, his operations are governed wholly by the prospect of return for his investment in the form of interest upon his capital. It is clear, therefore, (1) that public works of the first magnitude—arterial lines of communication and the like—which are, in the first place, for the public benefit, can present little of attractiveness to the investor, (2) that such works will never be made out of private money (any more than ironclad line-of-battle ships can be built, or coast defenses constructed, out of private money), and (3) that, as it cannot be denied that they are a necessity for the public welfare, the public, that is to say, the State, must of necessity provide the funds required for their construction and equipment; and this despite the instinctive tendency of the Anglo-Saxon race to depend in these matters upon private enterprise rather than upon governmental energy.

In the opinion of the writer, the advantages, direct and indirect, which would accrue to the citizens of the United States from the construction of the canal between New York and the Great Lakes would be so profound and far reaching, and the increase of the prosperity of the community would be so marked that the Central Government would be

Mr. Hunter. justified in forming the canal and in throwing it open to the trade of the world at the smallest possible toll charge; he claims that the case of the Manchester Ship Canal, and of the benefit which its working has been proved to have conferred upon the greatest manufacturing district on earth, furnishes a firm basis for this opinion, and he desires to thank the American Society of Civil Engineers and the authors of the papers under discussion for affording him an opportunity of expressing it.

Mr. Hoech. THEODOR G. HOECH, M. Am. Soc. C. E. (by letter).—The report of the New York Canal Commission on a barge canal of 12 ft. depth, and other studies of improved waterways between the Great Lakes and the metropolis on the Hudson River, after the able report of Major Symons recommending a barge canal 12 ft. deep, cannot fail to prove, even to a distant observer, the growing interest of the engineering profession in an improved waterway, and at the same time a sentiment for greater dimensions for this modern canal.

Not being able to check the figures presented, the writer may be allowed to point out some commercial features of North America's Atlantic slope affecting inland navigation.

The commercial power of the City of New York seems to be so great that the exporting merchants do not think of running ocean steamers up the Hudson River with the flood current to meet the canal boats as far inland as possible, as is the case in several European rivers. One could figure out, however, that the cheapest way of exporting western goods would be to carry them by barges or lake steamers, say, to the Hudson, and to transfer them there into large transatlantic steamers. It is to be considered a silent but great compliment to the harbor of New York that nobody advocates this waterway system, promising the cheapest rates from West to transatlantic ports.

New York's attractive and accumulating power seems also to have prevented the commerce from passing in greater degree through the Canadian canals of 14 ft. depth.

That dominant position of New York will, of course, be of deciding effect on the final solution of the waterway question, and the fact should not be overlooked that with a ship canal ocean steamers would pass the metropolis of the Atlantic Coast, and allow European freighters to run into the Great Lakes as far as Duluth, and take the grain of the West to Europe, without transfer and harbor expenses. It does not seem likely that New York will part with the many petty charges and profits derived from distributing and transferring goods.

One may safely presume that the City and the State of New York will construct a waterway satisfactory to their interests, which commercially will be of the same kind as the old Erie Canal—and will strengthen still more the great center on the Atlantic Coast. It is important to the city that all transfers between the Lake harbors and

New York should be done away with, while retaining the transfer business, and this condition can be satisfied by a large barge canal. Mr. Hoech.

In Germany, the number of barges—Seeleichter—is growing rapidly, and these barges run safely from the Rhine and the Dortmund-Ems Canal through the German Sea, and the Kaiser Wilhelm Ship Canal to the Baltic Sea—the Great Lake of Northern Europe. Besides these towed barges, twenty-five small steamers are running regularly from Koeln on the Rhine to London, Hamburg, Dantzig, etc.

The growing sentiment for greater dimensions for new canals may, in the writer's opinion, lead to a barge canal of 14 ft. depth, after some further discussion. This would give a fair field for competition and comparison between small freight steamers and large barges, and the results would be watched with great interest everywhere.

The advantages of a 14-ft. barge canal, which would take care of the business concerned for some time, may be stated as follows:

First, it would mean a great step in advance without changing the commercial position fundamentally, and the canal itself could be constructed in two parts: With a deeper canal from the Mohawk Valley to Lake Ontario in operation, steamers and barges could make use of the Welland Canal, allowing the time for building the Niagara Canal and the fleet of standard vessels.

Secondly, the fleets of steamers and barges would be enhanced in value on account of the two outlets to the ocean, and even a third one from Chicago to New Orleans, which can be hoped for.

Without strong reasons for a change, the dimensions of neighboring waterways, even when a little less profitable, should be accepted. The Canadian canals may keep their present dimensions for many years to come, and the ship canal from Chicago to New Orleans may not come at all, for two reasons. After a ship canal is built from the Great Lakes to the Atlantic Coast, no second ship canal would be needed to the Gulf, for no ocean steamer would choose such a long inland waterway in competition with cheap river steamers and still cheaper barges. But a deep barge canal from Lake Michigan to the Mississippi, and another one from Lake Erie to the Ohio River, would form, with the Canadian and the New York canals, a great system of uniform waterways opening half a continent to steamers and barges of standard dimensions.

ALFRED NOBLE, M. Am. Soc. C. E. (by letter).—The time required for the passage of a ship through the proposed Deep Waterway is perhaps as likely to provoke discussion as any point in Mr. Wisner's paper. The writer, having taken part in the investigation from which Mr. Wisner's conclusion resulted, will present an outline of it. Mr. Noble.

The movement of a ship through the artificial channels of the Deep Waterway would be obstructed by two different sets of causes: 1st. Reduction of speed due to the restricted dimensions and curvature of the

Mr. Noble. waterway and the meeting of ships therein; and 2d, delays for passage of locks.

Any restriction in a waterway, whether of depth or width, will cause a reduction in the speed of a ship passing through it. Lake St. Clair furnishes an example of a waterway where the restricted dimension is depth, the width being several miles. The logs of two ships, each loaded to within 2 ft. of the bottom of this lake, showed a reduction of speed of 16 to 18%, as compared with the speed of the same ships in the deep water of Lake Huron. While data on this point are not as numerous as might be desired, it was believed to be on the safe side to assume a reduction of 20%, which was adopted in the investigation referred to. New data, collected since the close of the investigation, confirm those used.

If the width of the waterway is also restricted, new retarding conditions result. When the ship moves forward its length, a volume of water equal to its displacement passes between the ship and the bottom and sides of the waterway to maintain the water level, creating a back current. If the speed of the ship through the water remains the same as in wide, shoal water, its speed past a fixed land point will be reduced by the speed of the back current. In the investigation this reduction was made, although the amount appeared to be somewhat excessive, because, in the first place, the proper reduction was already in part provided for by the adopted reduction of 20% for the effect of shoal water, and, in the second place, because the amount of water transferred from the bow of the ship to the stern was calculated as the product of its midship section by its length, without allowing for the reduced sections near the bow and stern. The calculated back flow was therefore too great, and the reduction of the speed of the ship as taken was too great.

To create the back flow and maintain it past the ship requires a head of water which opposes the movement of the ship. This, however, the investigation showed to be unimportant in the wide channel adopted for the waterway and with the speeds to be permitted in it.

These reductions of speed were applied in the case of a ship having a speed of 12½ miles per hour in the deep, open lake, resulting in speeds, with the same expenditure of power, varying from 8.2 miles per hour, in the ordinary section 215 ft. wide at the bottom, to 9.5 miles per hour in a channel 1 000 ft. wide at the bottom.

An investigation of this kind must be supported by experience, to be trustworthy, and therefore the actual movement of ships in other canals was examined with great care.

Manchester Canal.—The bottom width is 120 ft., the depth 26 ft., and the cross-section is about 4 400 sq. ft. in earth sections. In a paper presented to the VIIth International Congress on Navigation, in 1898, the Chief Engineer, Mr. W. H. Hunter, gave the dimensions

of a number of vessels which traversed the canal at speeds varying from 7.3 to 13.6 miles per hour. The slower rate was for a ship 255.3 ft. long, 33.7 ft. beam, drawing 17.6 ft. The largest ships traversing the canal are about 450 ft. long and 50 ft. beam. The writer accompanied one of the ships for several miles and carefully noted its speed. It was loaded to within a few inches of the canal bottom, and therefore steered badly, requiring the aid of tugs, but the speed reached 6 miles per hour. In the consideration of the permissible speed in a waterway, either from a theoretical or practical standpoint, the ratio of the area of the ship's immersed cross-section to that of the water, is a controlling feature. In the case of the ship just mentioned the area of the cross-section of the waterway was 3.7 times the immersed cross-section of the ship. This ratio will be designated r. Mr. Noble.

Suez Canal.—When opened, and for many years afterward, the bottom width of the Suez Canal was 72 ft., and ships were permitted to move at the rate of 6.2 miles per hour. The value of r for the larger ships was probably a little greater than 3. The bottom width of the standard section of the Deep Waterway is almost exactly three times as much, and the cross-section of the largest ships traversing it will probably not be as great. If the comparison be made by means of the ratio r, its value for the largest ships expected in the Deep Waterway will be 5.5, instead of 3, as at Suez. Whether the comparison of these ratios, or of bottom widths, be made, a speed of 8.2 miles per hour in the Deep Waterway will appear to be much safer than one of 6.2 miles in the Suez Canal.

As to the practicable rate of speed and degree of security attainable in an artificial waterway, no better example can be cited than the St. Clair Flats Canal, where serious accidents seldom or never occur. This is believed to be the most crowded waterway in the world. The maximum speed permitted by regulations is 8 miles per hour, but the largest freight steamers frequently pass at a higher rate; the largest passenger ships attain a rate of 11 to 13 miles per hour and a speed of 15.4 miles per hour is of record.

With these examples in view, the speed of 8.2 miles per hour in a straight waterway of 215 ft. bottom width appears moderate. The width is to be increased on all curves of less than 12 000 ft. radius, the widening being 1 ft. for each reduction of 200 ft. in the radius. A reduction of permissible speed, equal in miles per hour to

$$1.5 \times \text{degree of curve},$$

is made in the standard canal section, the reduction being reduced where the bottom width is greater. While this allowance is purely arbitrary, it is believed to be sufficient.

In the Suez, and other very narrow waterways, one of two meeting ships ties up. This is not found necessary in the artificial channels in the Great Lakes system, but the speed of both ships is reduced. In the

Mr. Noble. 300-ft. channels of the St. Mary's River, where the channel banks are submerged, the regulations provide that the speed of both ships shall be reduced to 6 miles from a permitted speed of 9, and, excepting in special localities where the current is very strong, this has been found sufficient. In the somewhat narrower channel of the Deep Waterway the assumed reduction of speed is to about 4 miles per hour. The number of meetings is deduced from an assumed traffic of 25 000 000 registered tons annually carried in ships averaging 2 500 tons net register, or 10 000 ships per year. This discussion thus far relates to channels in earth. Objection may be made that in channels in rock with rough sides such speeds would be dangerous. The reply to this would be that no such channels are contemplated. In rock the bottom width is to be increased and the sides made vertical, by channeling machines where the rock is sound, and by retaining walls where these are necessary.

The delays at locks will not be discussed here in like detail; briefly, the allowance for these delays is based on ample experience at the St. Mary's Falls Canal in the movement of ships and the other operations of locking. In addition to this a considerable period has been allowed arbitrarily for delays at lake points where, after storms, vessels may occasionally arrive more rapidly than they can be passed through the next lock, and for smaller delays at each intermediate lock.

In designing this waterway the view has prevailed that it would not be worth making unless it were adapted to more speedy navigation than the principal foreign canals. A much wider prism and better lock facilities are therefore provided, and it is confidently believed that the estimates of time required for passage through it will stand investigation.

Mr. Haupt. LEWIS M. HAUPT, M. Am. Soc. C. E. (by letter).—These papers open up a broad field for discussion, not only in engineering and economics, but in administration and statecraft.

Mr. Randolph's succinct discussion contains much in little, and is certainly a sagacious compromise, as it makes provision for the progressive expansion of the capacity of the waterway.

Experience has invariably demonstrated the inadequacy of rigid dimensions to keep pace with the growing demands of commerce and the necessity of a flexibility which will adapt itself to the rapid growth of tonnage.

As early as 1847 (before the Sault Canal was seriously thought of) the late J. J. Abert, Hon. M. Am. Soc. C. E., Colonel, U. S. Topographical Engineers, discussed the "Probable Increase" of the Lake trade, in an official report to Mr. Marcy, Secretary of War, in which he said:*

"It is difficult to approach this part of the inquiry without fear of

* Ex. Doc. No. 19. Thirtieth Congress. H. of Rep., First Session, Jan. 12th, 1848.

Mr. Haupt.

appearing to exaggerate. Those who knew these Lakes thirty years ago, and who know them now, will admit that existing facts have baffled human expectations, and that the wildest speculations of the imagination have been more than realized in the vast increase of their commerce. Then, if we examine into the elements of this increase, we can perceive no reason to doubt a less energetic action of these elements, for many years to come, than has been realized in the past."

That this forecast has been fully justified by subsequent events is confirmed by the statements of General O. M. Poe, whose investigations of the tonnage through the Lakes well qualified him to state the facts. He said:

"For thirty-five years I have watched the increase of the Great Lakes commerce, but neither I nor anyone else has been able to expand in ideas at the same rate. The wildest expectations of one year seem absurdly tame by the side of the actual facts of the next."

In view of this experience, it is somewhat surprising to read that so keen a statesman as Henry Clay opposed Hon. John Norvell's motion to construct this canal, in 1840, by the statement that it was "a work beyond the remotest settlement in the United States, if not in the moon."

Henry Clay evidently did not know of the latent resources of the Lake basin, nor the relative economies of water transportation—and even at the present time there is much misunderstanding as to the effect upon our industrial growth due to increased facilities.

It is well known that transportation seeks the line of least resistance (as measured by the total cost from mine to market). Tolls, transfer and terminal charges are as much an element of diversion as are physical obstacles. The Carnegie Company is now making efforts to ship direct from Lake ports *via* the Welland and St. Lawrence Canals, after making the overland portage by rail. Were the Lake Erie and Ohio River Ship Canal opened it would not only greatly facilitate this movement, but it would also augment the coal tonnage to Eastern Atlantic ports, and hence it would seem to be unwise not to make provision in the dimensions of the canal for not less than between 15 000 000 and 20 000 000 cargo-tons soon after its opening to tidewater. There is, no doubt, much truth in the contention of Major Symons, that if the freight once reached Lake Ontario it would not lock up over a high divide and longer route to maintain the City of New York as a commercial terminus, but would be switched off *via* the Montreal route; but, at the same time, that would seem to be no reason for denying to the densely peopled manufacturing and productive agricultural districts tributary to the Lakes the advantages of lower rates and more extensive markets. New York City is not the United States, and if that city may not share in the benefits (which the writer does not admit would happen), other sections should not be debarred from such advantages.

Mr. Haupt. It is said that "if built by the United States, the benefits should all accrue to the United States;" and, again, "its benefits should accrue only to the United States." The only route which permits this is that "along the line of the present Erie Canal," and hence the prism of lesser dimensions, with a barge system of carriers, is urged.

But can the beneficent influences of a canal or cheap water route be restricted to any country? Do not the waterways of France and Germany enable our exports and their imports to permeate every avenue of trade, and supply points not otherwise accessible at the same rates, to the benefit of our producers? If our shortest and cheapest route to our foreign markets lies through Canadian waters, is there any reason why our producers should not utilize them rather than pay for transfers and storage at both ends of a broken and circuitous route?

It should also be noted that the staple products of a State tributary to a deep water route are quoted higher than those at remote points and which require transhipments, so that the producer reaps the benefit of the cheaper route, while the consumer also buys at a lower price because of the economy in cost of carriage.

Money spent in the maintenance of an expensive system requiring, say, 60% of the gross income for operation when one is available which can be maintained and operated for one-half or one-third this cost, is extravagant. It has been estimated by Stevenson that the value of a commercial channel varies as the cube of the depth, while the economy of bulk movements is fully sustained by the rapid increase in the size of ocean freighters and the demand for deeper channels to carry them.

Many minor questions suggest themselves, as speed, ratio of beam dimension to section of canal trunk, number of trips per annum, cost of plant for a given tonnage movement, etc., which have been more or ess fully treated at the Navigation Congresses and by the members of this Society, so that it is not desirable to review them here, as the intention of the writer is merely to emphasize the need of making early provision for the largest movement, in view of the rapid increase of population and traffic in sight.

It does not appear to be appreciated that by the year 1930 our population will have almost doubled, and that we are increasing three times more rapidly than any of the civilized nations of the world whose statistics are available to the writer. Ordinary prudence would therefore demand that early provision be made to maintain our freedom of internal circulation, and the creation of external markets to provide employment for the next generation, and prevent degeneration and distress.

During the past century each decennial ratio has shown an average decrease of $\frac{1.9}{400}$ of the preceding ratio, yet the actual numerical increase of population has gone on in a geometrical ratio. The

increase during the past decade, for example, was nearly 14 000 000, or more than the entire increase from the discovery of America to 1830, and between 1920 and 1930 the increase should be about 25 000 000 souls. Another factor which will augment greatly the commerce through the enlarged canal will be the opening of the link between Chicago and the Mississippi River, now being urged by the Illinois River Valley Association, thus bringing the waterways of the Great Basin into unison with those of the Lakes during the season of navigation. This is not so remote as was the Sault Canal in 1840, and it is one of the great and urgent improvements required by the present generation, and which will promote the future of our great Republic. Let the American Society of Civil Engineers be found in the forefront of all these great movements to develop our commerce. Mr. Haupt.

RUDOLPH HERING, M. Am. Soc. C. E. (by letter).—The relative merits of a ship canal and a barge canal from Lake Erie to the Hudson River have evidently been insufficiently demonstrated to enable a decision to be reached at present—a strong array of facts and talented arguments still appearing on both sides. Mr. Hering.

One might draw the inference that therefore a fair balance exists between the two projects, and that the total advantages are nearly equal, although not identical. It seems more probable, however, that there is as yet insufficient knowledge regarding some of the facts which must control the enterprise, and that considerable time may yet elapse before they are obtained.

Regarding the cost of construction there should not be much difference of opinion, if a definite plan is once established. The probable revenue, on the other hand, is a speculative sum, and neither the authors nor some of the discussors seem to have supported their assumptions with sufficient evidence to satisfy either the capitalist or the legislator. It seems to the writer that here there is room for more facts.

Leading up to the revenue is the use that will be made of the canal; and this depends partly upon the tolls to be exacted and partly upon the speed which can be obtained.

In the Kaiser Wilhelm Canal, vessels capable of making 16 knots per hour in open sea, with a midship section equal to one-third of the canal, and using the same power, made only between 5 and 6 knots per hour. In the Suez Canal, the *Austral*, with 43 revolutions of the screw, and capable of making a speed of 11 knots in open sea, made only 5 knots per hour. We, therefore, cannot rely on much higher speeds than these for large vessels, even in a ship canal such as the one proposed, which fact, in addition to those mentioned, affects the probable revenue.

The suggestion that steamers from the Upper Lakes would go to Europe *via* the St. Lawrence River, that is, through Canada, instead

Mr. Hering. of *via* New York City, does not seem to the writer to argue against the ship canal, any more than a statement that the Michigan Central Railroad, passing from Buffalo to Chicago through Canada, would injure our national interests. Why should not a United States steamer, as well as a railroad train, pass through Canada? The section from Lake Ontario to New York would have to depend largely on the business intended for the Atlantic Coast. It may not be improbable that even the Gulf Coast could be supplied more economically by such a ship canal from the Lakes, and without transfer, than by the Mississippi Valley or by rail.

One reason, not already suggested, why private capital prefers investments in railroads to long canals, may be the fact that the capital required is very much larger. The investor is not much interested in the other fact, that the running expenses are much larger for railroads than for canals.

Mr. Rafter. GEORGE W. RAFTER, M. Am. Soc. C. E. by (letter).—Mr. Hinds writes approvingly of the continuously-descending canal from Buffalo to the Mohawk Valley, but it seems clear to the writer that, had he possessed more extensive information as to the difficulties of the route, this project would not have received his approval. In the end it will pass into the realm of impractical projects, which have only gained importance so long as the conditions were unknown.

The writer was Consulting Engineer to the Canal Committee appointed by Governor Roosevelt. This Committee began by assuming that a continuously-descending canal was in every way a meritorious project, and hence directed, first of all, that it be examined in detail. It soon appeared that there were two routes, as follows: (1) A southern route, which passed to the south of Seneca River; (2) A northern route, passing to the north of that stream; and finally, a route which was not continuously-descending, *via* Seneca River itself. The southern route is the one suggested by Elnathan Sweet, M. Am. Soc. C. E. On very casual examination it appeared certain that this route involved very great difficulties for the entire distance from Newark (the proposed point of diversion) to near Syracuse, a distance of over 50 miles. The country is sand and gravel, with absolutely no stone, while at Syracuse there is a very heavy cut to be made. The Montezuma marsh is also encountered. Even for a 12-ft. canal the estimate for 58 miles was $29 000 000.

The southern route having turned out to be exceedingly expensive, a route to the north of Seneca River was then examined. The region here is of the same general character as to the south, except that the Montezuma marsh is avoided. There would be a crossing of Seneca River, a short distance east of Baldwinsville, which would require a very deep and heavy foundation for an aqueduct. The estimated cost of this route was $22 000 000 for about 58 miles.

Mr. Rafter.

The high-level, continuously-descending canal, by either the northern or southern routes, having thus developed very great difficulties, the writer then proposed the Seneca River route, which in every particular answers the requirements. Space will not be taken to describe this route in detail. The State Engineer informs the writer that his report thereon will be published in the Annual Report for 1900, which will make it available to anyone interested. Attention, however, may be directed to the maps which accompany the report. They were compiled from the field sheets of the U. S. Geological Survey at a scale of $\frac{1}{45\,000}$, and exhibit, it is believed, every necessary feature.

In addition to these three routes, an extension of the Syracuse level, east and west, was examined. The total distance, carefully gone over, was fully 300 miles. Approximate profiles were plotted for the several routes, and it is believed that there are no longer any unknown problems in the country from Newark to Syracuse, all of which is set forth in the report in considerable detail.

The practical conclusion to be drawn from this examination is that for a canal of the dimensions of the proposed deep waterway, the project, while not absolutely impossible, becomes substantially impractical. There are ways by which the same result may be obtained so much cheaper as to render it exceedingly improbable that a canal will ever be carried out by the continuously-descending route.

The locations of the several canal lines are made in such a manner as to bury the canal below the present surface, as far as possible, thus insuring freedom from embankment breaks. By way of further insurance, embankments over 10 ft. in height have been designed with masonry core walls, in accordance with the best recent practice. Under 10 ft. in height they have been designed merely with puddle walls. In water-tight materials, the puddle walls have been omitted.

Some question may arise as to the necessity for masonry core walls in canal embankments. In the writer's opinion, they are necessary, in such large construction as is here proposed, as an absolute guarantee against breaks. A main line of transportation—a national highway—should not be subject to the possibility of broken embankments, with consequent delay of all traffic for 5 to 15 days.

In order to show the experience under this head on the New York State Canals, Table No. 10 is submitted. This table includes all breaks reported by the Superintendent of Public Works from 1888 to 1898, inclusive, except for the year 1890, for which the data are not at hand.

The most serious break was that at the Adams Basin on the Erie Canal, in 1888, which stopped all navigation between Rochester and Buffalo for 12 days.

In 1891, two breaks on the Erie Canal occurred at the same time—one at an aqueduct and the other a defective embankment—which required 10½ and 9 days, respectively, to repair.

Mr. Rafter.

TABLE No. 10.

Year.	Number of breaks reported.	Total detention, in days.	Percentage of a navigation season of 210 days.
1888	7	24	11.4
1889	4	14	6.5
1891	5	28	13.3
1892	5	7	3.3
1893	4	21	10.0
1894	6	10	4.8
1895	3	12	5.7
1896	2	6	2.9
1897	4	55	26.2
1898	11	74	35.2
Totals for 10 years	51	251	Average 12.0

In 1893, a break of an embankment of the Glens Falls feeder required 11 days to repair.

In 1896, there were two breaks, and the total detention was only 6 days. One of these breaks was on the Black River Canal and the other on the Eastern Division of the Erie Canal.

In 1897, a serious break occurred on the Forestport feeder, which required 38 days for repairs. These figures are included in the total detention for that year.

In 1898, the total number of breaks was eleven, causing a total detention of 74 days, of which 16 are included in the repairs of a break in the Forestport feeder embankment at the same point as the break in 1897. With the exception of two breaks on the Champlain Canal, all the other breaks of 1898 were on the Erie Canal, the total detention from the Erie Canal breaks being 53½ days. The Erie Canal breaks were all through embankments, and, with one exception, were at points where culverts pass under. The detail of the foregoing breaks may be obtained from the Annual Reports of the Superintendent of Public Works.

The statements given in the Superintendent's reports show that muskrats, by their burrowing, frequently cause embankment breaks. So far as known, masonry core walls are the only effectual remedy for this difficulty.

Breaks at and about the ends of locks form also a considerable proportion of the whole. The carrying of a concrete core wall into the embankments at the lower ends of locks would correct this difficulty.

A strong argument for the additional expenditure required to construct core walls in embankments may be drawn from the foregoing tabulation of the number of breaks, etc., from 1888 to 1898, inclusive. Even when cases like the Forestport feeder breaks, in 1897

and 1898, are deducted, the clear indication is that embankments without core walls are too dangerous for a great through transportation line such as is here proposed. Such a line, to be satisfactory to all the interests using it, must be so constructed as to be beyond the mutations of ordinary accidents. It would be difficult to make any time-limit contracts with the possibility of delays of 5 to 20 days caused by breaks. Mr. Rafter.

In considering the foregoing question on its merits it must be further remembered that doubling the quantity of water in the canal prism makes the destructive effect of breaks at least fourfold as great as at present. The cost of repairs and damages would, therefore, become very large.*

Taking the foregoing into account, the writer's view is that new work should include estimates for masonry core walls in all embankments over 10 ft. in height.

R. P. J. TUTEIN-NOLTHENIUS, M. Am. Soc. C. E. (by letter).—The writer agrees with Mr. North that all sea commerce is not carried in ocean leviathans, but that fair profits may be made by smaller vessels. Freighters of the same size as the steamers which run regularly from Cöln and Duisburg, on the Rhine, to different ports on the North Sea, and are said to pay a dividend of 13%, would probably carry freight advantageously from Duluth to New York if a canal of proper dimensions existed between the Lakes and the Hudson. Mr. Tutein-Nolthenius.

As will be seen by Table No. 11, which gives the dimensions of all the marine steamers now running on the Rhine, different companies build ships of different lengths, and each adheres to the dimensions once preferred, which seems to prove that, between certain limits, all types are profitable. For example, the Hamburg Steamship Company in 1885 preferred steamers about 230 ft. in length, and the new vessel built for it in 1899 is of almost the same size, while the Neptune Company (the largest owner of this kind of steamers), prefers vessels of less length, only increasing from 140 ft. in 1890 to 170 ft. in 1898.

When the Rhine is at an extremely low level—which seldom happens—these steamers cannot load to their full draft, and part of their freight is then stowed in barges, which the steamer tows alongside, until approaching the ocean. Then this supplemental cargo is transferred to the steamer, while descending the river, in order to lose no time by stopping. The barges remain in the river.

The great pecuniary advantage of not transferring freight in an intermediate port is proved by the fact that these companies make greater profit by loading their small marine craft at Cöln, and sending them directly, say to Hamburg, than by building large barges, such

* It would be interesting to know the cost of repairs to breaks and the ensuing damages for the last 40 or 50 years, or since the enlargement of 1835-62 was fully completed. Without having the figures at hand the writer has little doubt that, on the average, the cost of such breaks would have paid the interest on the first cost of properly constructed core walls.

TABLE No. 11.

Name of Rhine marine steamer.	Tonnage. Gross.	Tonnage. Net.	Length.	Beam.	Draft, loaded.	Depth to deck.	Year in which the vessel was built.	Number of voyages per year (out and back) = one voyage.	Destination.	Name of Owner.
			Ft.	Ft.	Ft.	Ft.				
Arion.....	290	170	118	20.6	9.6	11.8	1874	17	From Cöln to Bremen.	Dampfschifffahrt-Gesellschaft "Neptun," at Bremen.
Jason.....	344	197	140	22.4	10.10	12.5	1890	16	" " Lubeck.	" " " "
Iris.....	343	201	141	22.2	10.2	12.5	1890	16	" " "	" " " "
Flora.....	344	196	141	22.4	9.10	12.6	1891	20	" " Bremen.	" " " "
Themis.....	475	345	160	24	10.10	11.9	1891	13	" " Stettin.	" " " "
Saturn.....	478	346	159	23.4	10.10	11.9	1891	13	" " "	" " " "
Luna.....	468	279	160	24	11.2	12.5	1892	13	" " Dantzig.	" " " "
Rhea.....	492	293	160	24	12.6	13.4	1892	12	" " Königsbergen.	" " " "
Pax.....	513	301	160	24	11.10	12.6	1893	16	" " Hamburg.	" " " "
Tortuna....	513	304	160	25.3	11.6	11.11	1893	12	" " Dantzig.	" " " "
Stella.....	479	294	151	23.10	11.2	12.5	1895	13	" " "	" " " "
Thalia.....	465	286	151	24	11.6	16.1	1896	17	" " Hamburg.	" " " "
Pollux.....	518	319	165	25.9	11.6	15.1	1896	13	" " Stettin.	" " " "
Castor.....	519	320	164	24	10.2	16	1896	17	" " Hamburg	" " " "
Venus.....	626	361	171	26	13.2	16.10	1897	10	" " Königsbergen.	" " " "
Phädra.....	619	372	168	27	13.2	16.5	1898	9	" " Riga.	" " " "
Ariadne.....	620	373	168	27	12.6	16.5	1898	9	" " "	" " " "
Egeria.....	627	381	171	24.8	11.6	17.1	1898	12	" " Königsbergen.	" " " "
Ruhrort.....	389	232	161	24.1	9.6	11.10	1896	14	" " Hamburg.	Dampfschifffahrt-Gesellschaft "Hamburg," at Hamburg.
Düsseldorf..	158	42	107	20.10	10.2	11.10	1897	14	" " "	
Köln.....	400	242	161	24.1	10.2	11.10	1897	14	" " "	
Industrie....	839	513	236	27.9	11.6	14.8	1885	30	From Cöln to London, and sometimes from the Baltic to Rotterdam and Duisburg.	Rhein-Seeschifffahrt Gesellschaft at Cöln.
Energie.....	740	452	236	30.1	11.6	14.5	1887	30		
Rhenania....	874	655	207	33.9	12	14.5	1894	12	From Duisburg to different ports of the Baltic.	" " " "
Westfalia....	997	603	230	34	12	15.4	1899	12		
Elbing I.....	455	278	165	27.6	12.4	15	1893	10	From Duisburg to Memel and Elbing.	Elbinger Dampfschiff-Rhederei "F. Schichau" at Elbing.
Elbing II....	535	317	180	27.9	12.4	15.2	1895	10		

Mr. Tutein-Nolthenius.

Mr. Tutein-Nolthenius.

as usually carry the Rhine exports to Rotterdam, where the freight is transferred to ocean steamers. And yet these barges carry freight very economically, some being even 328 ft. long, 40 ft. broad and drawing 9 ft.; while the excellent condition of the Rhine estuary allows ocean steamers of any size to enter the inland port of Rotterdam.

Canals of sufficient dimensions are not only a boon to the counties they cross, but to the whole country at large, as they put an efficient check on railway tariffs. For this reason the Netherlands railway freight tariffs are not as high as those of Belgium and Prussia, where canals cannot compete as effectually with the iron road. While in Belgium two-thirds, and in Prussia even three-fourths, of the railway earnings are contributed by the freight traffic, in the Netherlands the revenues from freight-carrying do not amount to 50% of the total income of the iron roads.

Although the Dutch canals are efficient, they still leave much freight for the railroads, as shown in Table No. 12.*

Table No. 12 shows that in bulk the railways carry less than the waterways; in value, the first are favored, which is natural, as articles of great value more easily bear the higher railway rates.

In Table No. 13, the imports from Prussia during the same year, 1894, are analyzed, after abstracting that part of the imports which only traverses the Netherlands. It shows clearly that for the products the waterway is often chosen even though a greater number of consumers is reached more directly by rail; for strangers are mistaken in thinking that a Dutchman always lives near a canal; this is only true for parts of some provinces.

TABLE No. 12.

Total of imports and exports, 1894.	Tons of 1 000 kilograms = 2 204 lbs.			
	By rivers and canals.		By road.	
	Sailing vessels.	Steam vessels.	Common roads.	Railroads.
Imports from Prussia.....	3 017 585	156 727	42 662	3 236 217
Exports to Prussia.....	4 440 483	292 777	113 415	944 533
Imports from Belgium.....	1 962 431	135 149	54 597	1 020 076
Exports to Belgium.....	1 619 959	129 421	62 324	1 112 335

* Tables Nos. 12 and 13 are extracted from a study which the writer published in the Dutch *Economist*, of 1896. The chosen year, 1894, offers no peculiarities, but more recent years would probably show an increase in favor of the river transportation, as the total number of vessels entering and clearing at the Prussian frontier increased from 44 469 in 1894 to 46 089 in 1899, while the average size of the ships was almost doubled.

Mr. Tutein-Nolthenius

TABLE No. 13.

Imports from Prussia, for domestic use, 1894.	Tons of 1 000 kilograms = 2 204 lbs.		Value.	
	By river and canal.	By railroad.	By river and canal.	By railroad.
Coal and fuel (wood)	1 425 000	2 255 000	$3 550 000	$5 675 000
Building materials	167 000	82 000	850 000	125 000
Stone for river works (rip-rap)	488 000	.....	175 000	.....
Lumber	38 000	22 000	325 000	175 000
Ore	80 000	17 000	975 000	425 000
Manufactured metal	137 000	163 000	9 325 000	13 175 000
Fabrics	108 000	96 000	8 725 000	7 800 000
Agricultural products	43 000	52 000	3 500 000	2 275 000
Item from the Indies and the Orient	1 000	9 000	50 000	1 350 000
Manure	23 000	32 000	25 000	50 000
Unclassified freight	10 000	4 000	400 000	350 000
Totals	2 520 000	2 682 000	$27 900 000	$31 400 000

In conclusion, although it is evident that a canal from the Hudson to the Lakes will never offer such facilities as the Rhine, the writer is convinced that a canal better adapted to modern needs than the present Erie Canal (which, however, considering the period of original construction, is a masterpiece not equaled in Europe), will be of inestimable value to New York and to the counties it crosses. It will also stimulate the railways to still more economical management, and thus be a profit to everybody.

Mr. Wells.

Lionel B. Wells, M. Inst. C. E. (by letter).—In 1900, the tonnage of the Manchester Ship Canal reached 3 000 000 tons, of which 2 800 000 tons were in sea-going ships. The canal was constructed to accommodate ships of 3 000 to 4 000 tons, but ships of nearly 6 000 tons gross have been built for the express purpose of trading to Manchester, and do so.

The largest ship which has hitherto navigated to Manchester is the S.S. *Samoa*, 6 839 tons gross measurement. Her cargo included 20 000 bales of cotton.

The heaviest tonnage delivered on the quays from one bottom is between 6 000 and 7 000 tons. These large ships make 5 knots an hour in the canal. They are limited to this speed, but smaller ones are permitted to steam at 8 knots, when it is found by observation of the particular vessel that this is not injurious to the canal banks.

The writer has passed through the Suez Canal in a ship 329 ft. by 39 ft., drawing 21 ft. of water and having a gross tonnage of 2 200 tons. This ship, at sea, made 11 knots; in the canal she was driven at full speed and made 8 knots, a loss of 27.2 per cent. The regulation limits speed to 5.33 knots, but it is ignored when out of sight of the official. At 8 knots there was considerable wash, and undoubtedly the banks were injured.

Mr. Wells.

In 1885, the writer made some experiments on canal-boat propulsion in a canal 90 ft. wide on the surface, from 11 to 12 ft. deep, and having a sectional area of about 700 sq. ft.

The trial was made with a steam tug 76 ft. long, 12 ft. 8 ins. wide, having a draft amidships of 5 ft. 3 ins., and an immersed amidship-section of 60 sq. ft. The speed on a lake was 10¼ miles, and in the canal 7½ miles, a drop of 26.8 per cent.

Over the same course, a steam launch 79 ft. long, 7 ft. 9 ins. wide, with an immersed amidship-section of 30 sq. ft. was tried, and made 8¼ miles in the lake and 7½ miles in the canal, a drop of 9.09 per cent.

Mr. Cooley.

L. E. Cooley, M. Am. Soc. C. E. (by letter).—The following notes are designed to present some data on the relative value of three projects for a waterway from Lake Erie to the Hudson River, in respect to the over-sea trade only, as the conditions may be assumed at some time in the future and after such waterways have had time to develop their logical relation to the commercial movement.

The conditions are:

(1) Unlimited ocean navigation extended to the Lakes by works of the greatest practicable efficiency.
(2) Lake navigation extended to the seaboard.
(3) Barge navigation in barges of a dead freight capacity of 2 500 to 3 000 tons. The project now under consideration by the authorities of the State of New York, for barges of a capacity of 1 000 tons, is regarded as entirely inadequate and as holding out no better promise, under future railway conditions, than the Erie Canal has experienced under past and existing conditions.

The writer is twice on public record—in the report signed by the Advisory and Consulting Engineers to the Committee of Investigation of Expenditures under the "Nine Million Act" in 1898; and again, in 1899, in a letter to the "Roosevelt Committee," headed by General Greene (which letter, however, was mangled in publication).

A policy of progressive change and relocation was advised for the Erie Canal System, on the basis of locks for six canal-boats, or 330 x 36 ft., and an ultimate development for a barge navigation of the capacity herein mentioned.

The writer has a high opinion of the value of barge navigation, and for nearly fifteen years has advocated a barge route from the Lakes to the Gulf, with locks for six barges in one tow, on a depth of 14 ft. This has been the declared policy of the State of Illinois since 1889. Such a fleet would carry 15 000 tons with great economy. He has never considered, however, that the economy could approach that of the single hull of the same aggregate capacity in any trade for which the single hull is adapted.

Mr. Cooley.

1. THREE WATERWAYS.

(*A*) *Ocean.*

Lock, 660 x 72 x 32 ft., for ships of dead freight capacity of 21 000 net tons.

Equated length * of route, Chicago to New York, 1 900 miles, or 1 000 miles from Lake Erie.

First cost, $300 000 000.

Interest, maintenance and operation, 4%, or $12 000 000 per year.

(*B*) *Barge.*

Lock, 330 x 36 x 16 ft., for barges of dead freight capacity of 3 000 net tons.

Equated length * of route, Chicago to New York, 1 600 miles, or 700 miles from Lake Erie.

First cost, $100 000 000.

Interest, maintenance and operation 4%, or $4 000 000 per year.

(Barges capable of carrying 2 500 tons on 14 ft., the depth proposed for the Lakes and Gulf project, with locks for six barges and tow boat; capacity on 10 ft., 1 500 tons. The dimensions are fixed on the basis of six Erie Canal boats.)

(*C*) *Coast and Lakes.*

Lock depth of 21 ft. for boats having dead freight capacity of 6 700 net tons for sea-going model, and 8 000 tons for Lake model.

Equated length * of route, Chicago to New York, 1,800 miles, or 900 miles from Lake Erie.

First cost, $200 000 000.

Interest, operation and maintenance, 4%, or $8 000 000 per year.

2. EQUIVALENT LENGTH.

(Basis—Ocean Navigation.)

Barge.—Cost of carrying 1 mile by barge taken as equivalent to $2\frac{2}{3}$ miles by ocean carrier.

Coast.—One mile equivalent to $1\frac{3}{4}$ miles.

Lake.—One mile equivalent to $1\frac{5}{8}$ miles.

Rail.—One mile equivalent to 10 miles.

Terminal Equivalent.—This is taken as an equivalent to 2 500 miles of ocean carriage. This equivalent measures, in miles of actual carriage, the time in port, port charges, cost of stowing and discharging cargo; in fact, all costs from warehouse or other carrier at origin to warehouse or other carrier at destination, as distinguished from the actual cost *en route*.

* The equated length will vary with the normal speed in free water, the rate allowed in restricted channels, and the lock time, and will vary with the treatment. It involves a large field for good judgment.

Mr. Cooley.

TABLE No. 14.—EQUIVALENT OCEAN CARRIAGE, IN STATUTE MILES—CHICAGO TO LIVERPOOL.

(Ocean, New York to Liverpool, 3 500 + 2 500, or 6 000 statute miles.)

Means of transit.	Equivalent ocean.	Rate per ton.
Ocean (unlimited waterway)	7 900 miles	$1.580
Coast (21-ft. waterways)	11 775 "	2.355
Lake and ocean (one transfer)	11 425 "	2.285
Barge and ocean (one transfer)	13 767 "	2.593
Lake, rail and ocean (two transfers)	16 963 "	3.393

NOTE.—Rate taken at ¼ mill per ton-mile for the actual ocean carriage. This corresponds to $1.20 between New York and Liverpool. $1.40 between Buffalo and New York by rail, and $0.79¼ between Chicago and Buffalo by Lake, the due proportion of the intermediate charge being included. The basis of comparison is the sixth class of the official classification, this being adopted as the average of all freight for the purposes of apportionment in the Trunk-Line Association. Rates in the sixth class usually run about 20% above the grain rate. The average of Lake freight is considerably lower than sixth class.

(*1*) The barge can carry 1 500 miles rather than reship by ocean-carrier, and 2 600 miles rather than reship by Lake carrier. Therefore, it will do a direct business between all Lake points and all Atlantic Coast points permitted by navigable conditions. The theory of the Lakes and Gulf waterway is that barges can work between Lake points and points in the Gulf of Mexico and Caribbean Sea.

(*2*) Lake boats are not considered more suitable for over-sea work than barges, but, under suitable conditions, they will deliver direct to coast points. The sea-going model will be substituted for direct service to coast points remote from New York.

(*3*) The sea-going coaster can carry 3 033 miles beyond New York in competition with a Lake boat, re-shipping to an ocean carrier at New York. It would, therefore, reach some distance beyond the Nicaragua Canal. Such boats are also adapted to the trade of many minor foreign ports and will do a fraction of the over-sea business.

3. LENGTH OF ROUTE.

(*a*) Export wheat in 1897 was carried a mean distance of 4 400 miles from all Atlantic, Gulf and St. Lawrence ports. The opening of the Nicaragua Canal will increase the mean freight route; 4 400 miles may, therefore, be taken as a conservative measure from New York for all foreign freight movement *via* the proposed canal route; and for this estimate, all freight passing the Nicaragua Canal may be treated as foreign.

(*b*) The mean Lake route on wheat in 1897 was 877 miles, and this may be taken for all freight involving reshipment, for the over-sea trade, by rail or Erie Canal.

(*c*) The barge route will be peculiarly favorable to freight originating at, or destined for, Lake Erie ports which does not now find its

Mr. Cooley. way to water routes on account of their shortness. The mean Lake route for this condition is taken at 700 miles.

(*d*) The mean Lake route for the 21-ft. canal may be taken at 750 miles.

(*e*) The mean Lake route for the ocean carrier may be taken at 800 miles.

4. Over-Sea Movement.

TABLE No. 15.

	Relative.	Actual.
Lake, rail and ocean	*1.00	† 8 000 000
Barge and ocean	1.50	12 000 000
Lake and ocean	1.90	15 200 000
Ocean	3.00	24 000 000

* It is presumed, from the relative freight rate for over-sea trade, that the 21-ft. channel will not produce a large increase over the barge route, as reshipment is involved, except for small port service. The ocean channel is adapted to all craft, and gives Lake ports all the advantages of seaports, with full control over the movement in tributary territory, and will very greatly increase the movement.

† The grain reaching tide-water in the crop season of 1897 (Atlantic, Gulf and St. Lawrence) was 13 600 000 tons, and the Lake movement was 8 100 000 tons. The export (fiscal year, 1898) was 10 300 000 tons. Over 28 000 000 tons, about 70% of the foreign trade, was handled between Portland and Norfolk. The Lake grain in foreign movement was probably 6 000 000 tons. The total foreign movement (over-sea), using the Lakes, is taken at 8 000 000 tons, and is not assumed to increase with the continuance of existing conditions.

5. Value of Service in Over-Sea Movement.

TABLE No. 16.—(Compiled from Sections 3 and 4.)

Carrier.	Equivalent route.	Rate. (¢ mill.)	Saving per ton.	Tons moved. Millions.	Gross saving. Millions.	*Deduction 4% cost.	Surplus millions.	Dividend in surplus.
Lake, rail and ocean	17 985	$3.587	0.000	8.0	$0.000	0.0	0.0	0.0
Barge and ocean	13 133	2.627	0.960	12.0	11.520	4.0	7.520	7.5%
Lake and ocean	12 081	2.416	1.171	15.2	17.799	8.0	9.799	4.9%
Ocean	8 700	1.740	1.847	24.0	44.328	12.0	32.328	10.8%

* See Section 1 for capital, interest, maintenance and operation.

Note.—The Barge or Lake Canal will make a better showing in the domestic trade with North Atlantic ports. The Lakes and Gulf waterway will do the domestic work between the Lakes and the Gulf and Caribbean.

6. Lake Territory—Unlimited Ocean Navigation.

The proportion of the total for the United States is given for the following items:

TABLE No. 17.

Corn, oats and wheat, production, 1898	74.5%
(Same) shipped out of country	82.9%
Population in 1890	46.1%
Railway mileage, 1898	54.0%
Railway ton-mileage, 1898	55.2%
Freight originating on road, 1898	57.0%
Economic potential (capacity of soil)	42.0%

Mr. Cooley.

The foregoing shows the intimate relation that the Lakes, regarded as arms of the sea, would have to the freight production, and justifies the high assumption for traffic under the conditions of ocean navigation. Neither barge nor Lake navigation can command an area so extensive, draw so large a proportion of the freight, or create so much new traffic.

Explanation.

The assumption of 2 500 miles as the value of the transfer (measured in terms of the carrying rate for ocean navigation) is probably too low, and it certainly is much higher under existing conditions. The necessary cost of a transfer of grain may be taken at $\frac{5}{8}$ cent per bushel, including therein not only the intermediate charge, but that portion of the rate incident to the transfer. There is an average loss or shrinkage of 0.3 of 1% on grain, and more on package freight, especially cooperage. All these may be assumed at 25 cents per net ton for grain. This would represent 30 cents per ton for sixth class, or average for all freight, taking the usual tariff rates between grain and sixth class. This is equivalent to 1 500 miles, and leaves an entirely inadequate equivalent for time in port and port charges.

Lake freights are seemingly low. This is, in part, due to the way the rate is made up, and again, to the very low class of average Lake freight, and also to extraordinary terminal provisions for certain commodities. It is assumed that these special conditions will not obtain to the same extent in a foreign movement, and that any advantages cannot persist indefinitely.

Conclusions.

The foregoing presentation does not assume to be more than illustrative, and the writer will not feel called upon to defend it. The data for a proper and final discussion in these matters has nowhere been assembled and made accessible, nor has any accepted attempt been made to expound first principles. The writer speaks advisedly in this matter, and after many conferences with the master-minds in transportation problems. The relative value of different projects is not now capable of close determination.

The writer does not admit the validity of the dividend or commercial valuation as a measure of public utility in any public enterprise, and especially one that is recognized as out of the corporate field. He has, however, thought it expedient to follow the other parties to this discussion, and avoid an essay on that matter at this time.

The writer concludes that the barge proposition, of the capacity which he has given, is superior to the proposed extension of Lake navigation to the seaboard. He also concludes that ocean navigation extended to the Lakes is vastly more valuable than either, and will return more in proportion to the investment; and that such development will become an economic and political necessity.

Mr. Mayer. JOSEPH MAYER, M. Am. Soc. C. E. (by letter).—The objections to the statements made in the writer's paper refer to the practicable speed and safety of navigation in the proposed channel, the time necessary at terminals, the traffic capacity of the canal, and the amount of traffic to be expected.

As regards the first four, the writer referred to the United States Canal Commission as authority, and mentioned the experience in the St. Clair Flats Canal, which is of the same dimensions as the proposed ship canal and is dammed off from the shallow water at its sides.

In regard to the safety of navigation it has been suggested that the great length of the proposed canal, which has 102 miles of standard section and 102 miles of canalized river of much greater section, may make such a difference that the experience in the St. Clair Flats Canal is not to the point. Some facts taken from a paper* by *Sir* Charles Hartley, M. Inst. C. E., and from the discussion on that paper, are therefore added.

The Suez Canal was originally built with 72 ft. bottom width, and was 26 ft. 3 ins. deep. It was then deepened to 27 ft. 10 ins. at low water, and enlarged to a bottom width of 118 ft. with side slopes of 1 on 2 in some places, and 1 on 2½ in others; and with a berm on which rests the stone revetment near the water line for the protection of the slopes. A further widening may be undertaken in the near future. The stone revetment on one side is not in good condition. It is not intended to be permanent, and has not been built as well as that on the other side.

There are nine sidings 49 ft. wide and 2 460 ft. long, for the purpose of permitting vessels to pass each other. The length of the canal is 100 miles, 27 miles of which are lakes. The most northerly of these lakes and the longest is really a swamp dammed off from the canal. Eight miles of the Great Bitter Lake is deeper than the original canal, and permits free and unrestricted navigation for this distance. The other lakes are of very small extent. They are shallow and contribute little to the facilities of navigation.

Vessels going in the same direction are not permitted to pass one another. In spite of this rule, which makes the speed of all vessels nearly equal to the slowest, the average duration of transit is 17 hours 22 minutes, corresponding to a speed of 5.76 miles per hour. The regulations forbid a speed greater than 10 kilometers, or 6.21 miles, per hour. The slope protection in many places could not withstand a greater speed without harm. There are observing stations at considerable distances apart, where vessels are not allowed to pass before the lapse of the time corresponding to the maximum allowed speed. Mr. Lionel B. Wells, an English engineer who passed through the canal in 1898, in discussing *Sir* Charles Hartley's paper, stated that

* "A Short History of the Engineering Works of the Suez Canal," *Min. Proc.* Inst. C. E., vol. cxli.

the speed limitation is generally disregarded between observing stations, and that a speed of 8 knots an hour is frequently attained for short distances. Mr. Mayer.

The yearly traffic is about 3 600 vessels of 9 000 000 net tons. The tidal current in the southern section attains sometimes a velocity of 2.9 miles per hour.

Here is a canal which carries safely an enormous traffic, which is half the width of the proposed canal between the Lakes and New York, and in which there is a strong current. It has about the same length as the standard section of the latter. It does not appear reasonable, therefore, to doubt that the latter, with practically no current, will furnish a safe channel for a very much larger traffic.

The regulations on the Kaiser Wilhelm Canal between the Baltic and North Seas allow a speed of 9.3 miles (15 kilometers) per hour for vessels of less than 16 ft. 5 ins. draft, and 7½ miles (12 kilometers) per hour for vessels of larger draft. The standard cross-section of this canal is 4 520 sq. ft.

Mr. Fülscher, an engineer on this canal, in discussing *Sir* Charles Hartley's paper, states that vessels having a midship-section of not more than one-sixth of the cross-section of the canal suffer but little reduction of speed therein, as compared with their speed in open water.

The proposed canal between the Lakes and New York has a standard section of about six times the midship-section of the vessels intended to pass through it.

As appears from Mr. W. H. Hunter's discussion, 8 miles an hour is a speed frequently attained on the Manchester Ship Canal. Mr. Elnathan Sweet also substantially confirms the estimate of speed adopted by the writer, and bases his view on experiments made in recent years in Germany and France.

The writer has assumed a speed of 7 miles in the standard section and 9 miles in canalized rivers of much wider section. It is evident, from the facts stated, that speeds of 1 mile more than those assumed would have been more nearly in accord with them.

As regards the time spent at terminals, two cases have to be considered: Vessels loaded with bulk freights, like ore, coal or grain, can be loaded and unloaded by special machinery, which will be provided wherever a large amount of freight of this kind is to be handled. Experience on the Lakes shows that during the whole navigation season the average time spent in harbors is about 3 days per round trip, and the assertion that this time will be 10 days is based, not on theory or experience, but on unreasonable prejudice. Vessels loaded with miscellaneous high-class freight need spend not more than 1 week per round trip at terminals, as is shown by the records of the steamers of the American Line, which are scheduled to make one round trip between New York and Southampton every 3 weeks.

Mr. Mayer. The writer's estimates of cost referred to low-class bulk freights, and not to high-class freights. The difference in cost between the different classes, however, would not be more than on railroads.

Transfer of bulk freights to other vessels becomes superfluous as soon as a ship canal exists, as there is no necessity of adhering to the present expensive mode of transporting bulk freights from the West to Europe, or points on the Atlantic Coast which cannot be reached by canal-boats.

Regarding the capacity of the canal proposed, the estimate of the United States Canal Commission was quoted, and the calculation of the proper tolls was based on this estimate.

The capacity of the canal is governed by that of the locks. Present experience at Sault Ste. Marie, and the prevailing average size of Lake and ocean vessels, indicate for the locks proposed a much smaller capacity than that given. But it is evident that what is wanted is not the present capacity of the locks but that at the time when the amount of freight will have grown to be nearly equal to it. On the Suez Canal the average net tonnage of the passing vessels increased from 1 170 tons, in 1873, to 2 640 tons, in 1898. A large increase in the average size of vessels may be expected during the next 20 years, which would increase the capacity of the locks to the same extent. The present capacity of the Sault Ste. Marie locks, therefore, is no direct measure of what the locks on the proposed canal will be able to do. The following letter, received from Mr. Noble in answer to a request, will remove any doubts on this subject:

"CHICAGO, DECEMBER 1ST, 1900.

"Mr. JOSEPH MAYER,
"No. 1 Broadway, New York, N. Y.

"DEAR MR. MAYER,—The estimates of cost for the Deep Waterway are based on a single system of locks for single lifts and a double system where two or more lifts are combined. It is assumed that the cargo of a fully loaded freight ship is twice its net register, that its east-bound freight will be in full cargoes and its west-bound freight one-fourth as much. This makes the total freight one and one-quarter times the net register.

"The capacity of a single lock is determined by investigation to be 25 150 000 tons of freight in a navigation year; of the Lewiston double flight of six combined lifts, of 40 ft. each, the capacity is 35 801 000 freight tons. If the capacity of the single locks should be reached, by doubling them the capacity could be increased to 35 801 000 without increasing the facilities of systems of two or more lifts.

"In arriving at these amounts the following method was pursued:

"Ships are to be locked singly. The average tonnage, net register, is assumed to be 2 500, corresponding to 3 000 tons of freight for a round trip. This average tonnage is greater by far than the present average on the Lakes, but the ships now building (and for the last five years) for the freight-carrying trade exceed this very much. These large ships are crowding the others out, and by the time the Deep Waterway could be built the small ship will not be an appreciable

factor in the business. This tonnage is less than the present average at Suez, where it is increasing quite rapidly. Mr. Mayer.

"The size of type ship being thus arrived at, the theoretical time of lockage was calculated and the resulting tonnage per year. This was greater than could be handled practically. To obtain a coefficient the records of the St. Mary's Falls Canal were used.

"In 1894 all the business was done by one lock, locking generally two or more ships at once. The delays to ships awaiting lockage were large, reaching what may be considered an economical limit. It was determined what change in time of lockage would have taken place if ships had been locked singly; if the ships had been of the larger size assumed for the Deep Waterway; and if the time for filling and emptying the locks had been changed to conform to the time for the locks of the Deep Waterway. Correcting the tonnage actually passed at the St. Mary's Falls Canal to these changed conditions, we have the tonnage which a single lock of the Deep Waterways would have passed with the same sundry delays and the same idle time as occurred at the St. Mary's Falls Canal in 1894 and also with the same loss of time to shipping while awaiting lockage, which loss of time, as already stated, was considered to establish the economical limit of capacity of the waterway. This tonnage was found to be 0.621 of the theoretical calculated capacity, and was considered the best coefficient available. This coefficient was then applied to the theoretical calculated capacity of each system of lockage.

"While the method may not be wholly satisfactory, I do not know of any better one. At no other ship canal in the world has the lockage capacity been reached.

"The traffic capacity figure you have used is probably the one above given with single locks duplicated. I believe the capacity estimates to be safe, and with the information now existing I do not think anyone can successfully controvert them.

"Very truly yours,
"A. NOBLE."

A considerable difference in the real capacity of the canal, however, will not materially affect the argument for or against it, as there is no reasonable doubt that it will be very profitable as soon as the freight transported through it is equal to its capacity. The cost of increasing the latter is comparatively small.

A more important question is that of the amount of freight to be expected. Major Symons, in his report of 1897, has made an estimate in which he gives the amounts of the different kinds of freight which he expects for a canal 24 ft. deep on the Oswego Route, and the writer was fully aware of the fact when this paper was written, but did not mention the estimate, because no evidence is given in that report to support his mere assertions, and because the writer believes it to be impossible to make a detailed estimate of the probable traffic. If anybody had made an estimate 20 years ago of the probable traffic of the inter-lake channels, he would have either greatly underestimated the traffic, or have been laughed at as visionary. The idea that iron ore in any quantity would be transported to Pittsburg from the Lake Superior Region would have been found to be utterly ridiculous and as indicating the ignorance of the author of it.

Mr. Mayer. We now know, from experience on the Lakes, that a great reduction in the cost of transportation will develop an enormous amount of new business, and that it will deflect from distant and more expensive routes business already existing. The development of new business is a relatively slow process, requiring the establishment of new manufactures along the new route, while the deflection of existing business from other means of transportation will take place much more quickly. It is true that no such transport of heavy cheap goods over great distances takes place on the line of the future canal as exists now on the Lakes, but such transport would not exist along the lake routes if lake freight rates were the same as those by rail. To any careful observer the experience on the Lakes should show two things: First, that the new business developed by the lowering of freight rates is more than half the total. Second, that one cannot estimate in detail the new business which will be developed by a great reduction in cost of transportation.

One can with some approximation estimate the amount of existing transportation which will be transferred from other lines to one very much cheaper, but not the new transportation which will be created. It is evident that the canal proposed will obtain most of the grain and much of the provisions passing from upper lake points to the Atlantic coast and Europe; that it will obtain lumber in large quantities in both directions, besides salt, building stone, cement, brick, coal, iron ore, etc.; but to put definite quantities behind all these items, shows only the habit of their author to make positive statements in regard to subjects which he does not know.

The principal data which permit making an approximate estimate of the freight to be expected are furnished by the amount of freight obtained through the Lake rates, due to the improvement of the interlake channels, and by a comparison of the population and the number and importance of the harbors at the two ends of the lake region with those at the two ends of the proposed canal. Since the visible makes a greater impression on us than the invisible, we are apt to underestimate in such a comparison of an existing waterway with a proposed one the amount of freight which may be expected by the latter. The development of the iron mines in the Lake Superior Region is an accomplished fact, and their product goes at present largely to Pittsburg, therefore the argument is ready at hand that there are no iron mines on the new canal, and that the ore of the western mines does not go East. A considerable part will go East after the canal exists, and other sources of freight, as unknown now as the Lake Superior mines were twenty years ago, will arise as soon as heavy goods can be transported cheaply. The quantity of goods produced, transported and consumed is largely dependent on the size of the population which produces and consumes them, and the cheapness of transporta-

tion. And the possibility of transporting them by water from producer to consumer depends mainly on the existence of the appropriate channels and the number and importance of the harbors available. In comparing, therefore, the relative amount of freight which may be expected to pass through the Detroit River and the canal proposed, it is reasonable to take into consideration the populations and harbors connected by the two waterways. If this is done, and allowance is made for the natural increase of business during the next twenty years, it appears to the writer to be very moderate to estimate the then existing traffic through the proposed canal as equal to its capacity. Another argument against a ship canal on the Oswego Route is to the effect that Canada would build a ship canal to Montreal and deflect business in that direction. The larger part of the Dominion is chiefly interested in such a canal on account of the cheap transport to Europe and the United States which would be created thereby. A ship canal from Lake Ontario to Montreal would not materially cheapen transport to these countries after the American ship canal exists; it would be in a precarious position without control of a ship canal around Niagara Falls; and it would be such a ruinous undertaking for a small and poor country like Canada that it is an insult to our neighbors to think them capable of it. Mr. Mayer.

An estimate of the water supply needed at the summit level has been made by one of the discussors, and an objection against the Oswego Route is based thereon. This discussor asserts that the water supply needed at the summit level for each passing vessel is equal to the lock area multiplied by the sum of the lifts of all the locks.

It is reasonable to provide water supply for the full capacity of the canal. At each end of the summit level is a lock. From the passage of the sill by a vessel entering a lock from above, it takes a certain minimum time for this vessel to go to the lower level, move out, let another vessel pass in, rise up, move out, and the next downgoing vessel to advance to the sill. This period of time is required for one cycle of lock operation, and will pass one vessel down, another up. For a given lock area, it varies somewhat with the litt at the lock. One lock chamber full of water is required per cycle, or a volume equal to the area multiplied by the lift. The shortest time required for one cycle is found by observation of existing locks of similar size and lift. It is easy to estimate the influence of a change in lift on the time required for one cycle. The experience at the locks of Sault Ste. Marie allows, therefore, a very close estimate of this time to be made. From this the number of cycles per 24 hours is obtained, and the quantity of water needed for the two locks at the end of the summit level can easily be estimated. If the amount of leakage per 24 hours in the summit level and the two adjacent locks, and the evaporation per day, is added to the foregoing amount, the water needed at the summit level is known. The latter items are

Mr. Mayer. relatively small if the canal is well built. This is all the water needed at the summit. And if the two adjacent locks have the greatest lift of those on the same side of the summit, the water needed for lockages at the summit is sufficient for all lockages below, provided the level water surfaces between any two succeeding locks, in comparison with the lock chambers, are large enough to permit their filling twice without reducing the depth too much on the level. Any additional water needed, however, may be supplied below the summit level. The problem of water supply is therefore not so serious as it would be if the estimate mentioned were correct. The argument based on this difficulty becomes invalid on account of the gross error in the estimate of the water needed.

It has been suggested that it would probably be better to make the canal somewhat deeper and narrower. The writer's studies of the opinions of experts on the Suez Canal lead him to believe that the same area of cross-section as proposed by the Commission, with a depth of 24 ft., would be more satisfactory. At a depth of 21 ft., this would give a width of 192 ft. instead of 215 ft., and a bottom width of about 177 ft., with slopes of 1 on $2\frac{1}{2}$. The Commission, however, undoubtedly had good reasons for the cross-section selected. The width chosen by it substantially agrees with that proposed by the International Commission which investigated the proper means for increasing the capacity of the Suez Canal. But the depth recommended by that Commission was 9 meters, or 29 ft. 6 ins.

It has been correctly stated in the discussion that most of the ocean traffic is carried on in comparatively small vessels which could easily pass through the proposed canal, and that the large vessels are very few and play a small part in the freight traffic, especially in the coasting trade. It is evident, therefore, that for most of the traffic small vessels are more economical than very large ones. The assertion that vessels capable of passing through the proposed canal could not with economy cross the ocean is not supported by the facts.

Since the locks proposed would only be able to handle the traffic for at most twenty years, additional locks would soon be required which could then be built of a size and depth fit for the demands upon them. The canal could then be deepened should it appear to be desirable.

Another very important question touched by Mr. North, is the advisability of levying tolls. The writer does not deny the advantages which any section of the country would derive should the United States decide to pay a large part of the freight charges in that section only. This is substantially what the proposition of building at the expense of the United States a costly canal, which is to be toll free, amounts to. The prospect of inducing people of other sections of the country to vote for such a scheme, however, is very slender, and,

in order to secure a majority of votes, as proposed by Mr. Thomas Curtis Clarke, it would be necessary to couple it with other schemes beneficial to other parts of the country. The writer believes that the reduction in cost of transportation to be brought about by this canal is so large that the levying of proper tolls will not materially diminish the amount of freight which will pass through it; and it is ungenerous and inexpedient for the section of the country to be benefited to expect other sections, not so fortunately situated, to help in paying the yearly expenses incurred for it. Mr. Mayer.

For the purpose of illustrating from an economic standpoint the objections to a toll-free canal, a simplified example containing the essential features of the problem will be chosen.

There are quarries at a town, "A," where a cubic yard of stone costs $2; in another town, "B," are quarries where a cubic yard of equally good stone costs $3; the cost of transportation between the two places is $1.50 per yard. In this case each place uses its own quarries. The State intervenes and assumes two-thirds of the cost of transportation.

Then the town "B" will obtain stone from "A" at $2.50 per yard. The actual cost of this stone is $3.50 per yard, of which the State pays $1. The effect of the interference of the State is that more labor is spent for the stone used at the town "B" than before.

Stone has become actually dearer though it is nominally cheaper. Other effects will be produced, among them, losses to the quarry owners of the town "B" and gains to those of the town "A"; generally the latter effects alone are dwelt upon by advocates of such State interference. When the State omits to charge a reasonable toll for the use of a canal which it builds it not only enriches one section of the country at the expense of the rest, but it increases the production of things at a distance which could really be produced cheaper at home; it thereby increases the amount of labor which has to be expended for obtaining a given amount of goods. There are cases, however, and the Manchester Ship Canal seems to be one of them, where the principal benefit obtained by a canal is the lowering of excessive monopolistic freight rates on competing railroads, and where the attempt to levy tolls covering the yearly expenses of such canals would be destructive of their usefulness. The writer believes that a toll-free barge canal of large capacity, from the Lakes to New York, would have this effect. And the advocates of a barge canal wisely refuse to recommend tolls, as they would destroy most of its usefulness. A barge canal of small capacity, however, would not answer this purpose, as the railroads would find it to their advantage to let it have all the business it could carry, and ignore its freight rates.

The rail freight rates in this country, however, are only about one-third of those in England; the possible reduction is small, and the effect of a competing canal, therefore, very limited.

Mr. Mayer. The tolls proposed for the ship canal by the writer, however, are much smaller than the expenses incurred for transfer on a barge canal; the former is therefore superior, as it pays its way and offers larger benefits to the public.

The writer's criticisms of the barge canal project were mainly directed against the arguments of the New York State Canal Commission, which attempts to prove that a barge canal is desirable, independent of the incidental advantages of lower railroad freight rates due to it.

Theoretically, it would be best, in canals which are not used to their full capacity, to increase the amount of freight using the canals by varying the tolls for the different kinds of merchandise, taking into account origin and destination, in proportion to the value of the service rendered by the canal, charging on no part of the freight more than the traffic will bear. For a canal which is used to its full capacity a uniform toll is the best in every respect. The value of the service rendered by the proposed canal is greatest for freight which saves transfers. The principle of charging in proportion to the number of transfers saved seems to be, therefore, the nearest practicable approach to an ideally perfect system of tolls.

The tolls proposed by the writer were calculated for an annual expense of $9 000 000, allowing for 3% interest. If the United States can obtain the money for construction at 2% interest, the annual expense will be only $7 000 000, and the tolls given would be sufficient, even if the estimate of traffic is considerably in excess of the facts.

Mr. Thomas Curtis Clarke's remarks are interesting, and the writer thinks that the scheme recommended by him for consideration is worthy of careful investigation, which is likely to show that it is not only feasible but economically desirable, weighing the advantages to be gained with the cost of their attainment. If the scheme were as well known in all its aspects, and if its economic merits were equal to those of the canal here considered, the writer would think it advisable to combine the two; but, for the present, he would prefer that those most interested in the eastern canal would come forward, frankly declaring their willingness to pay for the yearly cost by proper tolls, and advocating the passage of a law in Congress providing for its construction on these terms.

It has been asserted that Lake vessels would be very uneconomical on the ocean. The writer does not believe that the equivalents, on the size of which the whole argument depends, have inherent probability, in the face of the actual transportation charges on the Lakes and ocean. All the difference in these is explainable partly by the present small depth of lake channels, and partly by the smaller distances on the Lakes. If a canal of either 21 or 24 ft. depth existed, vessels would be built adapted to both Lake and ocean traffic, and

their disadvantages for either would be small in comparison with the savings accomplished by the avoidance of transfers. Mr. Mayer.

The superiority of a canal 30 ft. deep can only be shown by assuming an enormous traffic. Though the writer believes that such a traffic will exist on this route in the near future, he does not think that after this discussion it is possible to convince a sufficient number of people to induce action by Congress in favor of such a canal. The smaller canal will be more profitable at the beginning, and for this purpose does not require an amount of traffic which appears improbable to the majority of judges.

The capacity proposed by the United States Canal Commission for the beginning is, as appears from Mr. Alfred Noble's letter, 25 000 000 tons, which can be increased to 36 000 000 tons by doubling the single locks. If one of the two rows of locks, in the places where there are several locks in a chain, were made 30 ft. deep, the canal could be easily increased to 30 ft. depth whenever the traffic required an increase of capacity beyond the 25 000 000 tons. This ought to unite all the advocates of a ship canal, because it will fulfill the wishes of those in favor of a 30-ft. canal as soon as the growth of traffic shows this to be advantageous.

The purpose of this paper was to make it evident by the discussion which it would call forth, that the preponderant judgment of those who have studied this question is in favor of a ship canal. Mr. Wisner's paper and the writer's have accomplished this purpose. The objections which have been raised against a ship canal have been effectively answered in the discussion, by those most competent to do so; and many who were either ignorant of, or indifferent to, the subject, have been convinced or made more enthusiastic in favor of an enterprise which is the most important engineering work of the near future.

GEORGE Y. WISNER, M. Am. Soc. C. E. (by letter.)—It is gratifying to note the advance which has been made in general knowledge on the waterway problem since the discussion in the *Transactions* of the Society for June, 1898.* Mr. Wisner.

The present papers attack the problem from entirely different standpoints and by different methods, and arrive at practically the same results; while the discussions, which have been contributed by some of the ablest waterway engineers in the world, although not in harmony as to the actual dimensions which should be adopted, in no way controvert the facts on which the conclusions in the papers were based.

One of the principal issues raised in the writer's paper was that estimates of transportation rates by different routes should in each case include all items relative to the cost of construction, maintenance

* *Transactions*, Am. Soc. C. E., Vol. xxxix, page 273; "The Economic Depth for Canals of Large Traffic," by Joseph Mayer, M. Am. Soc. C. E.

Mr. Wisner. of route, and the cost of moving the expected commerce between terminals, which in any way affect the fixed charges properly chargeable to the rates of the respective routes.

Major Symons calls attention to the writer being a strong advocate of ship canals; yet, in all of his own estimates for transportation rates on his proposed barge canal, he assumes the waterway to be constructed and maintained at public expense, and then attempts to show the great saving in transportation rates, as compared with those on the routes where all fixed charges for construction and maintenance must be taken care of in the transportation rates.

The writer has endeavored to treat each route in exactly the same manner, and wherever fluctuating values have been used, the conclusions drawn are based on results from which the errors from such sources are practically eliminated; and if such methods may be termed those of an advocate, then those adopted by Major Symons need qualifying with a much stronger phrase.

It is immaterial whether the routes be constructed and maintained at public expense, or whether the fixed charges be made a part of the transport rate; the result to the producers of the country is the same. If the route is worthy of being constructed at public expense, it is fair to presume that the people of the whole country are to be benefited, and whatever saving may accrue in transportation rates from Government construction indirectly becomes a tax on the producers who use the line.

If, therefore, when all direct and indirect benefits are considered, a route cannot be shown capable of furnishing better rates with fixed charges included than its competitors, we must conclude that it is not worthy of construction.

In discussing the relative merits of these different proposed waterway routes, there are several important considerations which must not be lost sight of, if a correct and satisfactory solution of the problem is expected.

Existing knowledge as to the natural physical conditions of the routes, and the economical type of freight carrier which can be best utilized on the respective waterways in the transport of domestic and foreign commerce, must be fully considered, and all conclusions based on sentiment eliminated. While most of us would take great pride in such a grand achievement as that of extending the ocean waterways to the ports of the Great Lakes, we must inevitably answer the question, "Will it pay?" before asking the producers of the country to contribute their earnings for its construction. Not that the reduction in transportation rates must be such as to warrant a private company taking up the project for profit, but that the result to the whole country from direct reduction of rates and from resulting development of new commerce and industries will show a balance on the right side.

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Under the most favorable conditions to be expected, it will be at least ten years before either of the proposed waterway routes can be opened for business. The relative movement of domestic and foreign commerce which will exist after that period is the important factor on which both the type of waterway and of freight carrier must be based. At the present time about one-fourth of the wheat raised in the United States is exported; but from the increase in population which may safely be predicted, the present surplus of wheat raised will be required for domestic consumption within less than twenty years, and unless the annual yield can be increased, the export of grain must be eliminated from the calculation. Whatever the future may have in store for the grain business of the country, it is safe to say that the export of raw materials will greatly decrease, and the export of manufactured products largely increase in the near future, which implies a decreasing tonnage for the corresponding value of the commerce.

The Commissioner of Navigation gives the weight of exports from the United States in 1899 as follows:

TABLE No. 18.—Weight of Exports in 1899.

	Value.	Weight.	Value per ton.
		Tons of 2 240 lbs.	
Articles, in terms of weight.....	$904 827 281	25 569 729	$35
Articles estimated by experts.....	85 370 277	602 482	142
Lumber, boards and shingles.....	20 138 053	2 142 424	9
Miscellaneous manufactures.....	193 595 611	967 978	200
Total	$1 203 931 222	29 282 613	$41

It will be noted from Table No. 18 that the entire exports in 1899 only amounted to 29 282 613 gross tons, of which 25 569 729 tons, worth $35 per ton, were products, the export of which is likely to greatly decrease in the near future, and that 967 978 tons, worth $200 per ton, were manufactured products, the exports of which are increasing rapidly, and will be likely to comprise the principal part of our foreign commerce during the era for which the the waterway must be constructed. It therefore appears that the waterway, if constructed at all, should be more for the purpose of distribution of products for domestic consumption than for the accommodation of foreign commerce.

The economical dimensions for a waterway, and for the freight carriers which use it, are functions of the distance between freight terminals.

In ocean traffic, with from 3 000 to 5 000 miles between terminals, there is no question that larger ships will be economical, and will be built as fast as the improvement of the depth of terminal harbors will

Mr. Wisner. permit; but, on the Great Lakes, where the length of haul is limited to 1 000 miles or less, the limit in size of economical ship construction has already been reached, and with unlimited depth in the connecting channels of our lake waterways it will not pay to construct freight carriers for lake service alone any larger than those recently put in the service. This statement may seem somewhat strong, but the facts are that with a haul of less than 1 000 miles, the running expenses and fixed charges for ships over 480 ft. long, 52 ft. wide and 19 ft. draft, increase more rapidly than the profits from increase of carrying capacity.

Table No. 19, giving the dimensions, horse-power, speed, cost of ship, running expenses and cost of transport per ton-mile for different types of carriers, is based on actual lake practice and experience, and has been prepared from data furnished by Mr. Frank E. Kirby, who is doubtless the best authority in the world on this class of ship construction.

TABLE No. 19.

Dimensions and draft of ship, in feet.	Number of propellers.	Horse-power.	Speed per hour, in miles.	Carrying capacity, in net tons.	Pay roll and subsistence, per day.	Fuel, oil and waste, per day.	Cost of ship.	Cost of transport per ton-mile, in mills.
480 × 52 × 19...	Single	2 200	12.5	8 600	$60	$117	$387 000	0.128
480 × 52 × 23...	"	2 480	12.5	9 600	60	129	462 000	0.129
480 × 52 × 27...	"	2 800	12.5	11 760	62	143	554 400	0.121
500 × 54 × 27...	"	2 930	12.5	12 600	62	149	612 400	0.120
520 × 56 × 27...	Two	3 100	12.5	13 300	80	155	705 600	0.129
540 × 58 × 27...	"	3 200	12.5	13 980	80	159	771 400	0.130
550 × 60 × 27...	"	3 330	12.5	14 100	82	165	828 400	0.136
480 × 52 × 19...	Single	3 650	15.0	7 650	72	154	410 000	0.141
500 × 54 × 27...	"	4 850	15.0	10 000	76	204	661 200	0.151

It will be noted that for speeds of over 12.5 statute miles per hour the rate increases. If the lines of the vessel be adapted for obtaining greater speed with less power, the carrying capacity is decreased and the cost of carrying correspondingly increased. In computing the cost of transport per ton-mile the prices for labor and materials used are the same for each type of vessel, and, therefore, any variation which may arise from fluctuation of prices will be practically eliminated in the relative cost of transport for different types of ships.

The rate per ton-mile in Table No. 19 only includes those items which enter into the cost of carrying a ton one mile in the open lake. If we take into consideration the greater loss of time by the larger ships at terminals, the difference in cost of transport becomes still less, as shown by Table No. 20, which is a computation of transport rate for the first and fourth of the steamships given in Table No. 19.

TABLE No. 20.—TRANSPORT RATE PER TON. Mr. Wisner.

Dimensions of ship, and draft	480 × 52 × 19 ft.	500 × 54 × 27 ft.
Sailing time of round trip	7 days.	7 days.
Detention at terminals	3 days.	4 days.
Wages and subsistence	$600	$682
Fuel, oil and waste	909	1 183
Insurance on ship	477	830
Insurance on cargo	860	1 260
Miscellaneous expenses	30	40
Interest on investment	530	922
Deterioration, etc	530	922
Profit to shipowner	400	600
Total	$4 336	$6 439
Rate per ton	0.378	0.383

It will be noted from Tables Nos. 19 and 20 that the actual cost of moving a ton of freight 1 mile in the open lake is such a small percentage of the transport rate which must be charged to transact the business, that the length of haul and the amount of detentions at terminals are the important factors necessary for determining the economical dimensions for freight carriers, and, consequently, of the water routes to be used by them.

The rate computed in Table No. 20 is for ships adapted for both lake and ocean traffic, and used for lake traffic only. If freight be carried between Lake ports and New York in vessels adapted for only lake and waterway business, the actual savings on the Lakes by using the deeper-draft vessels would be less than one-half of the estimated fixed charges arising from the excess of cost of construction and maintenance of deeper channels, as stated in the writer's paper.

Since at least three-fourths of the freight carried on the Lakes is for domestic consumption, and because domestic traffic must increase much more rapidly than that with foreign ports, it seems evident that if any waterway be constructed it should be the one best adapted for the distribution of products and manufactures for domestic use.

It is generally conceded that to produce the best results, a waterway from the Lakes to the Atlantic should have dimensions which will permit the passage, at economical speeds, of ships best adapted for the traffic of the water routes connected.

It is also established from the cost of vessel construction on the Lakes, and the cost of transport in carriers of different speeds and dimensions, that the maximum size of steamer for economical transportation for hauls of less than 1 000 miles has been reached in lake practice; from which it appears that the only legitimate reason for constructing waterways more than 21 ft. deep will be solely to accom-

Mr. Wisner. modate foreign commerce between lake and foreign ports. The only saving to be obtained by the use of deeper waterways will be the cost of transfer at New York of products destined for foreign ports, and, since all traffic for the Atlantic coast trade would have to be transferred at New York, if carried in the larger ships, it is probable that the 19-ft. carrier would be more economical on the average, no matter how large the volume of commerce be assumed.

Without regard to the pleasing sentiments awakened by the proposition to open up a waterway from the Great Lakes to the seaboard, capable of accommodating the largest ocean freight carriers, the cold facts must be faced that the interests of the producers of the country are but the collective benefits to be derived by individuals, and unless it can be shown that the benefits to be secured from the construction of a new route, with all expenses included, will exceed those obtained from routes now in use, it will certainly never be built, no matter whether it be a $60 000 000 barge canal or a $300 000 000 waterway for ocean steamships.

Major Symons' statements that "Mr. Wisner has certainly jumped at a conclusion which is in no manner justified by the facts and present data," and that "his premises are entirely in error," seem peculiar, to say the least, when it is considered that he was well aware when he wrote his discussion that the St. Lawrence Canals were actually open for business for the entire navigation season of 1900.

It is technically true, as he states, that the Canadian Canals have not yet been entirely completed. Neither has the St. Mary's River improvement been fully completed, yet its commerce in 1900 was over 25 000 000 tons, while on the new St. Lawrence Canal only one fully loaded freighter is reported as having passed through. Why? Simply because, as stated in the paper, any waterway requiring transfers will fail to divert freight from the lake and railroad lines.

It is a notable fact that the steamers now being constructed for lake and ocean commerce through the Canadian Canals are for 20 ft. draft, it being considered more economical by the builders to lighter through the canals than to undertake transportation on the Lakes with ships of only 14 ft. draft.

The fact mentioned that "the Canadian Government and private parties have only just commenced work at Port Colborne and Montreal to prepare for the traffic which they expect to go through that commercial highway," is one for which the Canadian officials would like to have some legitimate reason. From the tenor of recent articles in Montreal papers, they evidently fear that the desired improvements may never materialize. It must also be borne in mind that the volume of freight for domestic consumption is so large that it practically controls the route followed by foreign trade in the same commodities, and must to a great extent prove a strong handicap for the Canadian route.

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The writer did not state that the Canadian canals were failures, as implied in Major Symons' discussion, but that they had thus far failed to divert traffic from established routes, and that any waterway of less depth than required for the passage of the best type of lake freighters cannot materially modify the transportation rates over existing routes.

Major Symons' comparison, of the effect of the enlargement of the Erie Canal, to the economy of an 8 000-ton steamer over one of 2 000 tons capacity, is not apropos.

The conditions which have caused the decline of traffic on the Erie Canal since 1880 cannot be eliminated by the construction of a waterway only 12 ft. deep, while the economy in the use of larger freight carriers on the Lakes depends simply upon the ratio of the running expenses and fixed charges, due to cost of ship, to the carrying capacity for steamers of different dimensions.

While not prepared to stand sponsor for the exact ratio of power required, respectively, for towing wooden and steel barges, the writer is surprised that an engineer of Major Symons' high standing should for one moment question the fact that the resistance of wooden surfaces to movement through water is largely in excess of that of steel, especially in the case of barges in canals and shallow rivers, where the bottoms cannot be maintained long in good condition except by constant repairs. The fact that wooden ships, with bottoms in the condition similar to that of barges in use on shallow streams, offer great resistance to movement through the water, is so well known that comment is unnecessary. If further evidence is needed to convince Major Symons of his error, a careful study of the resistance of wooden and steel conduits to the flow of water should certainly be sufficient.

If, as stated in the discussion, "one of the strongest arguments in favor of a barge canal following the general route of the Erie Canal, is that business upon it can be done in cheap, light, wooden boats," the project is certainly based upon a very weak foundation, for it is generally admitted that in vessel construction wood must give way to steel long before either of the proposed waterways can be constructed.

While it is true that the insurance rate on the present small canal boats is much less than that used in the estimates given in the paper, it should be noted that the risk on 1 000-ton barges in fleets of five or more, in a 12-ft. waterway and through the lakes and rivers traversed by the route, will be practically the same as that for a steamer having a tonnage equivalent to that of the fleet.

On page 80 of Major Symons' report on a ship canal from the Great Lakes to the navigible waters of the Hudson River, it is stated that:

"The ordinary rates of insurance on the Lakes are 3 to 3½% on steel vessels. No insurance man interviewed thinks that the rates would be less for vessels navigating the canal, while many announced that they would be higher."

Mr. Wisner. It is not safe to estimate for lake and river insurance at less than 4½%, but, admitting that this rate is too high, the discrimination is really in favor of the canal barge, for the reason that the cost of vessels per ton of carrying capacity is much greater for steamships than for barges, and, therefore, if a less rate be introduced in Tables Nos. 3 and 4 of the paper, the showing for the barge canal would be made even worse than stated.

Major Symons is in error when he states that the cargo insurance used in Table No. 3 of the paper is increased in some unexplainable manner, for the reason that exactly the same rate is used as in his report, but the amount is computed for the cargo carried in both directions, instead of making the shipper pay the insurance on the west-bound freight, as was done in the barge canal estimates. If forcing the shipper to pay all items which make estimated transportation rates undesirably large is a correct and fair procedure, the principle should be still further extended and make transportation practically free. Major Symons has apparently made use of this method, so far as the cost and maintenance of barge route and west-bound insurances are concerned, and it would be interesting to know why the same principle should not be applied in the estimates of railroad rates, by omitting all fixed charges for right of way, construction and maintenance of road.

In view of the large experience that Major Symons has had in connection with lake waterway and harbor improvements, it is surprising that he should not be aware that the barges he describes in his reports would be neither safe nor economical for lake transportation as compared with the large freighters now used in the lake service. If such was the case, we certainly ought to expect a much more crowded state of affairs in the Canadian canals than has ever existed.

In Major Symons' reports and discussion great importance is given the fact that "nearly all of the foreign-bound commerce of New York leaves the port as berth cargoes in the large ocean liners," and that anything like quick dispatch at the port of New York would require a complete upheaval and reversal of the methods of doing business in that great port.

To those interested in rapid and economical methods of transportation, this state of affairs is a strong argument in favor of adopting some other method of interior transportation, which will not perpetuate a system which, from excessive terminal detentions, great cost of transfer, and from diversion of traffic to less economical routes, adds materially to the cost of marketing the products of the country. The very fact that the differential rates on railroads to the seaboard all discriminate against New York is proof that New York harbor is the most economical natural outlet for exports, and, instead of perpetuating existing conditions, the efforts should be to increase the receipts of

freight, so that full cargoes can be furnished to ocean freighters. The commerce of New York is taken as berth cargoes for the simple reason that the volume received for export is not what the commanding position of the harbor and its unsurpassed natural conditions for interior communications entitle it to. Mr. Wisner.

With direct deep-water connection with the great producing centers of the Northwest, the railroads would have to discontinue differentials detrimental to the Port of New York, or lose much of their present traffic.

In either event, the export commerce of the port would increase and make it possible to establish methods which would give better dispatch than at present.

The statement that "the data furnished by Mr. Wisner are not at all sufficient to prove that the Great Lake freighters could with safety make a speed of 8 miles per hour in the canal proposed by the Board of Engineers on Deep Waterways" needs some substantiation before being accepted. What better proof is needed than that large freighters do make such speeds safely in similar canals every day?

The St. Clair Flats Canal is by no means the only restricted section of the Lake waterways, yet it is a notable fact that no accidents have occurred in these channels, except at sharp curves in strong currents, a condition which has been entirely eliminated in the plans proposed by the Deep Waterway Board.

Mr. Hunter's unqualified endorsement of the writer's estimate, that 8 miles per hour as an average speed would be certainly safe, should settle the question beyond all doubt.

The cross-section of the Manchester Canal is only four-fifths of that of the proposed waterway, yet experience has proved that speeds of 8 miles per hour are safe, and licenses permitting such speeds are now issued.

It is true that strenuous efforts are being made to have some of the narrow channels of the Lake waterways widened to 600 ft., where there are sharp curves and strong currents, but in the plans for the proposed deep waterway to the seaboard, sharp curves are eliminated and wide channels provided in rivers wherever strong currents exist.

In view of the facts that the speed of large freight carriers on canals of similar dimensions to those of the proposed deep waterway has been thoroughly established by actual safe practice, and that the time required at terminals has been fixed by experience extending over many years, the closing statement of Major Symons' discussion, that the estimates of Mr. Mayer and Mr. Wisner are theoretical possibilities, while those for his barge canal are reasonable probabilities, must be regarded as a strong exhibition of over-confidence in his own personal views.

The methods and rapidity of handling cargoes at terminals have

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changed but little in the past ten years, as shown by Table No. 21, which gives the types of large carriers built in 1890 and 1900, and the average time of detentions at terminals for those years.

TABLE No. 21.

Name of ship	*Matoa.*	*Gates.*
When built, and years compared	1890	1900
Length over all, in feet	290	500
Beam, in feet	40	52
Depth, in feet	21	30
Registered tonnage (gross)	2 311	5 085
Draft of water in channels, in feet	15.5	17.8
Average cargo, in gross tons	2 500	7 000
Average time at terminals per round trip, in hours	51	63

Mr. Mayer has covered the relative merits of government and private ownership of the proposed waterway so thoroughly that but little comment is necessary. His idea that a moderate toll might be allowed, to cover all expenses incurred by the nation, would be a serious mistake if put into practice, for the reason that the benefits derived from such a waterway will be as much of an indirect nature in building up new commerce as of direct value in decreasing the cost of transport, and, therefore, to obtain the maximum benefit, it should be made absolutely free to all.

Mr. Schenck is in error in supposing that loss of time from meeting vessels, and from reduction of speed on curves, has not been provided for in the estimates of time of round trips on the proposed 21-ft. waterway from the Lakes to the Atlantic. It is established beyond doubt that a speed of 8 miles per hour is absolutely safe on tangents, and in passing vessels a reduction to one-half speed has been used in the estimates, and for curves a reduction depending upon the degree of curvature, as stated by Mr. Noble.

Mr. North, in his able discussion, refers to "the determination to make 24 ft. the governing depth for the channels of the Great Lakes," as an argument against limiting the depth of the proposed waterway to 21 ft.

If it were a fact that the governing depth of the lake channels was to be fixed at 24 ft. or more, the writer would be fully in accord with Mr. North's views. No project for channels of over 21 ft. depth has ever been seriously contemplated, and, besides, it has been so thoroughly demonstrated that larger ships than those recently constructed will not be economical in the Lake service, that the shipping associations will probably ask the Government to fix the ultimate navigable depth of Lake waterways at 21 ft.

It is true that in contracts for the improvement of Lake waterways

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the payment for depths up to 23 ft. is sometimes provided for, but this simply arises from the fact that with some kinds of materials to be excavated, a clear navigable channel 21 ft. deep cannot be made without making occasional cuttings from 1 to 2 ft. deeper than required for navigation. Mr. North quotes the writer as authority for the statement that 40 000 000 tons of freight passed through the Detroit River in 1899. This is not correct. The actual volume carried on the entire chain of lakes was not far from 40 000 000 tons, as stated in the paper, while that passing through the Detroit River probably did not exceed 30 000 000 tons.

Table No. 22 shows the movement on the Lakes for the four leading commodities in 1898:

TABLE No. 22.

Commodity.	Total traffic, in net tons.	Eastward traffic, in net tons.
Grain (including flour).....	9 661 411	9 661 411*
Iron ores.....	13 650 788	11 028 321
Lumber.....	4 540 000	2 531 180
Coal.....	8 722 667	.....
Total.....	36 574 866	23 220 912

The report of the Committee on Canals of New York State gives the total entrances and clearances of registered tonnage at Lake ports in 1898 as 124 046 366 tons. Dividing this by two, the Committee obtains the total lake traffic as 62 023 000 tons.

This, however, is misleading, as to the actual volume of freight movement, which, in fact, was only about two-thirds of the volume indicated by the entrances and clearances at the Lake ports.

The entrances and clearances include those of all pleasure and passenger steamers entering and leaving the ports, which at many of the harbors is an important part of the tonnage. To assume, therefore, that the freight movement through the Detroit River will within a few years amount to 80 000 000 tons is wholly unwarranted by the present conditions or future probabilities.

It is true that the commerce of the St. Mary's River has approximately doubled every six years since 1882, but on the Detroit River the traffic has not doubled in the last twenty years,† instead of in the last six years, as quoted by Mr. North, and it must be remembered that this ratio cannot continue indefinitely, and, in fact, that the condition of the Lake grain and ore trade has already reached the stage where the annual increase of volume of traffic must be much less than heretofore. The unprecedented growth of commerce on the Great Lakes

* Including Canadian traffic through Georgian Bay and the Welland Canal.

† See Tunell's "Statistics of Lake Commerce," page 3.

Mr. Wisner. has been due to the development of cheaper transportation by the improvement of waterways, and the consequent bringing into existence of new manufacturing enterprises which otherwise could never have been started. While the free improvement of these waterways has often been criticised as undue favoritism of the competitors of the railroads, it is nevertheless true that the great prosperity of the railroad lines which parallel the water routes has been largely the indirect result of the new traffic which would never have existed had not cheaper transport been developed by waterway improvements.

The effect of waterway improvements on the movement of the Lake commerce is well illustrated by the relative volume of flour carried by vessels and on railroads during the period of channel improvements.

In 1866, the railroads carried approximately one-half of the flour between the upper Lake ports and the seaboard; in 1867, three-fourths; in 1877, 94%; and in 1881, 96 per cent.

The improvement of the St. Clair Flats Canal and the completion of the 15-ft. lock and canal at Sault Ste. Marie in 1881, turned the advantage again in favor of lake transportation, so that, in 1882, the Lake shipments of flour were 21%; in 1886, 38%; and in 1889, 48% of the total volume. It is hardly to be expected that this ratio will materially change in the future, for the reason that winter shipments by railroad are certain to continue, and that future reduction in water rates must be small after the completion of the 21-ft. waterway project, which depth, economical transportation, so far as Lake traffic is concerned, will not warrant increasing.

If a commerce of 80 000 000 tons annually through the Detroit River, with a corresponding increase of traffic to the seaboard, can be guaranteed after the completion of a 30-ft. waterway, the writer will heartily concur with Mr. North as to the desirability of such construction. Not that the cost of transport will be less for Lake traffic than with a 21-ft. waterway, but that the indirect benefits from developing a commerce of 80 000 000 tons will more than repay the additional cost of a larger waterway. Such a result cannot be reasonably expected, and it is probable that 20 000 000 tons between the Lake ports and the seaboard is a much safer prediction; for, with the route closed one-third of the year, the railroads would certainly continue to carry a large part of the commerce.

The total traffic on the New York canals in 1898 was 3 360 063 tons, of which 1 573 227 tons were through freight. The total freight traffic over the New York Central Railroad was 23 403 439 tons, of which only 4 153 084 tons were through freight. The total freight tonnage on the New York State canals and railroads was 67 141 146 tons, of which probably about 15 000 000 tons were through commerce between the freight centers of the country tributary to the Lakes and the seaboard.

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With a deep waterway completed and ready for business, it is probable that the railroads would still carry at least one-third of the through freight, leaving 10 000 000 tons available for waterway traffic.

It will not be safe to estimate that the through traffic will more than double in ten years after the completion of the waterway, indicating that 20 000 000 tons would be a maximum to be expected after ten years' operation of the waterway.

With an annual commerce of over 35 000 000 tons from the Lakes to the seaboard, the capacity of the waterway would have to be increased over that of the estimate of the Deep Waterway Board, which would materially add to the expense account of construction and maintenance.

The total entrances and clearances of New York Harbor are given by Mr. Corthell as 79 544 653 tons, from which it is evident that the actual export commerce from New York must be considerably less than the registered tonnage shown by the clearances of the port.

A safer estimate can be made by giving the port its proper percentage of the total exports of 29 000 000 gross tons from the entire country in 1899.

It is not fair to argue that because the deepening of the Lake channels from 16 to 21 ft. resulted in reducing freight rates 50%, that a deepening from 21 to 30 ft. will result in a corresponding decrease, for, as before stated, the economical dimensions of a freight carrier in waterways of unlimited depth is a function of the length of haul. The cost of ships, per ton of carrying capacity, increases rapidly with the dimensions of the ship, and the corresponding fixed charges increase faster than the saving in the cost of transport, and, therefore, where terminal detentions are large in comparison with the length of haul, the fixed charges from interest and insurance, in connection with the time of round trip, determine the maximum limit of economical construction.

With the Suez Canal the elements of the problem are entirely different. The vessels passing the canal make trips of several thousand miles, and, since the depth of canal is the controlling element for the entire trip, it will pay to increase the depth to 32.5 ft. with a traffic of only one-third of that of our Lake waterways, where, with a limited length of haul, 21 ft. seems to be an economical maximum depth for waterway improvement.

Referring to Mr. Monro's discussion, the writer by no means wishes to be understood as believing that the Canadian Canals will prove a failure only so far as they may be expected to divert traffic from the present lake and railroad lines, or to materially modify freight rates on such lines.

The Canadian Canals are achievements that their projectors may well be proud of, and in the future, as in the past, will continue to be

Mr. Wisner. important factors in the transportation of Canadian products and manufactures, which are almost certain to largely increase in the near future.

It is well known that barges of 10 ft. draft cannot safely navigate the Great Lakes except during the average quiet weather, and for this reason a barge canal capable of passing vessels of 16 ft. draft, or, say, 18 ft. deep, has been advocated.

Such a canal would shut out all traffic with the seaboard with the modern lake freighters, and, from the longer time required for passing fleets of barges through locks, would not have nearly as great a capacity as a 21-ft. canal. Such a canal would in no way aid in the development of ship construction on the Lakes, and, so far as the writer can see, would not produce any of the indirect benefits to be expected from a deeper waterway with large locks.

Observations to determine the relative speeds of large steamers in deep and shallow water and restricted channels have been continued since the writer's paper was written, with results which practically confirm those obtained by the Board of Engineers on Deep Waterways.

The observed speeds of five steamers running between known points on Lake Huron compared with those on Lake St. Clair, where the depth was only 2 to 3 ft. greater than the draft of the vessel, showed a reduction of speed of 16% in the shallow water. It should be remembered, however, that large steamers, with a speed of 10 miles per hour, run in a sort of trough produced by the motion of the boat and the action of the propeller, so that there is a "squat" of the ship of about 1 ft., making the depth of water beneath the keel that much less than indicated by actual draft of ship and depth of channel.

The observed speeds through the St. Clair Flats Canal and the restricted channels of the St. Mary's River show a little greater average than that obtained by the Board of Engineers on Deep Waterways, and indicate beyond question that 8 miles per hour is a perfectly safe speed in a waterway 21 ft. deep, and having a cross-section of 5 500 sq. ft.

The results of the observations to determine the loss of speed of ships in restricted channels, furnished by Mr. Lionel B. West, are very interesting and practically confirm the results determined and used by the Board of Engineers on Deep Waterways, and made the basis of estimate of speeds in the writer's paper.

John Kennedy, M. Am. Soc. C. E., has kindly furnished the writer with the results of some observations made on the St. Lawrence River to determine the amount that a ship is depressed in a channel when running, which indicates that for ships of 22 ft. draft and speeds of 15 miles per hour, the vessel drops, or "squats," about 1½ ft.

The observations made for the writer on ships in the lake channels

Mr. Wisner.

show a drop of about 1 ft. for a 10-mile speed, which, considering the difference in speed, is a close agreement with the results furnished by Mr. Kennedy, and indicates that a 19-ft. draft is the maximum that can be safely used in a 21-ft. waterway.

Mr. Mayer raises the question whether a waterway with the same area of cross-section as proposed by the Board of Engineers on Deep Waterways, and having a depth of 24 ft., would not be more satisfactory than a 21-ft. waterway.

It is essential for an economical waterway that it should have sufficient depth for the safe passage of the best freight carriers of the routes connected, and a width adequate for the safe passing of vessels without too much loss of time.

To make the depth 24 ft. would diminish the width of channel 30 ft., and make it unsafe for ships to pass each other except at very low speeds. It is difficult to steer vessels when moving slowly, especially with a strong beam wind, and the results of making the channel narrower would be frequent grounding of ships and a much longer time for passage between terminals.

It is noted that in two of the discussions the volume of water needed for the supply of the summit-level of the waterway is erroneously stated, and, in order to clear up any questions as to volume of supply necessary, the following analysis of the problem is submitted:

Let M_1 = Area of lock × lift of the lock west of summit-level;
M_2 = Area of lock × lift of the lock east of summit-level;
D_1 = Displacement of ship going east;
D_2 = " " " " west;
Q = Quantity of water used.

For vessels locking successively over the summit from west to east:

$$Q = M_1 + D_1 + M_2 - D_1 = M_1 + M_2.$$

For vessels locking successively from east to west:

$$Q = M_2 + D_2 + M_1 - D_2 = M_2 + M_1.$$

With vessels alternating regularly each way:

For vessel going east:

$$Q = M_1 + D_1 - D_1 = M_1.$$

For vessel going west:

$$Q = M_2 + D_2 - D_2 = M_2.$$

Average quantity used per vessel $= \frac{M_1 + M_2}{2}$.

For an entire season, the alternate lockings will average one-half the total number of lockages, making the supply of water needed one and one-half lockfuls for each ship passing the summit-level.

If the locks at the ends of the summit-level have a less lift than at other points on the waterway, M_1 and M_2 must be computed for the maximum lifts on each side of the summit.

It is interesting to note that, while the displacement of a ship has no

Mr. Wisner. effect on the volume of water necessary on a summit-level, the volume required on a down-grade canal, from the Lakes to the Atlantic, with the heaviest traffic to the East, will be less than would be the case with the ships loaded to full draft in each direction.

The recent report of the State Engineer and Surveyor of New York on the Barge Canal from the Hudson River to the Great Lakes gives the estimated cost of a 12-ft. Barge Canal from Troy to Buffalo as $71 600 000, instead of $58 895 000, estimated by the Committee on Canals, in 1899. This increase in the estimated cost of the Barge Canal will increase the fixed charges used in Table No. 3 of the writer's paper, and make the rate per ton for transport of wheat from Chicago to New York $1.42, instead of $1.39, as given in Table No. 3. From Table No. 4 the cost of transport with a 21-ft. waterway is found to be $1.09; and, using the new estimate given by the State Engineer and Surveyor for the completion of the 9-ft. project of 1895, it is found that the cost of transport from Chicago to New York with a 9-ft. Barge Canal will be $1.47.

The present average cost to transport a ton of grain from Chicago to New York by lake and Erie Canal line is $1.75, and by lake and rail line $2.25.

These different rates of transport, with the fixed charges which will be necessary to cover the maintenance, operation and interest on the cost of the routes, fix the volume of traffic which will be necessary to make the proposed routes more economical than existing lines of transportation.

The rates of $1.09, $1.42 and $1.47, for 21-ft., 12-ft. and 9-ft. waterways, are computed on the assumption that the traffic on the waterways annually will be 20 000 000 tons, 15 000 000 tons and 10 000 000 tons, respectively.

In order that the transport rates on the proposed waterways shall be as economical as over existing lines, the traffic would have to exceed 8 000 000 tons, 6 000 000 tons and 4 000 000 tons, respectively, on the 21-ft., 12-ft. and 9-ft. waterways, to enable making the rate as low as the present rate by lake and canal; and 6 000 000 tons, 3 000 000 tons and 2 000 000 tons, respectively, to make the rate as low as the present average rate by lake and rail.

The total traffic on the Erie Canal in 1899 was 2 419 000 tons, of which only about one-half was through freight.

With waterways only 9 ft. and 12 ft. deep, requiring transfer delays and charges at Buffalo, and competing with the existing 14-ft. Canadian Canals, which will likely be increased to a depth of 21 ft. in the near future, it is extremely improbable that the through traffic which would be developed would warrant any better rates than those over existing lines.

With a 21-ft. waterway, allowing through bills of lading between

the lake ports and the seaboard, and quick passage for the economical type of freight carrier for the lake business, a commerce of 8 000 000 tons annually would certainly be available by the time the waterway could be completed, and would probably reach double that volume in a very few years. Mr. Wisner.

The papers and discussions of the proposed waterways have thus far been confined to routes between Lake Erie and the Hudson River. If, however, the question be put on the broader basis of the best and most economical route between the Atlantic and the ports of the entire country tributary to the Great Lakes, whether belonging to the United States or Canada, it is probable that an entirely different route from either of those under consideration would have the preference.

From Sault Ste. Marie to the head of ocean navigation on the St. Lawrence, at Montreal, through Georgian Bay, French River, Lake Nipissing and the Ottawa River is only 615 miles, or 25 miles farther than from The Sault to Buffalo by the lake route. The Ottawa route would have only about 30 miles of restricted canal section on the entire line, and could be constructed for less than three-fourths of the cost of a 12-ft. barge canal between Buffalo and the Hudson River.

A waterway from Montreal to New York for the distribution of domestic commerce could be constructed for about $100 000 000, or over $60 000 000 less for the entire waterway from The Sault than a similar waterway would cost from Lake Erie to New York.

The distance from Sault Ste. Marie to New York by the Canadian route would be about 100 miles less than by the routes from Lake Erie to the Hudson River, making a saving in distance of about 200 miles on each round trip between upper lake ports and New York.

The distance from upper lake ports to Liverpool would be about 800 miles less by the Ottawa route than by Lake Erie and a waterway through New York State.

There is a strong probability that a 21-ft. waterway will be constructed from Georgian Bay to Montreal in the near future, and, unless the people of New York take steps to open up deep water communication with the lake ports, either by a deep waterway to Lake Erie, or to connect with the Ottawa route at Montreal, the St. Lawrence River will become the outlet for the commerce between lake ports and the Atlantic.

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No. 891.

THE PRACTICAL COLUMN UNDER CENTRAL OR ECCENTRIC LOADS.

By J. M. MONCRIEFF, M. Am. Soc. C. E.

PRESENTED MAY 2D, 1900.

WITH DISCUSSION.

This subject has received the attention of many investigators, and, in consequence, numerous formulas have been laid before the engineering profession, with the object of providing a means of predicting or estimating the probable strength of columns as affected by their proportions. Nevertheless, it appears to the writer that no apology is needed for a fresh attempt in this direction.

In developing a theory of column resistance and its resulting formulas some of the more important points requiring to be kept in view are the following:

1. The theory should be based on correct principles and the formulas should be of correct form, without introducing refinements which have but little practical value or influence on the results.
2. The theory and formulas should have a wide range of application, to cover the conditions met in engineering practice,

and should not be limited to the case of columns under presumably central loads.

3. The reasoning and the formulas should be sufficiently simple to be understood by other than expert mathematicians, and should be suitable for use in every-day engineering practice.
4. The theory must show agreement with, and be a reasonable explanation of, the results of practical experiment.
5. The formulas should in practice be applicable generally to different materials, by simply introducing the values of the ordinary physical constants of strength and stiffness.
6. Empirical factors should be reduced to a minimum.

The simple theory and formulas which are set forth in this paper show a fair compliance with the foregoing requirements, and it is hoped that they may assist the practical engineer to a better understanding of the important subject of column strength.

The underlying principles upon which the reasoning is based are, (1) that a perfectly centered column of perfect material and straightness is an ideal conception seldom or never realized in practice, and (2) that the various disturbing influences preventing the realization are practically all capable, as regards their ultimate effect, of being represented by an equivalent eccentricity of loading.

Any theory based on these principles ought to be identical in its results with the theory of the ideal perfectly centered column of perfect material and straightness, when the factor representing eccentricity is reduced to zero.

The assumption of the principle of equivalent eccentricity receives practical justification in the records of the experiments of James Christie, M. Am. Soc. C. E., the late Charles A. Marshall, M. Am. Soc. C. E., M. Considère, and Professor Bauschinger, each of whom found that the physical axis of resistance in a column did not necessarily coincide with the geometrical axis, and in fact very frequently did not. Each of these experimenters made tests in which he practically felt for the physical axis in order to obtain a higher strength for the column under trial, and found that it was quite possible for a column to show higher strength when apparently loaded eccentrically, as compared with the strength when apparently loaded exactly over

the geometrical axis. Mr. Christie's and Mr. Marshall's principal tests were both, however, centered over the geometrical axis, and their attempts to feel for the physical axis were supplementary.

After the development of the theory and its resulting formulas, the first difficulty in its application to the case of columns under apparently central loading is the value to be assigned to the equivalent eccentricity, and a careful study of the records of nearly all the more important tests of columns was therefore undertaken, with the view of arriving at some idea of what that value should be.

It became at once apparent, from a comparison of the tests by different experimenters, that isolated tests or a set of tests covering only a small range in proportions, as measured by the ratio of length to radius of gyration, or having only a scanty number of tests at each ratio $\frac{l}{r}$, could in themselves afford no reliable basis for use in practical work, and the writer has no hesitation in saying that any general conclusions or formulas derived from such conditions are absolutely misleading.

It also became most clearly evident that any conclusions deduced from experiments must make full allowance for the possible or probable history of the material of the column during its manufacture and during its preparation for the testing machine.

Many of the causes of the divergence of practical experiments from the theoretic ideals have been incidentally alluded to in able papers and discussions on the subject of column resistance, but, very frequently, too much emphasis has been laid upon the variable nature of the material—excepting perhaps in the case of timber—and, on the other hand, too little has been credited to the probable history of the material, and to the influence of apparently insignificant initial curvature in the specimens, or small errors in setting in the testing machine.

The divergence alluded to may, with every probability of truth, be partly credited, in the case of wrought iron and steel, to the effects of the inevitable cold-straightening to which every bar, plate, or shape, turned out of the rolling mill, must be subjected before being fit for use in ordinary construction, and to a still greater degree before being put into a testing machine as a properly prepared specimen.

It cannot be too clearly realized that the material used in every-day construction is in anything but an ideal condition as regards freedom

from internal stresses, and as regards uniformity of elastic resistance in its detailed sections.

This is quite a different thing from assuming that material of similar history and of the same class varies very widely in its compressive strength, or in the value of the modulus of elasticity.

Striking instances of the influence of history have been given by *Sir* Benjamin Baker, Hon. M. Am. Soc. C. E., in the case of experiments which he carried out on solid, mild-steel columns, 30 diameters in length, showing that the resistance varied according to previous treatment, as follows:*

	Tons per square inch.
"Annealed	14.5
Previously stretched, 10 per cent	12.6
" compressed, 8 "	22.1
" " 9 "	28.9
Straightened cold	11.8"

There are also a number of references to the influence of history in Mr. James Christie's† papers on the strength of iron and steel.

The effect of cold-straightening is, of course, to locally strain the material beyond the limit of elasticity, and, without this overstrain, the bar or plate could not be straightened. The result is that at certain points the modulus of elasticity, or, in other words, the stiffness of the fibers overstrained in tension, is lowered very greatly as regards resistance to compressive stress, and the fibers overstrained in compression are affected similarly as regards their resistance to tensile stress. In addition, permanent internal stresses, both tensile and compressive, are set up in the material, and these are neither imaginary nor insignificant.*

A direct consequence of this interference with natural conditions is that the "physical" axis, or the axis passing through the center of resistance of every section of the column, will not be coincident with the geometrical axis, and in forming a mental conception of the physical axis under these artificial conditions, we are driven to the conclusion that in practical work it can rarely, if ever, be a straight line.

If these deductions be extended to the case of short test-specimens under compression, how is it to be expected that accurate determina-

* *Minutes of Proceedings*, Institution of Civil Engineers, Vol. xcii, p. 44.

† *Transactions*, Am. Soc. C. E., Vol. xiii.

tions of the natural elastic compressive strength can be derived from specimens cut from a portion of the material which may previously have been subjected to cold-straightening. Other portions of the same piece may not have been in the straightening press, and it would then be reasonable to expect them to show a higher value of elastic strength.

In the case of built columns, the effect of the process of machine-riveting is another outside influence on the condition of the material which requires recognition. Every practical constructor knows that in riveting up a member by hydraulic machine-riveters, the various parts have a tendency to stretch out and creep past each other, sometimes in very different degrees, resulting in the members twisting or bending out of a straight line, and no clearer evidence can be adduced to prove the existence of somewhat heavy internal stresses in the finished work.

In symmetrical sections the effect of the riveting down one side will be apparently neutralized by the subsequent riveting on the other; but in an assemblage of plates and angle bars, or other sections, as already remarked, the plates and bars often stretch or creep in different degrees, so that, although the resulting member may be quite straight and free from twist, this will be no proof of the non-existence of severe artificial internal stresses.

Among the instances of this which have occurred in the writer's experience, one, in connection with a bridge over the River Tyne, England, may be mentioned.

In order to guard against this creeping tendency, the writer specified that the large columns of the river piers should have their butt-joint ends machined over the full section, after having been riveted up in lengths in the contractors' yard, to ensure that the butts should bear on each other for the full sectional area of the members.

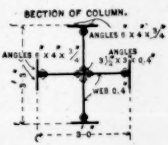

FIG. 1.

These columns are of cruciform section, as shown in Fig. 1, and the total length of each consists of three lengths of about 27 ft. By some oversight, however, in the case of some of the columns, the specified requirement was not carried out in the contractors' yard, and instead, the ends of each individual bar and plate were machined to a very good fit before rivet-

ing was commenced. Tacking rivets were then put in to hold the various parts in their correct relative positions, and the riveting was proceeded with throughout the length of 27 ft., and on its completion it was found that the angle bars and plates had crept past each other in varying degrees, so that the previous careful fitting was of no avail, and the joint had to be re-dressed by hand. The web-plate ends were also found to be hollow to the extent of $\frac{1}{16}$ in. in the half-width of column between the angle bars, and this undoubtedly points to the fact that internal stresses must exist in the column as made.

The columns appeared to be quite straight and true as a whole, and the material of the plates and angles was all made and tested under the same specified requirements of tensile strength, 28 to 32 tons (2 240 lbs.) per square inch, with ultimate elongation of at least 20% in 8 ins. The material is open-hearth acid-process steel, throughout.

In the case of cast-iron columns, of course, cold-straightening or machine-riveters do not enter as disturbing influences, but it is hardly necessary to point out the probably similar influences due to the hidden defects and internal stresses known to exist more or less in all castings.

The other influences to which the writer has referred, *i. e.*, the presence of initial curvature or small errors in setting specimens in the testing machine, have by no means an insignificant value in the results obtained in experiments on "centrally" loaded columns.

Mr. James Christie, in his paper* entitled "Experiments on the Strength of Wrought-Iron Struts," remarked that his specimens were practically straight, but careful measurements revealed the existence of small amounts of initial curvature. These measurements are recorded in his tables of results, and the calculations of deflection hereafter given will show that they had an appreciable influence on the strength of the specimens. Mr. Charles A. Marshall's tests also give valuable evidence in the same direction.

The importance of apparently very small errors in setting the columns in the testing machine also received most definite practical demonstration in the tests by Mr. Christie and Mr. Marshall, and also in those by Professor Bauschinger, since moving the specimens very small distances from their original positions had great influence on the results.

* *Transactions*, Am. Soc. C. E., Vol. xiii.

In view of these prejudicial influences, it can be no matter for surprise that such wide differences exist between results obtained under apparently identical conditions, by the same experimenter, on the same material and with columns of precisely the same proportions. It is surprising, however, that attempts are made constantly to express the average results of such wide differences by a single-line formula.

What would be thought of an engineer who used wrought iron and mild steel indiscriminately, in a bridge or other structure, after striking an average between the tensile strengths of the two materials as a basis upon which to determine the sections of the tension members? This would be no more erroneous and misleading than the present practice of using the average strength of columns of any particular class of material and of any one value of such proportions as are usual in practice. It is surely a much more rational and scientific procedure to ascertain within what limits we may reasonably expect the strength of columns to lie, and then to base our estimate of probable strength on the lower limit so derived.

No reference is here intended to be made to what would be deemed defective columns, in any respect, but only to columns which are in every practical sense believed to be above suspicion. The enunciation of the principle that the strength of columns of any given material cannot be represented by any single-line formula, but must be expressed by an area within which the results of experiments may be expected to lie, was, the writer believes, first made by Professor T. Claxton Fidler, M. Inst. C. E.,* ascribing the variations in column strength to variations in the modulus of elasticity.

In plotting the results of the tests of various experiments upon the diagrams accompanying this paper, the writer has endeavored to show every test wherever possible, provided no defects or special circumstances were involved.

In the case of Professor Tetmajer's tests, the records of a considerable number of the results, as given in his tables, were the averages of two specimens, so that those diagrams on which Tetmajer's tests are plotted do not show the full range of the tests, and the differences between maximum and minimum results would be actually somewhat more than shown. In all other cases, however, it is believed that every result plotted on the diagrams refers to a single experiment.

* "On the Practical Strength of Columns, and of Braced Struts," *Minutes of Proceedings*, Institution of Civil Engineers, Vol. lxxxvi (1886).

As far as the writer is aware, Hodgkinson's tests of columns of Low Moor, No. 3, cast iron and of wrought iron, with both ends rounded or both ends flat, are shown here complete and in their full number (within the limits of length of the diagrams) for the first time.

Hodgkinson's other experiments on cast-iron columns are also shown on a diagram to which further reference will be made, but these tests were on specimens of various kinds of cast iron, and by themselves could form no reliable guide, although, together with Hodgkinson's experiments on Low Moor, No. 3, cast iron, they have formed practically the only basis for the design of cast-iron columns for the last fifty years; and extensive tables of the strength of cast-iron columns, based on them, and calculated from Gordon's or Rankin's formulas, are to be found in nearly every pocketbook published for the use of engineers.

The diagrams have all been plotted to such proportions as to show clearly the differences between the various tests and also their relation to the calculated curves derived from the writer's formulas. It would have been easy to have shown an apparently better agreement by adopting other proportions for the diagrams.

The writer proposes to deal with the theory and resulting formulas in the first place, and afterward to compare them with the results of nearly all the more important series of tests hitherto made upon columns of cast iron, wrought iron, steel and timber.

Theoretical Principles.

Any column with the smallest eccentricity of loading or the smallest amount of initial curvature will immediately begin to deflect under load, and the deflection will increase in a much more rapid degree than the increase of load, and every increase in deflection tends still further to increase deflection. It is this last fact which makes a column of moderate length so exceedingly sensitive to small deviations in loading or to small initial bends.

In order to arrive at an expression for the strength of any column, it is therefore necessary to develop first a formula to express the deflection of that column under given conditions. The deflection of a column is caused solely by the bending moments imposed upon it, and the laws of deflection are therefore the same as for a beam under transverse stress.

The probable deflection of a solid beam subjected to bending moments can be determined very simply, with all necessary practical accuracy, if the relations existing between the stress diagram and the resulting deflection are made use of.*

These relations apply with equal correctness to the case of a column. Let $P\ Q$ in Fig. 2 represent a cantilever of uniform section, and let $P\ Q\ R$ be the diagram of bending moments—either regular or irregular in outline. Let I = moment of inertia of the cross-section of the cantilever, and E = modulus of elasticity of the material used.

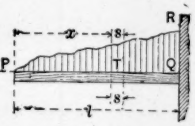

FIG. 2.

Take sections at distances x and $x+s$ from the point P at the extreme outer end of the cantilever, and let it be required to find the deflection of the point P below the point T at the center of s, caused by the stress existing between the two sections, which are further supposed to be exceedingly close together, so that s is very small compared to x. This being so, the bending moment on the cantilever between the two sections may, without appreciable error, be assumed as uniform for the length s.

Then, if M be the bending moment at the sections considered, and c, c' the distances of the extreme fibers from the neutral axis of the sections, the resulting stress in the fibers will be

$$f = \frac{Mc}{I} \text{ and } f' = \frac{Mc'}{I},$$

the common formula for the stress in a solid beam.

The resulting extension—or compression—of these extreme fibers will be $\lambda = \frac{fs}{E}$ and $\lambda' = \frac{f's}{E}$, and the point P will be deflected below the point T, as shown in Fig. 3 by dotted lines, and it is clear that, if we reduce s to an exceedingly small quantity, $\frac{\delta}{x} = \frac{\lambda}{c} = \frac{\lambda'}{c'}$, since the angle subtended by δ is very small; and therefore

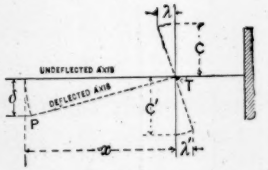

FIG. 3.

$$\delta = \frac{\lambda x}{c} = \frac{fsx}{Ec} = \frac{Mcsx}{IEc} = \frac{Msx}{IE}.$$

*"Theory of Solid and Braced Elastic Arches," by W. Cain, M. Am. Soc. C. E., 1879, and "Continuous-Girder Bridges," by T. C. Fidler, *Minutes of Proceedings*, Institution of Civil Engineers, Vol. lxxiv, 1883.

We have here dealt only with the deflections resulting from the bending moment and stress existing between the two sections, at distances x and $x + s$ from the point P, but the same relation holds for all other sections, and the total deflection caused by the bending moments along the whole length l of the cantilever will be $\Sigma\,\delta = \Delta =$ the sum of the values of $\frac{M s x}{E I}$ between the points P and $Q = \Sigma \frac{M s x}{E I}$.

The numerator of this quantity is simply the moment of the area of the diagram of bending moments around the extreme point P, and the equation may be put into the form

$$\Delta = \frac{A X}{E I},$$

where A = area of diagram of bending moments;

X = distance of center of gravity of this area from the point P.

It is to be remembered that the deflection to be dealt with in beams is always practically very small, compared to the length of the beam, and the above reasoning is not intended to be applied to absurd and improbable extremes. The relations deduced would not, of course, apply to a girder with a deep braced web under heavy shearing stresses, but on a solid beam section the deflection due to shearing stress is very small compared to that due to transverse bending; and, in the case of columns, whether solid or with braced webs, the shearing stresses are again much less than are usual in beams.

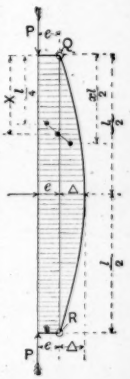

FIG. 4.

It is a simple matter to apply the foregoing relations to the case of a column under eccentric load, and with round or perfectly free pivoted ends. Let $Q\,R$ (Fig. 4) represent the axis of a column of length l acted upon by forces P acting at a distance e from the axis at its ends, with a resulting deflection Δ at the center of the column's length.

The column deflection is exaggerated in the figure, for the sake of clearness, and the length l marked thereon appears to be considerably shorter than the true length of the curved axis of the bent column, but the difference in an actual column under test would be very small, and would not affect the reasoning to any appreciable extent.

The diagram of bending moments will then be as shown in the

figure included between the curve of the bent column and the line of action of the end forces $P\,P$, and, treating each half of the column as a cantilever, the deflection from the tangent to the bent column at its center will be $\Delta = \frac{A\,X}{E\,I}$ when equilibrium is established between th bending moments produced by the eccentric load with the resulting deflection, and the internal moment of resistance of the column sections.

In order to solve this equation for any given case, it is necessary, theoretically, to know the exact character of the curve of the bent column, as the bending moment at any point $= P\,(e + \Delta')$, where Δ' is the deflection from the chord line at that point, but we are mainly concerned with the deflection and bending moment at the point where these are maxima, *i. e.*, at the center of the column length, and the central deflection is the very quantity that is sought.

Practically, however, it is of small importance to know the precise nature of the curve, and a considerable divergence from theoretic accuracy in our knowledge of its true character makes but a trifling difference to the final result, as may be proved easily by assuming various outlines for the curve.

If the bending moment were perfectly uniform for the full length of the column, the curve taken by the latter would be part of a circle, and if the eccentricity of loading were indefinitely small, the curve would be a curve of sines. The actual curve is somewhere between these, and depends on the amount of eccentricity, and, as the deflection of columns in actual test or practice is very small in comparison with the column length, it is sufficiently accurate for all practical purposes to assume the curve to be a parabola, which will, under actual conditions, differ in an exceedingly small degree from the curves of both of the extreme conditions mentioned.

Under any circumstances, the area of the diagram of bending moments for the half length of a column, loaded as shown in Fig. 4, will be

$$A = \left(P \times e \times \frac{l}{2}\right) + \left(P \times \frac{\Delta\, y\, l}{2}\right),$$

when y is a coefficient expressing the mean deflection in terms of the maximum deflection Δ at the center of the column length, and the moment of this area around the extreme end Q of the column will be

$$A\,X = \overline{\left(P \times e \times \frac{l}{2}\right)}\,\frac{l}{4} + \overline{\left(P \times \frac{\Delta\, y\, l}{2}\right)} \times \frac{x\, l}{2},$$

where x is a coefficient in terms of $\frac{l}{2}$, expressing the distance from Q of the center of gravity of the portion of the moment diagram included between the chord and the curve of the bent column, *i. e.*, that portion of the moment diagram due to column deflection.

Then
$$A\,X = \frac{P\,l^2\,e}{8} + \frac{P\,l^2\,\Delta\,y\,x}{4}$$
$$= \frac{P\,l^2}{8}(e + 2\,\Delta\,y\,x),$$

and
$$\frac{A\,X}{E\,I} = \Delta = \frac{P\,l^2}{8\,E\,I}(e + 2\,\Delta\,y\,x);$$

and therefore
$$8\,E\,I\,\Delta = P\,l^2\,(e + 2\,\Delta\,y\,x),$$

from which
$$\Delta = \frac{P\,l^2\,e}{8\,E\,I - 2\,P\,l^2\,y\,x} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Now, making use of the assumption that the curve of the bent column is a parabola, the corresponding values of y and x will be

$$y = \frac{2}{3}, \text{ and } x = \frac{5}{8};$$

and substituting these values in General Equation (1), we have as a practical result for the deflection of columns under eccentric load

$$\Delta = \frac{P\,l^2\,e}{8\,E\,I - (2\,P\,l^2 \times \frac{2}{3} \times \frac{5}{8})} = \frac{P\,l^2\,e}{8\,E\,I - \frac{5}{6}\,P\,l^2} \cdots\cdots\cdots \quad (2)$$

An inspection of these formulas at once shows that any column, of whatever material, with both ends round, and with the eccentricity of loading reduced to an exceedingly small degree, in fact, to as small an amount as we can form any conception of, so long as it has a positive value, would tend to have an infinite deflection, and therefore fail absolutely, as soon as the denominator of the right-hand member of the equation becomes zero.

Then, using Equation (1), Δ would have an infinite value when

$$8\,E\,I = 2\,P\,l^2\,y\,x;$$

and the ultimate load would therefore be

$$P = \frac{8\,E\,I}{2\,l^2\,y\,x}.$$

Under the assumed ideal condition of perfect central loading, the curve of the column when bent being a curve of sines, the values y

and x would each be $\frac{2}{\pi}$, and substituting these in the equation, we have as the ultimate load of the ideal column with both ends round

$$P = \frac{\pi^2 E I}{l^2} = \frac{9.87 E I}{l^2}$$

or Euler's formula.

If Equation (2), based on the curve of the column being assumed to be a parabola, were used in this way to estimate the ultimate load, instead of General Equation (1), with the correct values of y and x, the result would be

$$P = \frac{9.6 E I}{l^2},$$

so that by applying the assumption as to the curve of the column being a parabola, even to the ideal extreme, we would make an error on the safe side of only 2.73% below the theoretic truth in the estimate of ultimate load, and a small amount of eccentricity, such as we may reasonably expect in presumably centrally-loaded columns in actual practice, would reduce this already small error to still smaller and practically inappreciable dimensions.

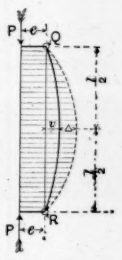

FIG. 5.

It is clear, therefore, from the foregoing, that the formulas for deflection, whether in the general form (1), or in the suggested practical form (2), are based on correct theoretic principles, and are of correct form.

We have dealt hitherto with the case of a column supposed to be absolutely straight before loading, and it remains to be seen what influence a small amount of initial curvature would have on the deflection.

Let r, Figs. 5 and 6, represent the versed sine of an initial curvature, whether outwardly visible or not, in the axis of an eccentrically loaded column. It may have positive value, Fig. 5, or negative value, Fig. 6, relatively to the eccentricity e with which the load is imposed.

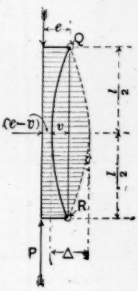

FIG. 6.

Let y_1 and x_1 be functions with regard to the area enclosed between the initial curve of the column and its chord line, of similar character to y and x, already adopted with regard to the curve of the column resulting from stress, or let y_1 and x_1 bear similar relations to v and l as y and x bear to Δ and l.

Then, whatever be the precise character of the initial curve and the deflection curve, using the same reasoning as before, we have

$$\frac{A\ X}{E\ I} = \frac{P\ l^2\ (e + 2\ y\ x\ \Delta \pm 2\ y_1\ x_1\ v)}{8\ E\ I} = \Delta$$

= central deflection due to stress,

and therefore $\Delta = \dfrac{P\ l^2\ (e \pm 2\ y_1\ x_1\ v)}{8\ E\ I - 2\ P\ l^2\ y\ x}$ ..... (3)

This would give a minus value to Δ, if the quantity $2\ y_1\ x_1\ v$ should happen to have the minus sign, and at the same time be greater in value than e, but this would simply mean that the deflection would take place in the opposite direction to that in which e alone would cause it to bend. It must be kept in view that we are dealing with small amounts of initial curvature, shown in exaggeration in Figs. 5 and 6, for the sake of clearness.

In the case of Fig. 5, where the initial curvature is positive with regard to the eccentricity, *i. e.*, acts with the eccentricity to increase the deflection, the diagram of bending moments increases with the load imposed from the area bounded by the straight line joining the end forces $P\ P$, and the initial curve of the column, to the area bounded by the line joining $P\ P$ and the curve of the deflected column.

The bending moment at the center of the column length is primarily $P\ (e + v)$, increasing, as the column deflects under load, to $P\ (e + v + \Delta)$. In the case of Fig. 6, the conditions are altered by the initial curvature acting against the eccentricity, and the primary bending moment at the center of the column length will be $P\ (e - v)$, increasing to $P\ (e - v + \Delta)$, or $P\ (e - v - \Delta)$, depending on whether the influence of e or v happens to be greater in producing the deflection Δ, since Δ must of necessity take the sign of the more influential of these quantities.

Now comparing Formula (3) with Formula (1), it will be seen that an initial curvature has an influence similar to an equivalent value of eccentricity of loading, and the term $(e \pm 2\ y_1\ x_1\ v)$ may be replaced by a single term represented by ε in those columns for which we cannot determine precisely the correct value e and v, such as the presumably centrally loaded column of ordinary material.

Substituting the symbol ε for e in Formula (2), we then have

$$\Delta = \frac{P\ l^2\ \varepsilon}{8\ E\ I - \frac{5}{6}\ P\ l^2} \text{ .....} (4)$$

for the deflection of practical columns apparently straight, and apparently centrally loaded.

It must at once be recognized that it is practically impossible to assign any value for ε beforehand, for any particular column, for reasons already given.

The foregoing reasoning will explain how it may easily happen that two columns of identical dimensions and of identical material, as far as we are able to determine, might give very different results in the testing machine, by one having the internal v acting with the accidental e, and the other having its internal v acting contrary to its accidental e, since the value of e might be very appreciable in the first case, and in the second case, if $-2\,y_1\,x_1\,v$ happened to be equal to $+\,e$, the value of ε would be zero, and the column would probably show a high ultimate strength.

We are now in a position to deduce a formula to express the maximum stress in a column under eccentric load, and to extend it to the case of the practical column under presumably central load.

Referring to Fig. 4, the bending moment at the center of the column length is

$$M = P\,(e + \Delta) = \frac{I f_b}{c} = \frac{a\,r^2 f_b}{c},$$

and therefore

$$f_b = \pm\,\frac{P\,(e + \Delta)\,c}{a\,r^2},$$

where r is the radius of gyration of the column section in the direction in which the column bends, and f_b represents the unit stress caused by the bending moment alone at a distance c from the neutral axis, and a = the sectional area of the column.

The direct compression on the column section at the same time is $+f_d = \frac{P}{a}$ = average load per square inch on the sectional area of the column, and the total stress in the extreme fibers will therefore be

$$F = \pm f_b + f_d = \pm\,\frac{P\,(e + \Delta)\,c}{a\,r^2} + \frac{P}{a},$$

and now, substituting the value of Δ from Equation (2),

$$F = \frac{P}{a} \pm \left(\frac{P\,(e + \dfrac{P\,l^2\,e}{8\,E\,I - \frac{5}{6}\,P\,l^2})\,c}{a\,r^2} \right) =$$

$$\frac{P}{a}\left\{ 1 \pm \frac{c}{r^2}\left(\frac{P\,l^2\,e}{8\,E\,I - \frac{5}{6}\,P\,l^2} + e \right) \right\}$$

$$= f_d \left\{ 1 \pm \frac{c\,e}{r^2}\left(\frac{f_d\,l^2}{8\,E\,r^2 - \frac{5}{6}\,f_d\,l^2} + 1 \right) \right\} \ldots\ldots\ldots\ldots (5)$$

From Equation (5) it is probable that an expression may be deduced to give the average load per square inch f_a corresponding to a given maximum or minimum stress F with varying values of the other factors, but the writer has found it much simpler and easier to deal with the value of the ratio $\frac{l}{r}$ corresponding to a given value of maximum or minimum stress F, average stress f_a, modulus of elasticity E, and a given value of $\frac{c\,e}{r^2}$.

Efforts have been made, in connection with most column formulas, to determine the value of f_a for a given value of $\frac{l}{r}$, but the writer has never found, in his own practice, any advantage in this, and it is equally convenient to be able to determine the value of $\frac{l}{r}$ corresponding to a given value of f_a. Either way is, as a matter of practice, equally suitable for the purpose of laying down a curve to express the strength of varying proportions of columns.

From Equation (5), the general expression for the maximum stress produced in a column, we have

$$F = f_a \left\{ 1 \pm \frac{c\,e}{r^2} \left(\frac{f_a\, l^2}{8\,E\,r^2 - \frac{5}{6} f_a\, l^2} + 1 \right) \right\}$$

and, dividing the factor $\frac{f_a\, l^2}{8\,E\,r^2 - \frac{5}{6} f_a\, l^2}$ by r^2,

$$F = f_a \left\{ 1 \pm \frac{c\,e}{r^2} \left(\frac{f_a \frac{l^2}{r^2}}{8\,E - \frac{5}{6} f_a \frac{l^2}{r^2}} + 1 \right) \right\},$$

and using the symbol R to represent $\frac{l}{r}$, we have

$$\begin{aligned} F &= f_a \left\{ 1 \pm \frac{c\,e}{r^2} \left(\frac{f_a\, R^2}{8\,E - \frac{5}{6} f_a\, R^2} + 1 \right) \right\} \\ &= f_a \left\{ 1 \pm \frac{c\,e}{r^2} \left(\frac{f_a\, R^2 + 8\,E - \frac{5}{6} f_a\, R^2}{8\,E - \frac{5}{6} f_a\, R^2} \right) \right\} \\ &= f_a \left\{ 1 \pm \frac{c\,e}{r^2} \left(\frac{48\,E + f_a\, R^2}{48\,E - 5 f_a R^2} \right) \right\} \quad \ldots\ldots\ldots\ldots (6) \end{aligned}$$

and now using the + sign in the brackets to determine the maximum fiber stress F_c

$$F_c = f_a \left(1 + \frac{c\,e}{r^2} \left\{ \frac{48\,E + f_a\, R^2}{48\,E - 5\, f_a\, R^2} \right\} \right)$$

from which we have, by simple algebraic transformation, the value of R corresponding to any fixed value of the maximum compressive stress F_c,

$$\text{or } R = \sqrt{\frac{48\,E}{5\,F_c + f_d\left(\frac{c\,e}{r^2} - 5\right)}\left[\frac{F_c}{f_d} - 1 - \frac{c\,e}{r^2}\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Similarly, using the — sign in the brackets in Equation (6) to determine the minimum fiber stress (not necessarily tensile) F_t.

$$F_t = f_d\left\{1 - \frac{c\,e}{r^2}\left(\frac{48\,E + f_d\,R^2}{48\,E - 5\,f_d\,R^2}\right)\right\}$$

which is easily transformed to

$$R = \sqrt{\frac{48\,E}{5\,F_t - f_d\left(\frac{c\,e}{r^2} + 5\right)}\left[\frac{F_t}{f_d} - 1 + \frac{c\,e}{r^2}\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

It should be noted here that F_t only becomes tensile when the maximum tensile fiber stress f_b caused by bending is greater than the direct compressive stress f_d, and it should also be noted that F_t must be given its proper sign to correspond with its character, *i. e.*, + when compressive and — when tensile, irrespective of the fixed signs shown in Equation (8).

The precise use of these equations (7) and (8) is as follows:

Let it be assumed that in a given section of column, we decide that a certain value of maximum compressive stress F_c, or a certain value of minimum stress F_t, is not to be exceeded; these values being inserted in Formulas (7) and (8), together with the value of E and $\frac{c\,e}{r^2}$, corresponding to the material used and the section of column and eccentricity of loading actually adopted, we have at once the value of R corresponding to different values of f_d, the direct load per square inch.

Both of these formulas reduce to exceedingly simple terms on the insertion of the physical constants E, F_c, or F_t, and the proper value of $\frac{c\,e}{r^2}$, as will be seen later in their application.

Formulas (4), (5), (6), (7) and (8) are all general expressions applicable to the case of columns with both ends free, and of any given material and form of section, and with any given value of eccentricity of loading probable in practical work.

Professor William Cain* came to the conclusion that, with an ideal column, perfectly centrally loaded, up to the value given by Euler's

* "Theory of the Ideal Column," *Transactions*, Am. Soc. C. E., Vol. xxxix.

formula, a very small increase to this load insures failure of the column. A very similar conclusion can be drawn by applying Formula (4) to a given example, say $E = 30\,000\,000$, $I = 10$ ins.^{4}, $l = 300$ ins., and assume $e = 0.001$ in., corresponding to an accuracy of loading far beyond practical possibilities.

$$\text{Then } \Delta = \frac{P l^2 e}{8 E I - \frac{5}{6} P l^2} = \frac{P \times 90\,000 \times \frac{1}{1000}}{2\,400\,000\,000 - 75\,000 P}$$

and if P be taken in units of 10 000 lbs., this reduces to

$$\Delta = \frac{12 P}{32\,000 - 10\,000 P}.$$

Working this out for the various values of P, we have the results shown in Table No. 1.

TABLE No. 1.

P, in pounds.	Δ, in inches.	P, in pounds.	Δ, in inches.	P, in pounds.	Δ, in inches.
0	0.0000	31 600	0.0948	31 970	1.2788
20 000	0.002	31 650	0.1085	31 975	1.5348
25 000	0.0043	31 700	0.1268	31 980	1.9188
28 000	0.0084	31 750	0.1524	31 985	2.5588
29 000	0.0116	31 800	0.1908	31 990	3.8388
30 000	0.018	31 850	0.2548	31 992	4.7988
30 500	0.0244	31 900	0.3828	31 994	6.3988
31 000	0.0372	31 950	0.7668	31 996	9.599
31 500	0.0756	31 960	0.9588	32 000	∞

These results are instructive, and it hardly needs a calculation of maximum fiber stress to show how great is the effect of the small additions near to the ultimate load of 32 000 lbs.

The value of I assumed $= 10$ ins.$^4 = a r^2$, and nearly corresponds to a rectangular solid section 3.307 ins. square, with area nearly 10.94 sq. ins., and least radius of gyration $= 0.955$ in. nearly, so that the ratio $\frac{l}{r} = \frac{300}{0.955} = 314$.

The maximum compressive fiber stress

$$F_c = \frac{P(e + \Delta) c}{a r^2} + \frac{P}{a} = \frac{P(0.001 + \Delta) \times 1.6535}{10} + \frac{P}{10.94 \text{ sq. ins.}}$$

from which we have, when

$P =$	$\Delta + e =$	F lbs. per square inch $=$
31 990	3.8398	23 200
31 992	4.7998	28 300
31 994	6.3998	37 000
31 996	9.6	53 600
32 000	∞	∞

A most interesting feature of these figures for maximum fiber stress is the theoretic assurance which they give as to the capacity of long columns to resist fatigue, even when loaded nearly up to the crippling point; and if the material of the column dealt with in the example be assumed to have a compressive elastic limit of, say, 40 000 lbs. per square inch, it will be seen that the column would be quite uninjured by an infinite number of loadings within 10 lbs. of its ultimate supporting power. This, of course, would only hold good if the load were imposed without the slightest dynamic effect, or impact.

This completes the investigation of what may be termed "the elementary column," of which, columns with fixed ends, flat ends and pin ends, may be considered as merely modifications.

It will be noticed that the formulas in each case include a term $\frac{c}{r}$ dependent on the form of column section, and the writer at one time hoped to find practical verification of the influence of form of section in the published results of experiments, but the influence of other factors is too great and the number of tests on any one form of section is too small to enable this to be done as yet.

Again, the number of experiments carried out with a value of eccentricity sufficiently great to make it a paramount factor is very small, and in the great mass of tests hitherto made, the endeavor has been to impose the load "centrally"; we must, therefore, substitute ε (see page 347) for e in the formulas when applying them to experiments under presumably central loads. Under these circumstances, it appears justifiable, in our present state of knowledge, to consider the factor $\frac{c\,\varepsilon}{r^2}$ as a constant, of which the value for centrally loaded columns must be determined from available experimental records.

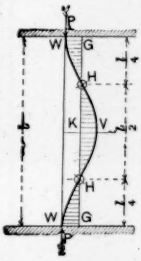

FIG. 7.

Attention will be given, next, to the fixed-ended column shown in Fig. 7, the section being assumed to be uniform, as before. When under load the column $W\ W$ will bend in a double reverse curve similar to that shown in the figure, and the central portion of the column $H\ H$ will behave similarly to, and be subject to, the same laws as a free-ended column. The question to be solved, in the first place, is as to the proportion of the total length of a fixed-ended column, which will act

as a free-ended column. This proportion is frequently stated to be one-half, without any reasons being stated.

In determining this it is convenient to consider one-half of the column only (since the other will necessarily behave in a precisely similar manner) as shown in Fig. 8, to a larger scale.

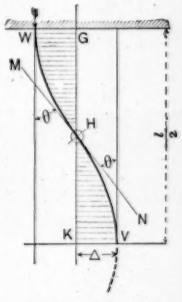

FIG. 8.

Referring again to Fig. 3 and its descriptive context it was shown that

$$\delta = \frac{M\ s\ x}{E\ I}$$

and therefore $\frac{\delta}{x} = \frac{M s}{E I}$ = the tangent of the angle of slope set up at the end of the length x by the stresses in the portion s of the cantilever considered; and as this angle of slope is in practice exceedingly small, its tangent will practically represent the angle in circular measure with all necessary accuracy, and the sum of all the exceedingly small angles of slope, for the full length of the cantilever, will then be

$$\Sigma \frac{\delta}{x} = \Sigma \frac{M s}{E I} = \frac{A}{E I} = \frac{\Delta}{X} = \Theta,$$

or the angle of slope at the extreme end of the cantilever is proportional to the area of the curve of bending moments.

As before, this reasoning and its results apply equally well to the bent column.

In Fig. 8 the tangent $M N$ to the curve of the bent column at the point of contrary flexure H is common to both portions of the curve, and the slope of each portion is, therefore, the same at this point, since the tangents at W and V remain vertical and parallel to each other, in consequence of the fixity of the end W and the symmetry of the whole column around the point V. The area of the bending moment diagram $G\ H\ W$ must, therefore, be equal to the area of the bending moment diagram $K\ H\ V$. Further, as the column section is assumed to be uniform, and there is no bending moment at the point H, where the two portions $W\ H$ and $H\ V$ react upon each other in simple compression and shear, the curvature of the two portions at corresponding points on either side of H must evidently be identical, inasmuch as both are of the same section, subject to the same forces and subject to the same laws of flexure.

From these conditions of equal curvature and equal area of bending moment diagram, it results that the length $G\,H$ must be equal to the length $K\,H$, and each of them will, therefore, be one-fourth of the total length of the column. Also the portion $W\,G$ of the total deflection must equal the portion $K\,V$, and the bending moment at the fixed ends W will equal the bending moment at the center V.

It is thus determined that the length of $H\,H$, Fig. 7, acting as a free-ended column, is one-half the total length L of the fixed-ended column, or, in other words, a fixed-ended column carrying a given load is twice the length of a free-ended column of the same section and having similar stresses.

This result is based on assumptions of perfect straightness before bending, perfectly homogeneous material, and perfect fixity of ends. In practical work some divergence will undoubtedly occur, which will require to be allowed for by an assumed equivalent eccentricity of loading, as in the case of the simple free-ended column already dealt with, and we therefore have, for the fixed-ended column:

$$R = 2\sqrt{\frac{48\,E}{5\,F_c + f_d\left(\frac{c\,\varepsilon}{r^2} - 5\right)}\left[\frac{F_c}{f_d} - 1 - \frac{c\,\varepsilon}{r^2}\right]} \quad \text{for failure by compression,}$$

$$\text{or } 2\sqrt{\frac{48\,E}{5\,F_t - f_d\left(\frac{c\,\varepsilon}{r^2} + 5\right)}\left[\frac{F_t}{f_d} - 1 + \frac{c\,\varepsilon}{r^2}\right]} \quad \text{for failure by tension.}$$

In actual practice the true fixed-ended column rarely, if ever, exists. It is difficult, even in experiments in a testing machine, to comply with the conditions necessary to ensure absolute fixity of ends, and in ordinary construction the difficulty is increased greatly.

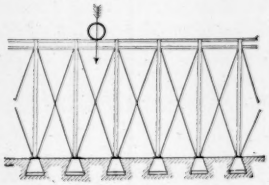

Fig. 9.

The writer has found that the vaguest ideas are sometimes held as to what is required to realize fixed ends in a column. A consideration of simple examples will probably exhibit this matter in a clearer light.

Assume, in the first instance, that we have, as in Fig. 9, a series of stiff gantry girders, 2 ft. deep by 10 ft. span, riveted securely to the heads of columns 30 ft. high, firmly braced together to preserve their

verticality. Assume, also, that the foundation blocks on which the column rests are very rigid, that the columns have large well-bolted bases, and that the ratio $\frac{l}{r}$ of these columns is very large, and the columns therefore slender in proportion.

Then the imposition of load on any span will cause deflection in the girder, and the ends of the girder will deviate from the vertical to a slight degree, but the relative stiffness of the girders themselves, as compared with the column, being high, the approximation to ideal fixity of ends would, practically speaking, be of a high degree.

In the second instance, Fig. 10, let the columns be spaced at 30 ft. centers, retaining the same depth of girder, 2 ft., and merely increasing the girder sections to obtain the same value of working unit stress, while increasing the radius of gyration of the columns to provide much greater stiffness of column. Under these conditions, the deflection of the girder under load, and consequently the slope of the ends of the girders where they are securely riveted to the column heads, would be increased largely, and the columns would be subjected to heavy bending stresses in addition to their direct load. These columns would be much less heavily stressed if they had pin-joint connections to the girders, and the apparent fixity of end, given by a secure riveted connection, would actually be accompanied by severely prejudicial secondary stresses.

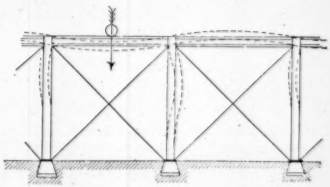

FIG. 10.

The conclusion derived from these examples is that in practical work the degree of approximation to fixity of ends depends entirely on the relative stiffness of the column and the other members of the structure attached to it; and the estimation of this degree of fixity demands the most careful consideration on the part of the engineer.

One of the advantages claimed for riveted connections in bridge work is that the compression members are thereby made into fixed-ended columns, and can be accorded higher stresses in consequence.

A portion of a riveted main girder of **N**-type is shown in Fig. 11, with connections of web members for two panel points, and the writer would ask whether the top boom fixes the ends of the vertical posts, or do the posts fix the ends of the panel lengths in the top boom (which is a column between panel points), or are we to rely upon the stiffness of the diagonal tension members to fix both?

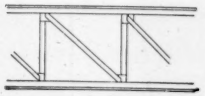

FIG. 11.

The last can hardly be considered a reasonable assumption, as the most heavily loaded portions of the top boom are at the center of the span, where also the lightest diagonals are found, and as regards the posts fixing the ends of the panel lengths of the top boom, this also is out of the question, as the stiffness of the posts is usually small as compared with the stiffness of the boom, and if we consider the top boom as fixing the ends of the vertical posts, under which class of columns are we to place the top boom panel lengths? They could not be considered as fixed-ended, and as they would have to perform the additional duty of fixing the vertical post ends, they could not be considered to be as favorably circumstanced as a round-ended or pivot-ended column. Here, again, we have to give consideration to relative stiffness of parts.

It is common knowledge that heavy secondary stresses exist in the connections of various members in a riveted structure, but it is not commonly recognized that these very secondary stresses may totally destroy any imaginary fixity of ends in the compression members, and actually place the members under worse conditions of stress than if pivoted end-bearings were adopted.

The writer is neither seeking to depreciate the practical value of the riveted connections nor to advocate either pin or pivoted end connections, but only wishes to point out the erroneous principles on which designs are frequently based.

As far as the writer is aware, no attempt has hitherto been made to arrive at a rational basis for the strength of flat-ended columns, although the greater number of tests of columns have been made with this class of end-bearing. As a rule, the assumption has been made that they act in precisely the same manner as fixed-ended columns, and column formulas to cover both in one expression are frequently given. This is quite erroneous, both from a theoretical point of view,

and from the evidence of actual experiments. With flat ends, no tensile stress can be developed at the ends, and with fixed ends it has been shown that the bending moment at each end is theoretically equal to that at the center of the column.

It is clear, then, that so long as no tensile stress is set up in a column with flat ends, it will behave as a fixed-ended column, that is, up to the point of loading at which the stress in the column at its ends and center is as shown in Fig. 12, when a small increase of load on the column will most probably cause failure.

FIG. 12.

If, then, we use the formula for minimum stress F_l, and make F_t equal to zero, we obtain the value of R at which a given value of f_d will produce a minimum stress zero, and we will thus determine the value of R corresponding to incipient tensile stress, and the formula with F_t inserted as of zero value, will give what may be called the critical value,

$$R = 2\sqrt{\frac{48\,E}{0 - f_d\left(\frac{c\,\varepsilon}{r^2} + 5\right)}\left(0 - 1 + \frac{c\,\varepsilon}{r^2}\right)}$$

$$= 2\sqrt{\frac{48\,E\left(-1 + \frac{c\,\varepsilon}{r^2}\right)}{-f_d\left(\frac{c\,\varepsilon}{r^2} + 5\right)}}$$

$$= 2\sqrt{\frac{48\,E\left(1 - \frac{c\,\varepsilon}{r^2}\right)}{f_d\left(\frac{c\,\varepsilon}{r^2} + 5\right)}} \left\{ \begin{array}{l} \text{when tension is incipient in flat-ended columns.} \end{array} \right\} \quad \ldots (9)$$

Here, again, it is necessary to determine from experimental results what value must be given to the factor $\frac{c\,\varepsilon}{r^2}$, if the formula is to be used to determine ultimate strength.

It may be urged that the load producing incipient tension in any given flat-ended column may not be the ultimate load; but, as soon as tension is attempted to be set up at the ends of a flat-ended column, the column will be in a highly unstable condition, and the ends will begin to rotate on their bearing faces. This is most readily seen by considering the case of a fixed-ended column in which tensile stresses have been set up at the ends and center.

If the fibers in tension at the ends were cut, so as to transform the column into a flat-ended column while under load, we would naturally expect the column to alter its curvature immediately, and largely increase its deflection, with the result that it would probably fail immediately, or with a comparatively small additional load. The substantial truth of this, in practice, is most clearly evident in Mr. Christie's experiments, as will appear later.

By plotting the curves for the two conditions, one for failure by maximum compressive stress, and the other for the critical condition of incipient tension, it is made evident that with any given section of column, up to a certain value of R, dependent on the eccentricity of loading and modulus of elasticity, no tension can be set up in the column, whatever the load, and flat-ended columns below this limiting proportion behave in every sense as fixed-ended columns, while beyond this point the strength will fail more or less rapidly.

The writer believes that the value of the difference in the strength of fixed and flat-ended columns is here dealt with in a rational manner for the first time.

With regard to pin-ended columns, it is quite useless to theorize with the view of showing their superiority to round or pivot ends, owing to the fact that their behavior under load, even in a testing machine, depends very largely on the closeness of the fit between pin and hole, upon the smoothness or otherwise of the bearing surfaces, upon the diameter of the pin in relation to the radius of gyration, and upon the presence, either accidental or premeditated, of a lubricating medium.

In actual practice, the vibration in a railway bridge, caused by the passage of the load, and the movements of the members relatively to each other under the common variations of stress, must undoubtedly go very far to destroy the friction upon which depends the superiority with which this type of strut is often credited over those with round ends.

There is as yet no satisfactory and conclusive evidence that in practical work the pin-ended column can fairly be credited with this greater strength, and the practice of imposing higher stresses on account of the pin ends is open to grave question.

This matter may be viewed from another standpoint, that of the advantages claimed for the pin joint as compared with the riveted con-

nection. Among these so-called advantages are freedom from secondary stresses and greater certainty of realizing the ideal condition of centrality of loading on the various members.

Any additional strength accompanying the pin-bearing type of column can only be obtained when frictional resistances are set up in the bearing, preventing rotation, and thus bringing into play a moment of resistance to bending on the column end, and this moment of resistance in turn can only be developed by subjecting the other members assembled on the same pin to secondary bending stresses in order to realize a partial fixity of column ends.

In any case, the additional resistance due to partial fixity of ends in the pin-ended column, if it actually exists in practical construction, must be obtained at the expense of the other members on the same pin, and is largely dependent on the stiffness of those members. The question may fairly be raised whether or not it would be consistent practice to make an allowance for the secondary stresses in these other members, if we rely on these secondary stresses to provide the column with increased resistance.

Comparison of the Formulas with the Experiments.

Deflection Formula (4) for Round-Ended or Pivot-Ended Columns.

$$\Delta = \frac{P l^2 \varepsilon}{8 E I - \frac{5}{6} P l^2}.$$

The starting point for the whole of the foregoing theory and formulas was the development of this expression for the deflection of a column, and it has already been pointed out that, in practical work, some of the controlling influences cannot be made subject to actual observation, depending as they do on internal conditions arising from past history, accidental errors in setting, etc., etc.

Nevertheless, it is important to have some definite knowledge as to whether the deflection formula bears characteristic features having practical agreement with the results of actual observations in experiments.

In order to make a comparison it was necessary to make numerous trial calculations with different estimates of the values of those factors which are primarily unknown, and the process of comparison therefore consisted of fitting the calculated values given by the formula to the observed deflections, so that it might be seen whether curves

plotted with loads as abscissas and deflections as ordinates have the same character by calculation and observation.

The unknown factors for which the values have had to be estimated are the following:

1. The modulus of elasticity..... E,
2. The equivalent eccentricity..... ε,
3. Any small amount of initial curvature capable of observation, V, but not always noted in records of test and making the total deviation of the column from a straight line $= D = \Delta \pm V$.

It was found that the influence of each of these three factors is so great that very small deviations from the estimated values finally adopted destroy the agreement between calculation and observation, and as, in these estimated values, we are already dealing with very small quantities, it is to be noted that the small deviations referred to would be incapable of being observed in any ordinary experiment.

The examples selected for purposes of comparison have been taken from the tests of round-ended columns of Low Moor No. 3 cast iron, made by Mr. Eaton Hodgkinson,* and from the tests of round-ended wrought-iron columns, made by Mr. James Christie.†

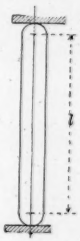

FIG. 13.

These examples have been chosen solely on account of the fullness of the records of deflections. In each case the effective column length has been taken as being the distance between the centers of the hemispherical ends, as shown in Fig. 13.

Examples from Hodgkinson's Tests of Low Moor, No. 3 Cast Iron.

Test No. 5, of Table I.‡—Solid, cylindrical, hemispherical-ended column, 60.50 ins. length over all, 0.99 in. diameter, say 59.50 ins. effective length.

I = moment of inertia of section = 0.0472 in.^{4}

Estimated values: $E = 14\,500\,000$ lbs., modulus of elasticity;
$\varepsilon = 0.055$ in., equivalent eccentricity of loading;
$V = +0.07$ in., probable initial curvature escaping observation.

* Recorded in the *Philosophical Transactions* of the Royal Society of London, for 1840.

† *Transactions*, Am. Soc. C. E., Vol. xiii.

‡ *Philosophical Transactions*, Royal Society, London, 1840.

P = load, in pounds.	(Δ Inches.	+ V) Inches.	= D calculated. Inches.	D observed. Inches.
515....	0.0253	+ 0.07	= 0.0953.....	0.05
655....	0.0360	+ 0.07	= 0.1060.....	0.10
991....	0.0756	+ 0.07	= 0.1456.....	0.14
1 183....	0.1160	+ 0.07	= 0.1860.....	0.19
1 471....	0.2525	+ 0.07	= 0.3225.....	0.32
1 615....	0.4425	+ 0.07	= 0.5125.....	0.52
Ult. load. 1 663....	0.5690	+ 0.07	= 0.6390.....	$\propto$ failed.

Test No. 20, of Table I.—Solid, cylindrical, hemispherical-ended column, 60.50 ins. length over all, 1.97 ins. diameter, say 58.50 ins. effective length.

$$I = 0.74 \text{ in.}^4$$

Estimated values: $E = 13\,600\,000$ lbs., $\varepsilon = 0.0675$ in., $V = 0$.

P = load, in pounds.	(Δ Inches.	+ V) Inches.	= D calculated. Inches.	D observed. Inches.
3 355....	0.0109	+ 0	= 0.0109.....	bent.
7 386....	0.0287	+ 0	= 0.0287.....	0.02
12 970....	0.0686	+ 0	= 0.0686.....	0.07
19 943....	0.1943	+ 0	= 0.1943.....	0.20
21 035....	0.2360	+ 0	= 0.2360.....	0.23
22 127....	0.2936	+ 0	= 0.2936.....	0.28
23 219....	0.3740	+ 0	= 0.3740.....	0.37
24 311....	0.5000	+ 0	= 0.5000.....	0.50 to 0.52
24 857....	0.5940	+ 0	= 0.5940.....	0.60
Ult. load. 25 403....	0.7230	+ 0	= 0.7230.....	$\propto$ failed.
(26 000)....	0.9350	+ 0	= 0.9350.....	....

Test No. 1, of Table VIII.—Hollow, cylindrical, hemispherical-ended column (Fig. 14), 90.75 ins., length over all, 1.78 ins. external diameter, 1.21 ins. internal diameter, say 89 ins. effective length.

Core center 0.19 in. out of center of external circle of column (ascertained after fracture). Center of area 0.163 in. out of center of external circle of column.

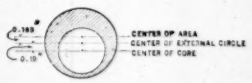

FIG. 14.

$$I = 0.31 \text{ in.}^4$$

Estimated values: $E = 17\,500\,000$ lbs., $\varepsilon = 0.1$ in., $V = -0.02$ in.

P = load, in pounds.	(Δ Inches.	$-$ V) Inches.	= D calculated. Inches.	D observed. Inches.
2 237....	0.06186	— 0.02	= 0.04186.....	0.03
2 813....	0.0897	— 0.02	= 0.0697.....	0.07
3 317....	0.12208	— 0.02	= 0.10208.....	0.11
3 821....	0.1664	— 0.02	= 0.1464.....	0.16
4 325....	0.23049	— 0.02	= 0.21049.....	0.20
4 829....	0.33153	— 0.02	= 0.31153.....	0.32
5 333....	0.51441	— 0.02	= 0.49441.....	0.49
Maximum. 5 585....	0.67549	— 0.02	= 0.65549....	not observed; column not allowed to break.
(6 000)....	1.2471	— 0.02	= 1.2271.....	

Test No. 5, of Table VIII.—Hollow, cylindrical, hemispherical-ended column (Fig. 15), 90.75 ins. length over all, 2.23 ins. external diameter, 1.53 ins. internal diameter, therefore, say 88.52 ins. effective length.

Core center was 0.135 in. out of center of external circle of column (ascertained after fracture). Therefore center of area was 0.12 in. out of center of external circle.

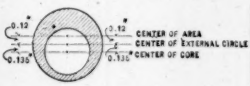

FIG. 15.

$$I = 0.8863 \text{ in.}^4$$

Estimated values: $E = 14\,500\,000$ lbs., $\varepsilon = 0.175$ in., $V = -0.025$ in.

P = load, in pounds.	(Δ Inches.	$-$ V) Inches.	= D calculated. Inches.	D observed. Inches.
2 237....	0.0347	— 0.025	= 0.0097.....	0.01
4 325....	0.0796	— 0.025	= 0.0546.....	0.06
6 341....	0.1415	— 0.025	= 0.1165.....	0.12
8 357....	0.2375	— 0.025	= 0.2125.....	0.22
9 365....	0.3085	— 0.025	= 0.2835.....	0.28
10 373....	0.4040	— 0.025	= 0.3790.....	0.37
11 381....	0.5475	— 0.025	= 0.5225.....	0.55
12 137....	0.7080	— 0.025	= 0.6830.....	0.69
Maximum. 12 389....	0.7525	— 0.025	= 0.7275....	not observed; column not allowed to break.
(13 000)....	(0.9950)	— 0.025	= (0.9700)...	

Test No. 15, of Table VIII.—Hollow, cylindrical, hemispherical-ended column (Fig. 16), 90.75 ins. length over all, 3.36 ins. external diameter, 2.61 ins. internal diameter, say 87.39 ins. effective length $= l$.

Core center, after fracture, found to be 0.067 in. out of center of external circle of column. Therefore, center of area was 0.105 in. out of center of external circle.

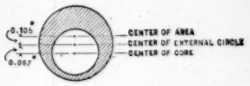

FIG. 16.

$$I = 3.915 \text{ ins.}^4$$

Estimated values: $E = 12\,000\,000$ lbs., $\varepsilon = 0.16$ in., $V = +0.035$ in.

P = load, in pounds.	(Δ Inches.	+ V) Inches.	= D calculated. Inches.	D observed. Inches.
3 355	0.0116	+ 0.035	= 0.0466	bent.
16 115	0.0727	+ 0.035	= 0.1077	0.09
18 667	0.0895	+ 0.035	= 0.1245	0.13
21 729	0.1128	+ 0.035	= 0.1478	0.15
24 148	0.1341	+ 0.035	= 0.1691	0.17
28 986	0.1872	+ 0.035	= 0.2222	0.24
33 824	0.2625	+ 0.035	= 0.2975	0.30
37 701	0.3445	+ 0.035	= 0.3795	0.38
41 632	0.4680	+ 0.035	= 0.5030	0.48
43 597	0.5550	+ 0.035	= 0.5900	0.59
45 563	0.6650	+ 0.035	= 0.7000	0.67
47 528	0.8175	+ 0.035	= 0.8525	0.87
48 511	0.9140	+ 0.035	= 0.9490	0.90
49 494	1.0320	+ 0.035	= 1.0670	1.07
Ult. load. 50 477	1.1750	+ 0.035	= 1.2100	∞

Test No. 7, of Table III.—Solid, rectangular pillar with hemispherical ends, 60.5 ins. length over all, 1.54 x 1.56 ins. nearly square, say 59 ins., effective length = l.

$$I = 0.475 \text{ in.}^4$$

Estimated values: $E = 13\,250\,000$ lbs., $\varepsilon = 0.06$ in., $V = -0.015$ in.

P = load, in pounds.	(Δ Inches.	− V) Inches.	= D calculated. Inches.	D observed. Inches.
2 141	0.010	− 0.015	= − 0.005	column bent.
4 465	0.025	− 0.015	= + 0.010	0.015
6 481	0.043	− 0.015	= + 0.028	0.02
11 169	0.130	− 0.015	= + 0.115	0.11
13 565	0.257	− 0.015	= 0.242	0.23
14 461	0.360	− 0.015	= 0.345	0 35
14 909	0.438	− 0.015	= 0.423	0.44
15 357	0.552	− 0.015	= 0.537	0.52
Ult. load. 15 581	0.632	− 0.015	= 0.617	∞

Examples from Christie's Tests of Wrought-Iron Struts with Hemispherical Ends.*

Test No. 204.—T-bar; 1 in. x 1 in. x 99.25 ins. length over all, 1-in. balls and plates, 98.25 ins. effective length = l.

$$I \text{ (least)} = a\,r^2 = 0.3 \text{ sq. in.} \times (0.26)^2 = 0.02028 \text{ in.}^4$$

Estimated values: $E = 30\,000\,000$ lbs., $\varepsilon = 0.05$ in., $V = +0.025$ in.

P = load, in pounds.	(Δ Inches.	+ V) Inches.	= D calculated. Inches.	D observed. Inches.
100	0.0118	+ 0.025	= 0.0368	0.05
200	0.0296	+ 0.025	= 0.0546	0.05
300	0.0590	+ 0.025	= 0.0840	0.08
400	0.1169	+ 0.025	= 0.1419	0.15
500	0.2843	+ 0.025	= 0.3093	0.30
Ult. load. 550	0.5964	+ 0.025	= 0.6214	∞

Test No. 205.—T-bar; 3 ins. × 3 ins. × 82.0625 ins. length over all, 2-in. balls and plates, 80.0625 ins., effective length.

$$I = a\,r^2 = 2.53 \text{ sq. ins. x } (0.62)^2 = 0.9725 \text{ in.}^4$$

Estimated values: $E = 25\,000\,000$ lbs., $\varepsilon = 0.02$ in., $V = 0.035$ in.

P = load, in pounds.	(Δ Inches.	+ V) Inches.	= D calculated. Inches.	D observed. Inches.
500	0.0003	+ 0.035	= 0.0353	0.03
5 000	0.0038	+ 0.035	= 0.0388	0.04
10 000	0.0091	+ 0.035	= 0.0441	0.05
15 000	0.0168	+ 0.035	= 0.0518	0.05
20 000	0.0293	+ 0.035	= 0.0643	0.06
25 000	0.0526	+ 0.035	= 0.0876	0.09
30 000	0.1122	+ 0.035	= 0.1472	0.15
34 000	0.3380	+ 0.035	= 0.3730	not recorded;
Ult. load. 34 110	0.3560	+ 0.035	= 0.3910	column failed.

* *Transactions*, Am. Soc. C. E., Vol. xiii.

Test No. 206.—T-bar; $2\frac{1}{2}$ ins. × $2\frac{1}{2}$ ins. × 82.375 ins. long over all, 2-in. balls and plates, 80.375 ins. effective length.

$$I = a\,r^2 = 1.73 \text{ sq. ins.} \times (0.55)^2 = 0.523325 \text{ in.}^4$$

Estimated values: $E = 32\,000\,000$ lbs., $\varepsilon = 0.035$ in., $V = +\,0.05$ in.

P = load, in pounds.	(Δ Inches.	+	V) Inches.	=	D calculated. Inches.	D observed. Inches.
500	0 00086	+	0.05	=	0.0508	0.03
3 000	0.0058	+	0.05	=	0.0558	0.05
6 000	0.0133	+	0.05	=	0.0633	0.06
9 000	0.0238	+	0.05	=	0.0738	0.07
12 000	0.0392	+	0.05	=	0.0892	0.10
15 000	0.0636	+	0.05	=	0.1136	0.12
18 000	0.1097	+	0.05	=	0.1597	0.16
20 000	0.1718	+	0.05	=	0.2218	0.22
21 000	0.2270	+	0.05	=	0.277	0.30
Ult. 21 500	0.2660	+	0.05	=	0.3160	$\propto$ failed.
(22 000)	(0.3200	+	0.05)	=	(0.3700)	....
(23 000)	(0.5120	+	0.05)	=	(0.5620)	....

Test No. 208.—T-bar; $1\frac{1}{2}$ ins. × $1\frac{1}{2}$ ins. × 81.1875 ins. long over all, on 1-in. balls and plates, therefore, effective length = 80.1875 ins.

$$I = a\,r^2 = 0.53 \text{ sq. in.} \times (0.32)^2 = 0.054272 \text{ in.}^4.$$

Estimated values: $E = 30\,000\,000$ lbs., $\varepsilon = 0.05$ in., $V = 0.03$ in.

P = load, in pounds.	(Δ Inches.	+	V) Inches.	=	D calculated. Inches.	D observed. Inches.
200	0.00538	+	0.03	=	0.0354	0.02
600	0.01965	+	0.03	=	0.0497	0.05
800	0.29380	+	0.03	=	0.0594	0.06
1 000	0.04188	+	0.03	=	0.0719	0.07
1 200	0.05840	+	0.03	=	0.0884	0.09
1 400	0.08135	+	0.03	=	0.1114	0.12
1 600	0.11525	+	0.03	=	0.1453	0.15
1 800	0.17100	+	0.03	=	0.2010	0.20
2 000	0.27750	+	0.03	=	0.3075	0.30
Ult. 2 200	0.5696	+	0.03	=	0.5996	∞ failed.
(2 250)	(0.74000	+	0.03)	=	(0.7700)	....
(2 300)	(1.04200	+	0.03)	=	(1.0720)	....

Test No. 211.—T-bar; 2 ins. × 2 ins. x 63.1875 ins. long over all, on 1-in. balls and plates, therefore, effective length = 62.1875 ins.

$$I = a r^2 = (0.95 \text{ sq. in.}) \times (0.43)^2 = 0.175655 \text{ in.}^4$$

Estimated values: $E = 27\,000\,000$ lbs., $\varepsilon = 0.10$ in., $V = 0.02$ in.

	P = load, in pounds.	(Δ Inches.	+ V) Inches.	= D calculated. Inches.	D observed. Inches.
	500....	0.00533	+ 0.02	= 0.0253.....	0.03
	2 000....	0.02455	+ 0.02	= 0.0446.....	0.05
	3 000....	0.04110	+ 0.02	= 0.0611.....	0.06
	4 000....	0.06175	+ 0.02	= 0.0818.....	0.08
	5 000....	0.08860	+ 0.02	= 0.1086.....	0.10
	6 000....	0.12460	+ 0.02	= 0.1446.....	0.15
	7 000....	0.17600	+ 0.02	= 0.1960.....	0.20
	8 000....	0.25450	+ 0.02	= 0.2745.....	0.27
	9 000....	0.38900	+ 0.02	= 0.4090.....	0.40
	9 500	0.50100	+ 0.02	= 0.5210.....	not recorded.
Ult. load.	9 510....	0.50400	+ 0.02	= 0.5240.....	$\propto$ failed.
	9 550....	0.51600	+ 0.02	= 0.5360.....	....

Test No. 217.—T-bar; 1 in. × 1 in. × 45¼ ins., long over all, on 1-in. balls and plates, therefore, effective length = 44.25 ins.

$$I = a r^2 = 0.3 \text{ sq. in.} \times (0.26)^2 = 0.02028 \text{ in.}^4$$

Estimated values: $E = 18\,500\,000$ lbs., $\varepsilon = 0.037$ in., $V = 0.008$ in.

	P = load, in pounds.	(Δ Inches.	+ V) Inches.	= D calculated. Inches.	D observed. Inches.
	100....	0.0025	+ 0.008	= 0.0106.....	0.01
	400....	0.0123	+ 0.008	= 0.0203.....	0.02
	600....	0.0215	+ 0.008	= 0.0295.....	0.03
	800....	0.0342	+ 0 008	= 0.0422.....	0.04
	1 000....	0.0533	+ 0.008	= 0.0613.....	0.06
	1 200....	0.0832	+ 0.008	= 0 0912.....	0.10
	1 400....	0.1415	+ 0.008	= 0.1495.....	0 15
	1 600....	0.2968	+ 0.008	= 0.3048.....	0.29
	1 700....	0.5410	+ 0.008	= 0.5490.....	0.55
Ult. load.	1 750....	0.8675	+ 0.008	= 0.8755.....	$\propto$ failed.
	1 800....	2.0250	+ 0.008	= 2.0330.....	....

With regard to the value of the modulus of elasticity E, as estimated for the foregoing comparative examples, it may be mentioned that Mr. Hodgkinson found from transverse bending tests of the Low

Moor No. 3 cast iron, that E ranged from 13 585 530 to 14 251 950 lbs., and Mr. James Christie* found from bending tests of the wrought iron upon which he experimented, that E ranged from 19 164 000 to 33 631 000 lbs.

The value of E, as found by bending tests, is necessarily that to which we must refer in dealing with column strength and stiffness, as the modulus of elasticity only enters into the column formula on account of the bending moments exerted on the column, and not at all in connection with direct compressive stresses.

It may be noted here that the value of E obtained from the transverse bending tests on ordinary cold-straightened wrought-iron or steel bars will depend upon the position of the points at which the straightening press has been applied. If the straightening is done near the center of the span, the value of E may reasonably be expected to come out very low, while the influence of any straightening done near the ends of the bars will have comparatively little influence on the results obtained.

In direct tensile tests the position of the points of straightening will have no influence on the results, which will only be affected by the amount of straightening to which the bar has been subjected. This will explain the much greater uniformity in the results obtained by direct tension, as compared with those obtained by transverse bending tests, and also with those obtained from compression tests where the slightest latitude is given for the specimen to act as a column, and where the material has had to be cold-straightened.†

GENERAL FORMULAS FOR THE RELATION OF COLUMN PROPORTIONS TO COLUMN STRENGTH.

Round or pivoted ends—

Formula (7) $$R = \frac{l}{r} = \sqrt{\frac{48\,E}{5\,F_c + f_d\left(\frac{c\,\varepsilon}{r^2} - 5\right)}\left[\frac{F_c}{f_d} - 1 - \frac{c\,\varepsilon}{r^2}\right]}$$

for failure by compressive stress,

or (8) $$R = \frac{l}{r} = \sqrt{\frac{48\,E}{5\,F_t - f_d\left(\frac{c\,\varepsilon}{r^2} + 5\right)}\left[\frac{F_t}{f_d} - 1 + \frac{c\,\varepsilon}{r^2}\right]}$$

for failure by tensile stress.

* "The Strength and Elasticity of Structural Steel," *Transactions*, Am. Soc. C. E., Vol. xiii.

† See results of Mr. Christie's tests in "The Strength and Elasticity of Structural Steel," *Transactions*, Am. Soc. C. E., Vol. xiii.

Fixed ends.... R = twice the values given by Formulas (7) and (8).
Flat ends..... R = same as fixed ends, keeping in view that in Formula (8) F_t is to be made zero, resulting in Formula (9).

Hinged or Pin ends. $R =$ { in upper limits, same as for fixed ends. / in lower limits, same as for round ends.

Attention must here be drawn to the fact that Formulas (7) and (8) are not actually two different formulas, but are only simple algebraic transformations of one and the same general formula (6), referred to the two conditions of failure by compressive stress and by tensile stress, the only other modifications necessary for their application to any of the types of column previously given being due to the conditions of end fixing as determining the relative length of columns of the same strength, but with different end conditions.

It will be noticed that there are three factors in the formulas to which it is necessary to assign values, and these are:

(1) The value of E = modulus of elasticity;

(2) The value of F_c or F_t = the maximum fiber stress;

(3) The value of $\frac{c\,\varepsilon}{r^2}$.

Careful study of the great variations shown in actual tests, and of the comparisons made between actual and calculated deflections, apparently indicate very great difficulty in assigning any fixed values for these quantities for general application, but it must be remembered, that it is not practically possible to predict the precise strength of any given column, and that being so, it only remains to endeavor to determine the limits between which we may expect the column strength to lie. With this in view, using approximately normal values of E as obtained by tests in direct tension or compression, assigning values to F_c approximately as shown by the higher tests of short columns, and to F_t as given by direct tests of tensile strength, the writer has found that the lower limit of column strength is given fairly by the formulas when $\frac{c\,\varepsilon}{r^2} = 0.6$; and it also appears that the upper limit given by the formulas, when $\frac{c\,\varepsilon}{r^2} = 0.15$ is rarely exceeded.

These values are applicable alike to cast iron, wrought iron, mild steel, hard steel, and several kinds of timber, as will be seen on reference to the various diagrams of column tests appended.

It is interesting to note that in the case of solid, round columns, the value $\frac{c\,\varepsilon}{r^2} = 0.6$ for lower-limit strength corresponds to an equivalent eccentricity of (0.3 × radius of gyration), and applying this to a bar of 1 in. diameter, where $c = 0.5$ in., and $r = 0.25$ in., the value of ε would only be (0.3 × 0.25 in.) = 0.075 in.

Of this amount the possible errors in setting the test specimen may be only a small part, and if we assume that initial external curvature of bar, irregularities in the line of physical axis, and the effects of cold-straightening account for say 60% of the total value of ε, the remaining 40%, or 0.03 in., would be the extreme permissible amount of error in setting the specimen, and it becomes apparent how important such small errors are in experiments on columns, and how very carefully the testing must be carried out in order to develop even the lower-limit strength shown by the accompanying diagrams.

This also impresses the mind with the danger of generalizing from the results of any single series of tests where the number of tests is not very large.

Strictly speaking, the theory and resulting formulas of this paper apply only when the loads are such as will not stress the material of the column beyond the elastic limit, a condition which applies equally to the common theory of flexure of solid beams under simple bending. This, however, forms no bar to the extension of the application of the formula for elastic beam strength to cases of ultimate strength where the elastic limit is exceeded, provided proper recognition is given to the fact that the ultimate maximum fiber stresses apparently developed are not true values of tensile or compressive strength, but are always much higher than obtained from tests under direct stress. The reasons for this are now well known and need not be dealt with here.

There is, therefore, considerable justification for the application of the column formulas to a comparison with experiments on ultimate strength, which, in fact, form the only available basis of reference to which the engineer can appeal.

Differences of opinion exist among engineers as to whether columns should be designed with regard to the ultimate strength or with regard to the maximum fiber stress developed by the working load. The former is by far the more common practice, owing to the existing state of knowledge as to the principles of column strength, and this renders

it necessary for any theory and formulas, proposed for use in practice, to be compared with available experimental evidence.

In the writer's view, the more rational method is to design columns so as to ensure that given maximum fiber stresses will not be exceeded under the working load, while at the same time taking care to refer to experimental results, in order to see that a sufficient margin is provided against failure by instability in the longer lengths.

It would be reasonable to expect that the values of F_c and F_t, being maximum fiber stresses, would, when referred to tests carried to ultimate failure, partake somewhat of the nature and value generally accorded to the corresponding maximum apparent fiber stresses determined from tests of ultimate transverse strength of simple solid beams. It is necessary, however, to keep in view that it is hardly possible that they can have such high apparent values as in beam tests, owing to the rapidly accumulating bending moment developed in columns by the increasing deflection, which must evidently be intensified by any extra yielding of the extreme fibers due to their elastic limit being exceeded, and thus accelerating the deflection.

In the case of beams, of course, the increase in deflection accompanying an increasing load has no influence whatever on the bending moment.

The writer has not found it necessary for "centrally" loaded columns of wrought iron, mild or hard steel, and timber, to deal with the condition of failure by tensile stress, excepting as regards incipient tension in flat-ended columns.

The maximum compressive stress in a column of symmetrical section always has a greater value in pounds per square inch than the maximum tensile stress, and the difference between the compressive and tensile strengths must be considerable before tension becomes the controlling influence. This is the case with high-class cast iron, such as was used by Hodgkinson in his tests of Low Moor, No. 3 iron, and the curves for failure by tension are therefore plotted on the diagrams showing these tests.

In order to discover, if possible, any special features accompanying any particular set of tests, the writer has assigned a separate diagram to the principal sets of experiments where the number and range of tests were sufficiently great to justify it, and where this was not the case, the individual tests have been given distinguishing symbols to assist in arriving at a correct judgment.

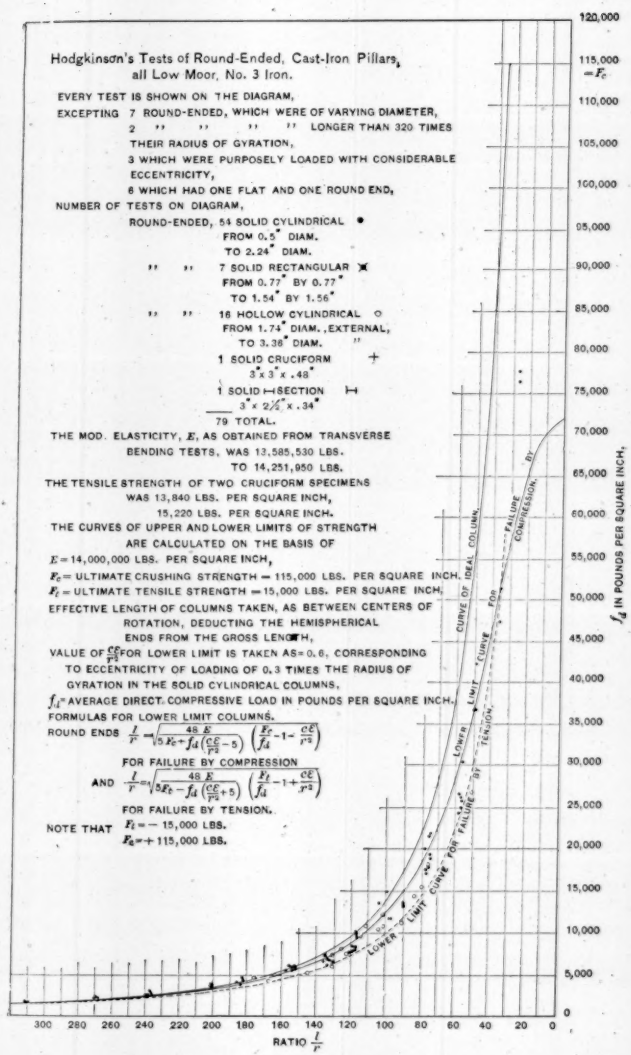
Hodgkinson's Tests of Round-Ended, Cast-Iron Pillars, all Low Moor, No. 3 Iron.
EVERY TEST IS SHOWN ON THE DIAGRAM,
EXCEPTING 7 ROUND-ENDED, WHICH WERE OF VARYING DIAMETER,
2 " " " " LONGER THAN 320 TIMES THEIR RADIUS OF GYRATION,
3 WHICH WERE PURPOSELY LOADED WITH CONSIDERABLE ECCENTRICITY,
6 WHICH HAD ONE FLAT AND ONE ROUND END,
NUMBER OF TESTS ON DIAGRAM,
ROUND-ENDED, 54 SOLID CYLINDRICAL ● FROM 0.5" DIAM. TO 2.24" DIAM.
" " 7 SOLID RECTANGULAR ✖ FROM 0.77" BY 0.77" TO 1.54" BY 1.56"
" " 16 HOLLOW CYLINDRICAL ○ FROM 1.74" DIAM. EXTERNAL, TO 3.38" DIAM. "
1 SOLID CRUCIFORM + 3" x 3" x .48"
1 SOLID H-SECTION H 3" x 2½" x .34"
79 TOTAL.
THE MOD. ELASTICITY, E, AS OBTAINED FROM TRANSVERSE BENDING TESTS, WAS 13,585,530 LBS. TO 14,251,950 LBS.
THE TENSILE STRENGTH OF TWO CRUCIFORM SPECIMENS WAS 13,840 LBS. PER SQUARE INCH, 15,220 LBS. PER SQUARE INCH.
THE CURVES OF UPPER AND LOWER LIMITS OF STRENGTH ARE CALCULATED ON THE BASIS OF
E = 14,000,000 LBS. PER SQUARE INCH,
F_c = ULTIMATE CRUSHING STRENGTH = 115,000 LBS. PER SQUARE INCH.
F_t = ULTIMATE TENSILE STRENGTH = 15,000 LBS. PER SQUARE INCH.
EFFECTIVE LENGTH OF COLUMNS TAKEN, AS BETWEEN CENTERS OF ROTATION, DEDUCTING THE HEMISPHERICAL ENDS FROM THE GROSS LENGTH,
VALUE OF $\frac{cc}{r^2}$ FOR LOWER LIMIT IS TAKEN AS = 0.6, CORRESPONDING TO ECCENTRICITY OF LOADING OF 0.3 TIMES THE RADIUS OF GYRATION IN THE SOLID CYLINDRICAL COLUMNS,
f_d = AVERAGE DIRECT COMPRESSIVE LOAD IN POUNDS PER SQUARE INCH.
FORMULAS FOR LOWER LIMIT COLUMNS.
ROUND ENDS $\frac{l}{r} = \sqrt{\frac{48\,E}{15\,F_c + f_d\left(\frac{cc}{r^2}-5\right)}\left(\frac{F_c}{f_d}-1-\frac{cc}{r^2}\right)}$
FOR FAILURE BY COMPRESSION
AND $\frac{l}{r} = \sqrt{\frac{48\,E}{5F_t - f_d\left(\frac{cc}{r^2}+5\right)}\left(\frac{F_t}{f_d}-1+\frac{cc}{r^2}\right)}$
FOR FAILURE BY TENSION.
NOTE THAT F_t = − 15,000 LBS.
F_c = + 115,000 LBS.
CURVE OF IDEAL COLUMN.
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION.
LOWER LIMIT CURVE FOR FAILURE BY TENSION.
f_d IN POUNDS PER SQUARE INCH,
RATIO $\frac{l}{r}$
120,000
115,000 = F_c
110,000
105,000
100,000
95,000
90,000
85,000
80,000
75,000
70,000
65,000
60,000
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0

FIG. 17.

As has already been remarked, the writer has been unable to obtain practical verification of the precise influence of form of section from existing records of experiments. To demonstrate this influence by practical evidence it would be necessary to make a large number of new experiments on sections having widely different values of $\frac{c}{r}$, and at the same time to impose the loads with a comparatively large and predetermined value of eccentricity, in order to overshadow the relative influence of what has been dealt with in the foregoing pages as "equivalent eccentricity."

All the diagrams of column tests accompanying this paper have been made self-explanatory as far as possible.

They are as follows:

Cast Iron.

Fig. 17.—Representing 79 tests of round-ended columns of Low Moor, No. 3 cast iron, by Mr. Hodgkinson.

The tests are plotted to effective lengths, an allowance having been made for the rounding of the ends by taking them as hemispherical of the same diameter as the bars. The ends were not actually hemispherical in every case, some being somewhat more pointed, but no error of importance is involved in the assumption made.

Fig. 18. Representing 96 tests of flat-ended and disc-ended columns of Low Moor, No. 3 cast iron, by Mr. Hodgkinson.

It will be noticed in this set of tests on flat-ended columns, that the longer columns, from a ratio of 80 upward, do not show so low a strength as is indicated by the lowermost dotted curve for incipient tension, but failure at this critical point in flat-ended columns is evidenced so strongly in the case of tests of wrought iron and steel that the higher results obtained by Hodgkinson in these tests on cast iron must be attributed partly to his extreme care, and partly to the comparative fewness of tests of each length.

These two sets of tests have been abstracted from Mr. Hodgkinson's remarkably careful records.* They are the only published records of tests of cast-iron columns made in a consistent and scientific manner on one grade of material of known character. The values of load per square inch, radius of gyration and ratio $\frac{l}{r}$, have been calculated by the writer from Mr. Hodgkinson's figures.

* *Philosophical Transactions*, Royal Society of London, 1840.

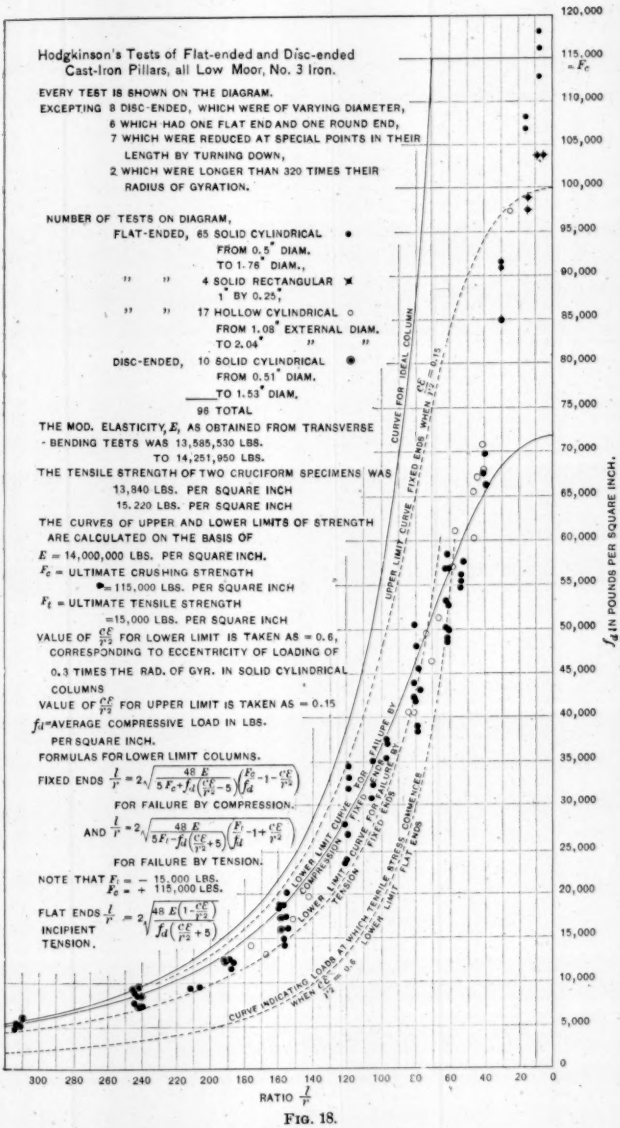
Hodgkinson's Tests of Flat-ended and Disc-ended Cast-Iron Pillars, all Low Moor, No. 3 Iron.
EVERY TEST IS SHOWN ON THE DIAGRAM.
EXCEPTING 8 DISC-ENDED, WHICH WERE OF VARYING DIAMETER,
6 WHICH HAD ONE FLAT END AND ONE ROUND END,
7 WHICH WERE REDUCED AT SPECIAL POINTS IN THEIR LENGTH BY TURNING DOWN,
2 WHICH WERE LONGER THAN 320 TIMES THEIR RADIUS OF GYRATION.
NUMBER OF TESTS ON DIAGRAM,
FLAT-ENDED, 65 SOLID CYLINDRICAL FROM 0.5" DIAM. TO 1.78" DIAM.,
" " 4 SOLID RECTANGULAR 1" BY 0.25",
" " 17 HOLLOW CYLINDRICAL FROM 1.08" EXTERNAL DIAM. TO 2.04" " "
DISC-ENDED, 10 SOLID CYLINDRICAL FROM 0.51" DIAM. TO 1.53" DIAM.
98 TOTAL
THE MOD. ELASTICITY, E, AS OBTAINED FROM TRANSVERSE BENDING TESTS WAS 13,585,530 LBS. TO 14,251,950 LBS.
THE TENSILE STRENGTH OF TWO CRUCIFORM SPECIMENS WAS 13,840 LBS. PER SQUARE INCH 15,220 LBS. PER SQUARE INCH
THE CURVES OF UPPER AND LOWER LIMITS OF STRENGTH ARE CALCULATED ON THE BASIS OF
E = 14,000,000 LBS. PER SQUARE INCH.
Fc = ULTIMATE CRUSHING STRENGTH = 115,000 LBS. PER SQUARE INCH
Ft = ULTIMATE TENSILE STRENGTH = 15,000 LBS. PER SQUARE INCH
VALUE OF cE/r² FOR LOWER LIMIT IS TAKEN AS = 0.6, CORRESPONDING TO ECCENTRICITY OF LOADING OF 0.3 TIMES THE RAD. OF GYR. IN SOLID CYLINDRICAL COLUMNS
VALUE OF cE/r² FOR UPPER LIMIT IS TAKEN AS = 0.15
fl = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
FORMULAS FOR LOWER LIMIT COLUMNS.
FIXED ENDS l/r = 2√(48E / (5Fc + fl(cE/r² − 5)) (Fc/fl − 1 − cE/r²))
FOR FAILURE BY COMPRESSION.
AND l/r = 2√(48E / (5Ft − fl(cE/r² + 5)) (Ft/fl − 1 + cE/r²))
FOR FAILURE BY TENSION.
NOTE THAT Ft = − 15,000 LBS., Fc = + 115,000 LBS.
FLAT ENDS l/r = 2√(48E(1 − cE/r²) / (fl(cE/r² + 5)))
INCIPIENT TENSION.
CURVE FOR IDEAL COLUMN
UPPER LIMIT CURVE, FIXED ENDS, WHEN cE/r² = 0.15
LOWER LIMIT CURVE FOR COMPRESSION FIXED ENDS CALCULATED BY
LOWER LIMIT CURVE FOR TENSION FIXED ENDS CALCULATED BY
LOWER LIMIT, FLAT ENDS
CURVE INDICATING LOADS AT WHICH TENSILE STRESS COMMENCES WHEN cE/r² = 0.6
Fc = 115,000
fl IN POUNDS PER SQUARE INCH.
RATIO l/r

FIG. 18.

Fig. 19.—Representing 76 tests of cylindrical, flat-ended columns of cast iron of various kinds, by Mr. Hodgkinson.*

One test by Mr. Charnock, at Bradford Technical College, England, on a hollow cast iron column, with flat flanged ends.†

One test by Professor John Goodman, at Yorkshire College, Leeds, England, on a hollow column with flanged ends.‡

Fourteen tests on cylindrical, hollow columns, with flat ends, made by the New York City Department of Buildings.§

Fourteen tests on hollow columns with flat and flanged ends, at Watertown Arsenal.‖

Total number of tests on diagram, 106.

The sectional areas, loads per square inch, radii of gyration and values of $\frac{l}{r}$, have all been calculated by the writer from the figures given in the records, except for the New York tests, for which the radii of gyration and ratio $\frac{l}{r}$, only, were calculated.

Most of the Watertown tests were on tapered columns, and all the areas, loads per square inch and radii of gyration calculated for these refer to the section at the middle of the column length, which will explain the divergence from the figures for ultimate strength per square inch given by Professor Lanza.

The results plotted on Fig. 19 refer to tests of columns of various kinds of cast iron. Hodgkinson's tests alone cover 17 different irons, of widely different compressive strength, in short specimens. No information is given as to the physical characteristics of the irons used in the other tests plotted on the diagram.

A number of the columns of this (1857) series by Hodgkinson were subjected to more than one test. They were made long at first, and after being tested they were cut down into shorter columns and re-tested, a circumstance still further adding to the difficulty of deriving any definite laws of strength from these tests.

Regarded as a means of determining the influence of column proportions on ultimate strength, the results of any of the tests plotted on Fig. 19 are of little value, and a most cursory consideration will

*Abstracted from Mr. Hodgkinson's paper in *Philosophical Transactions*, Royal Society of London, 1857.

† *Engineering*, February 28th, 1896.

‡ *Engineering*, September 11th, 1896.

§ *Engineering News*, January 13th, 1898.

‖ Reports, 1887-1888, and Lanza's "Applied Mechanics."

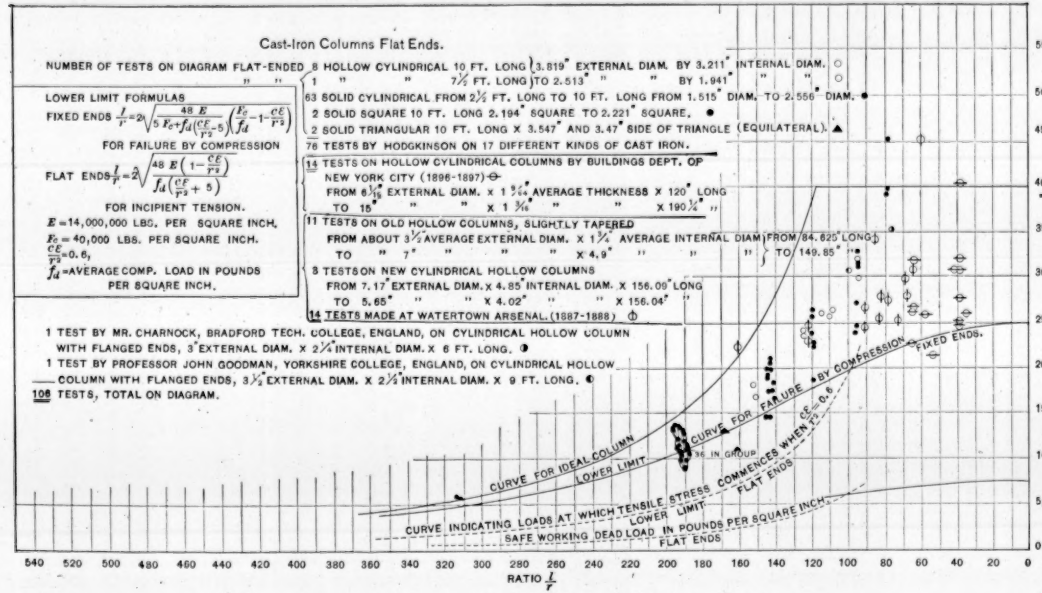
Cast-Iron Columns Flat Ends.
NUMBER OF TESTS ON DIAGRAM FLAT-ENDED 8 HOLLOW CYLINDRICAL 10 FT. LONG 3.819" EXTERNAL DIAM. BY 3.211" INTERNAL DIAM.
" " 1 " " 7½ FT. LONG TO 2.513" " " BY 1.941" " "
63 SOLID CYLINDRICAL FROM 2½ FT. LONG TO 10 FT. LONG FROM 1.515" DIAM. TO 2.556" DIAM.
2 SOLID SQUARE 10 FT. LONG 2.194" SQUARE TO 2.221" SQUARE.
2 SOLID TRIANGULAR 10 FT. LONG X 3.547" AND 3.47" SIDE OF TRIANGLE (EQUILATERAL).
76 TESTS BY HODGKINSON ON 17 DIFFERENT KINDS OF CAST IRON.
14 TESTS ON HOLLOW CYLINDRICAL COLUMNS BY BUILDINGS DEPT. OF NEW YORK CITY (1896-1897)
FROM 6½" EXTERNAL DIAM. X 1 5/16" AVERAGE THICKNESS X 120" LONG
TO 15" " " X 1 3/16" " " X 190¼" "
11 TESTS ON OLD HOLLOW COLUMNS, SLIGHTLY TAPERED
FROM ABOUT 3½ AVERAGE EXTERNAL DIAM. X 1¼" AVERAGE INTERNAL DIAM. FROM 84.625" LONG
TO " 7" " " " " X 4.9" " " " TO 149.85" "
8 TESTS ON NEW CYLINDRICAL HOLLOW COLUMNS
FROM 7.17" EXTERNAL DIAM. X 4.85" INTERNAL DIAM. X 156.09" LONG
TO 5.65" " " X 4.02" " " X 156.04" "
14 TESTS MADE AT WATERTOWN ARSENAL. (1887-1888)
LOWER LIMIT FORMULAS
FIXED ENDS $\frac{l}{r}=2\sqrt{\frac{48\,E}{5\,F_c+f_d\left(\frac{cE}{r^2}-5\right)}\left(\frac{F_c}{f_d}-1-\frac{cE}{r^2}\right)}$
FOR FAILURE BY COMPRESSION
FLAT ENDS $\frac{l}{r}=2\sqrt{\frac{48\,E\left(1-\frac{cE}{r^2}\right)}{f_d\left(\frac{cE}{r^2}+5\right)}}$
FOR INCIPIENT TENSION.
E = 14,000,000 LBS. PER SQUARE INCH.
F_c = 40,000 LBS. PER SQUARE INCH.
$\frac{cE}{r^2}$ = 0.6,
f_d = AVERAGE COMP. LOAD IN POUNDS PER SQUARE INCH.
1 TEST BY MR. CHARNOCK, BRADFORD TECH. COLLEGE, ENGLAND, ON CYLINDRICAL HOLLOW COLUMN WITH FLANGED ENDS, 3" EXTERNAL DIAM. X 2¼" INTERNAL DIAM. X 6 FT. LONG.
1 TEST BY PROFESSOR JOHN GOODMAN, YORKSHIRE COLLEGE, ENGLAND, ON CYLINDRICAL HOLLOW COLUMN WITH FLANGED ENDS, 3½" EXTERNAL DIAM. X 2¼" INTERNAL DIAM. X 9 FT. LONG.
106 TESTS, TOTAL ON DIAGRAM.
CURVE FOR IDEAL COLUMN
LOWER LIMIT
CURVE FOR FAILURE
96 IN GROUP
BY COMPRESSION
FIXED ENDS.
$\frac{cE}{r^2}$ = 0.6
COMMENCES WHEN
FLAT ENDS
CURVE INDICATING LOADS AT WHICH TENSILE STRESS
LOWER LIMIT
FLAT ENDS
SAFE WORKING DEAD LOAD IN POUNDS PER SQUARE INCH.
55,000
50,000
45,000
40,000
$=F_c$
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 19.

show the absurdity of attempting to generalize with regard to any of the three principal sets of tests of this diagram. Nevertheless, these are the most important tests of cast-iron columns yet made, from the point of view of the engineer, representing, as they do, all the experimental evidence at present available to justify the confidence of the designer using this material in its commoner qualities without any definite knowledge or check upon its physical characteristics.

It is worthy of note that prior to the Watertown tests of 1887 and 1888, there was absolutely no published experimental evidence existing of the strength of common grade cast-iron columns of the proportions of length to radius of gyration in most common use. Hodgkinson's tests (1857) did not give results on columns of shorter lengths than 79 or 80 times the radius of gyration, and his tests in the 1840 paper were on iron of a comparatively high class. This want has, to a slight extent, been filled by the Watertown tests and the New York Building Department tests.

It is surprising to think of the enormous number of cast-iron columns which have been put into use without any justification for the loads imposed on them, except a simple faith in Hodgkinson's, Gordon's and Rankine's formulas, and in the numerous tables calculated therefrom and published in engineering pocketbooks and treatises.

The writer does not pretend that the curves plotted on Fig. 19 have any other than a purely accidental correspondence with the experimental results shown on the diagram.

The curves calculated from the formula refer to material of certain fixed characteristics, while the experiments plotted on Fig. 19 were made on cast iron of widely different grades.

At the same time the curves follow definite laws, and may serve as a basis of reference. The writer, in his own practice, would not care to count upon higher ultimate strengths for common cast-iron columns than are given by the lowest curves on the diagram.

Wrought Iron.—Round or Pivoted Ends.

Fig. 20.—Representing 33 tests of round-ended columns by Mr. Christie.* These tests have all been plotted to lengths measured from center to center of hemispherical ends. Mr. Christie gives values of R based on extreme lengths.

* *Transactions*, Am. Soc. C. E., Vol. xiii.

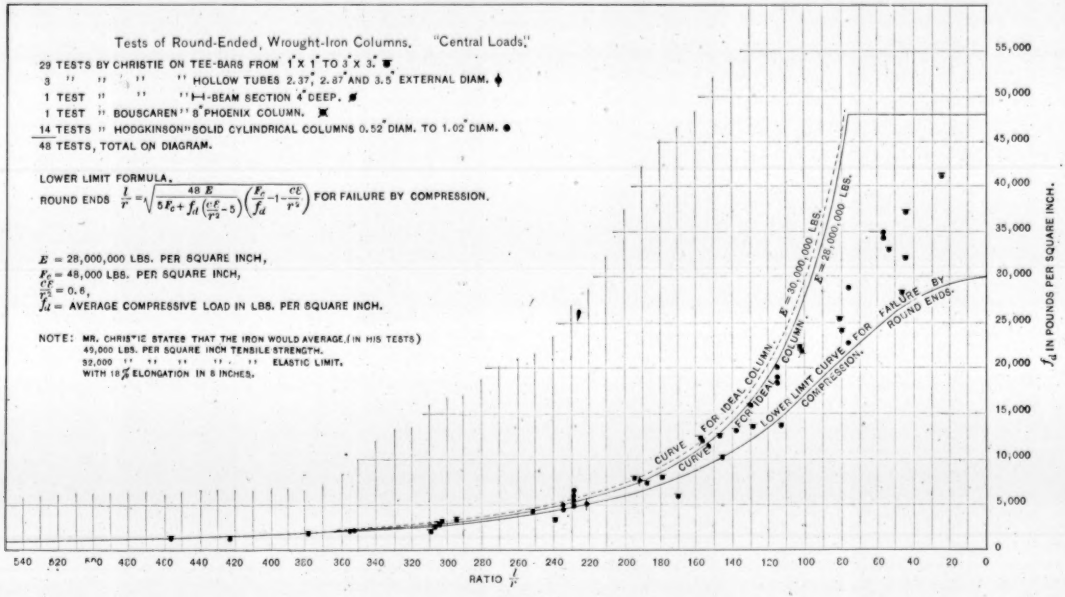
Tests of Round-Ended, Wrought-Iron Columns. "Central Loads."
29 TESTS BY CHRISTIE ON TEE-BARS FROM 1" X 1" TO 3" X 3".
3 " " " " HOLLOW TUBES 2.37", 2.87" AND 3.5" EXTERNAL DIAM.
1 TEST " " " I-BEAM SECTION 4" DEEP.
1 TEST " BOUSCAREN " 8" PHOENIX COLUMN.
14 TESTS " HODGKINSON " SOLID CYLINDRICAL COLUMNS 0.52" DIAM. TO 1.02" DIAM.
48 TESTS, TOTAL ON DIAGRAM.
LOWER LIMIT FORMULA,
ROUND ENDS $\frac{l}{r}=\sqrt{\frac{48\,E}{5F_c+f_d\left(\frac{cL}{r^2}-5\right)\left(\frac{F_c}{f_d}-1-\frac{cL}{r^2}\right)}}$ FOR FAILURE BY COMPRESSION.
E = 28,000,000 LBS. PER SQUARE INCH,
F_c = 48,000 LBS. PER SQUARE INCH,
$\frac{cL}{r^2}$ = 0.6,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
NOTE: MR. CHRISTIE STATES THAT THE IRON WOULD AVERAGE (IN HIS TESTS)
49,000 LBS. PER SQUARE INCH TENSILE STRENGTH.
32,000 " " " " ELASTIC LIMIT.
WITH 18% ELONGATION IN 8 INCHES.
CURVE FOR IDEAL COLUMN. E = 30,000,000 LBS.
CURVE FOR IDEAL COLUMN. E = 28,000,000 LBS.
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION.
FOR FAILURE BY ROUND ENDS.
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 20.

In the case of Tests Nos. 227, 228 and 229 the ratio $\frac{l}{r}$ has been recalculated from the lengths given in Mr. Christie's Table No. 6. The values of $\frac{l}{r}$ for these three tests given in this table do not agree with the lengths and radii of gyration.

The 33 tests by Mr. Christie shown on this diagram are Nos. 200 to 229, inclusive, of his Table No. 6, and Nos. 287, 290 and 293 of his Table No. 8.

One test of a round-ended column of large size by L. F. G. Bouscaren,* M. Am. Soc. C. E. The ends of this column were portions of a sphere of 10½ ins. radius, and the effective length has, therefore, been taken to be 20¼ ins. shorter than the length over all.

Fourteen tests of round-ended columns by Mr. Hodgkinson.† Allowance for the round ends, in arriving at effective length, has been made in this case also. The sectional areas, loads per square inch, radii of gyration, and values of $\frac{l}{r}$, have been calculated by the writer from Mr. Hodgkinson's records.

Total number of tests on diagram, 48.

Fig. 21.—Representing 116 results, by Professor Tetmajer, on pivot-ended columns.‡ These results represent 210 experiments. Ninety-four of the results plotted represent in each case the average of two tests, while twenty-two of the results are for single experiments. The results, as plotted by the writer, are in every case for the load per gross square inch. Some of the specimens were compounded of two or four pieces riveted together, and in the diagram given by Professor J. B. Johnson in his "Materials of Construction," these results appear to have been plotted for load per net square inch (the rivet holes being deducted), and they have, therefore, too high a value. This error was repeated in the reproduction of the diagram by Mr. Marston in his discussion of Professor Cain's paper.§

Fig. 22.—This diagram is simply a combination of Figs. 20 and 21, and therefore includes practically the whole of the available experimental evidence as to the strength of wrought-iron columns with both ends free, and unconstrained, *i. e.*, with "round" or "pivot ends" and with "central" loading.

* *Transactions*, Am Soc. C. E., Vol. ix.

† *Philosophical Transactions*, Royal Society, London, 1840.

‡ "Tetmajer's Communications."

§ "The Ideal Column," *Transactions*, Am. Soc. C. E., Vol. xxxix, pp. 109 to 111.

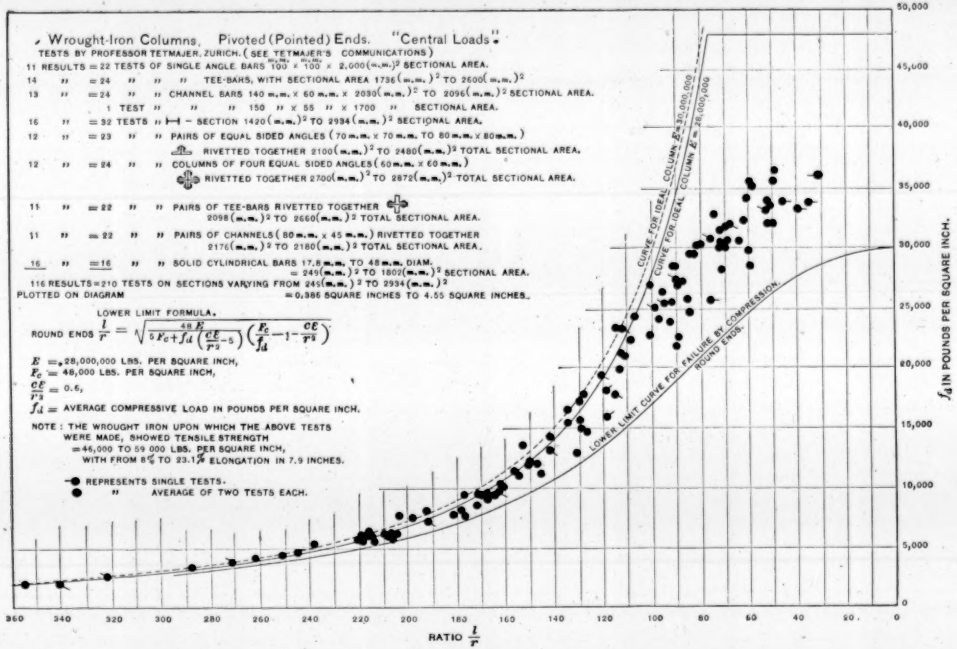
Wrought-Iron Columns, Pivoted (Pointed) Ends. "Central Loads."
TESTS BY PROFESSOR TETMAJER, ZURICH. (SEE TETMAJER'S COMMUNICATIONS)
11 RESULTS = 22 TESTS OF SINGLE ANGLE BARS 100 m.m. x 100 m.m. x 2,000(m.m.)² SECTIONAL AREA.
14 " = 24 " " " TEE-BARS, WITH SECTIONAL AREA 1736(m.m.)² TO 2600(m.m.)²
13 " = 24 " " CHANNEL BARS 140 m.m. x 60 m.m. x 2030(m.m.)² TO 2096(m.m.)² SECTIONAL AREA.
1 TEST " " " 150 " x 55 " x 1700 " SECTIONAL AREA.
16 " = 32 TESTS " ⊢ SECTION 1420(m.m.)² TO 2934(m.m.)² SECTIONAL AREA.
12 " = 23 " " PAIRS OF EQUAL SIDED ANGLES (70 m.m. x 70 m.m. TO 80 m.m. x 80 m.m.)
RIVETTED TOGETHER 2100(m.m.)² TO 2480(m.m.)² TOTAL SECTIONAL AREA.
12 " = 24 " " COLUMNS OF FOUR EQUAL SIDED ANGLES (60 m.m. x 60 m.m.)
RIVETTED TOGETHER 2700(m.m.)² TO 2872(m.m.)² TOTAL SECTIONAL AREA.
11 " = 22 " " PAIRS OF TEE-BARS RIVETTED TOGETHER
2098(m.m.)² TO 2660(m.m.)² TOTAL SECTIONAL AREA.
11 " = 22 " " PAIRS OF CHANNELS (80 m.m. x 45 m.m.) RIVETTED TOGETHER
2176(m.m.)² TO 2180(m.m.)² TOTAL SECTIONAL AREA.
16 " = 16 " " SOLID CYLINDRICAL BARS 17.8 m.m. TO 48 m.m. DIAM.
= 249(m.m.)² TO 1809(m.m.)² SECTIONAL AREA.
116 RESULTS = 210 TESTS ON SECTIONS VARYING FROM 245(m.m.)² TO 2934(m.m.)²
PLOTTED ON DIAGRAM = 0.386 SQUARE INCHES TO 4.55 SQUARE INCHES.
LOWER LIMIT FORMULA.
ROUND ENDS $\frac{l}{r} = \sqrt{\frac{48\,E}{5\,F_c + f_d\left(\frac{c\varepsilon}{r^2}-5\right)}\left(\frac{F_c}{f_d}-1-\frac{c\varepsilon}{r^2}\right)}$
E = 28,000,000 LBS. PER SQUARE INCH,
F_c = 48,000 LBS. PER SQUARE INCH,
$\frac{c\varepsilon}{r^2}$ = 0.6,
f_d = AVERAGE COMPRESSIVE LOAD IN POUNDS PER SQUARE INCH.
NOTE: THE WROUGHT IRON UPON WHICH THE ABOVE TESTS WERE MADE, SHOWED TENSILE STRENGTH = 46,000 TO 59,000 LBS. PER SQUARE INCH, WITH FROM 8% TO 23.1% ELONGATION IN 7.9 INCHES.
REPRESENTS SINGLE TESTS.
" AVERAGE OF TWO TESTS EACH.
CURVE FOR IDEAL COLUMN E = 30,000,000
CURVE FOR IDEAL COLUMN E = 28,000,000
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. ROUND ENDS.
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 21.

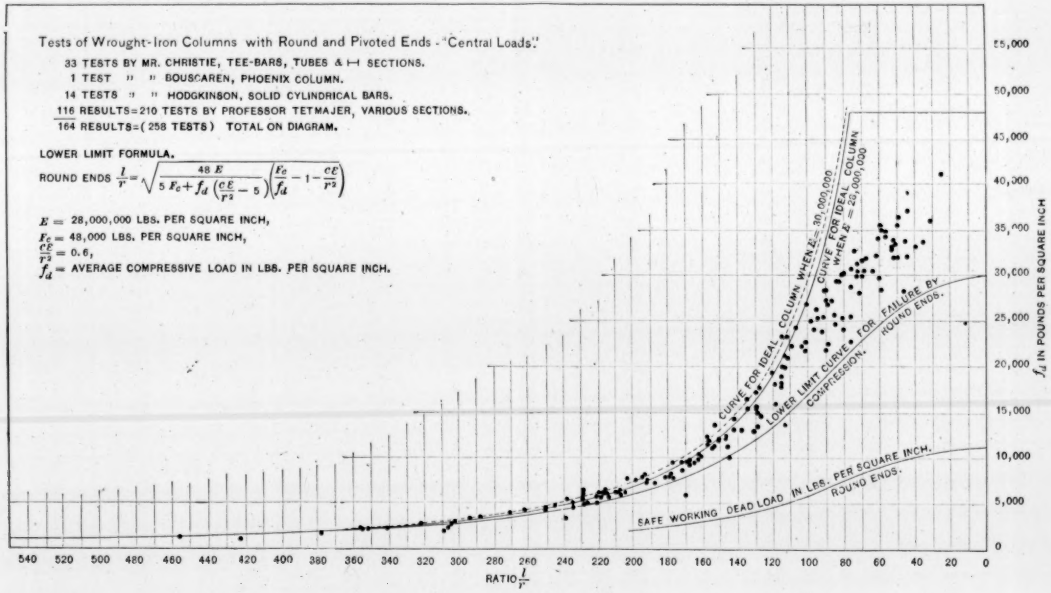
Tests of Wrought-Iron Columns with Round and Pivoted Ends - "Central Loads."
33 TESTS BY MR. CHRISTIE, TEE-BARS, TUBES & H SECTIONS.
1 TEST " " BOUSCAREN, PHOENIX COLUMN.
14 TESTS " " HODGKINSON, SOLID CYLINDRICAL BARS.
116 RESULTS=210 TESTS BY PROFESSOR TETMAJER, VARIOUS SECTIONS.
164 RESULTS=(258 TESTS) TOTAL ON DIAGRAM.
LOWER LIMIT FORMULA.
ROUND ENDS $\frac{l}{r}=\sqrt{\frac{48\,E}{5\,F_c+f_d\left(\frac{c\,E}{r^2}-5\right)}\left(\frac{F_c}{f_d}-1-\frac{c\,E}{r^2}\right)}$
E = 28,000,000 LBS. PER SQUARE INCH,
F_c = 48,000 LBS. PER SQUARE INCH,
$\frac{c}{r^2}$ = 0.6,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN WHEN E = 30,000,000
CURVE FOR IDEAL COLUMN WHEN E = 29,000,000
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. ROUND ENDS.
SAFE WORKING DEAD LOAD IN LBS. PER SQUARE INCH. ROUND ENDS.
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 22.

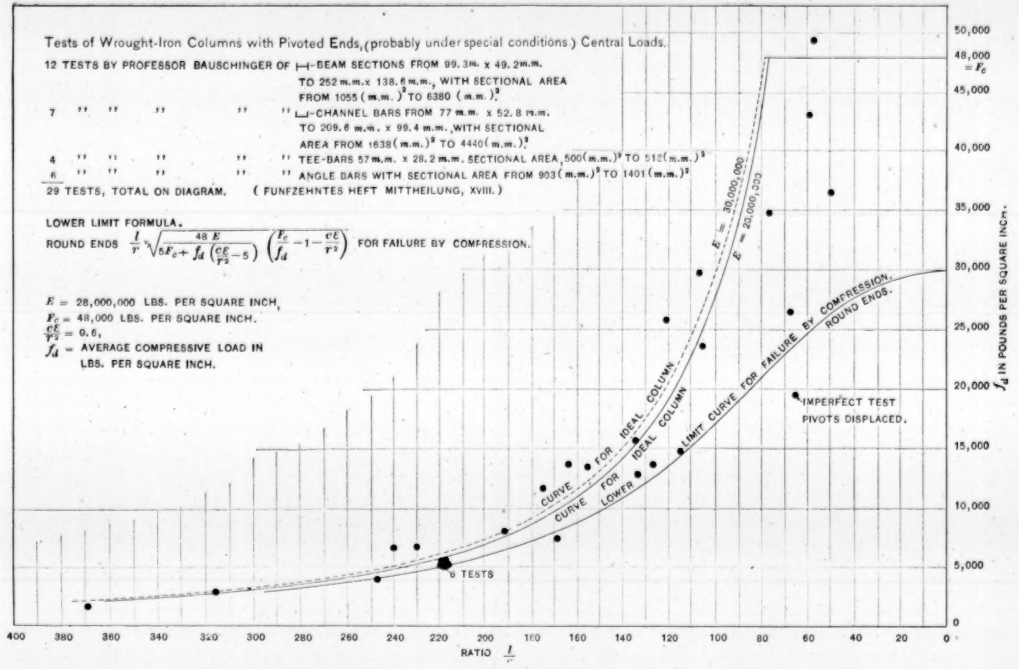
Tests of Wrought-Iron Columns with Pivoted Ends, (probably under special conditions.) Central Loads.
12 TESTS BY PROFESSOR BAUSCHINGER OF ⊢-BEAM SECTIONS FROM 99.3m.m. x 49.2m.m. TO 252 m.m. x 138.6 m.m., WITH SECTIONAL AREA FROM 1055 (m.m.)² TO 6380 (m.m.)²
7 " " " " " ⊔-CHANNEL BARS FROM 77 m.m. x 52.8 m.m. TO 209.6 m.m. x 99.4 m.m., WITH SECTIONAL AREA FROM 1638 (m.m.)² TO 4440 (m.m.)²
4 " " " " " TEE-BARS 57 m.m. x 28.2 m.m. SECTIONAL AREA, 500 (m.m.)² TO 512 (m.m.)²
6 " " " " " ANGLE BARS WITH SECTIONAL AREA FROM 903 (m.m.)² TO 1401 (m.m.)²
29 TESTS, TOTAL ON DIAGRAM. (FUNFZEHNTES HEFT MITTHEILUNG, XVIII.)
LOWER LIMIT FORMULA.
ROUND ENDS $\frac{l}{r} = \sqrt{\frac{48\,E}{5F_c + f_d\left(\frac{cc}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{cc}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
E = 28,000,000 LBS. PER SQUARE INCH,
F_c = 48,000 LBS. PER SQUARE INCH.
$\frac{cc}{r^2}$ = 0.6,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
E = 30,000,000
E = 28,000,000
CURVE FOR IDEAL COLUMN.
CURVE LOWER FOR IDEAL COLUMN
LIMIT CURVE FOR FAILURE BY COMPRESSION. ROUND ENDS.
IMPERFECT TEST PIVOTS DISPLACED.
6 TESTS
50,000
48,000 = F_c
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 23.

The experiments of M. Considère on pivot-ended columns cannot be made use of as bases of reference for practical work, on account of the attempt made to feel for the "physical axis," or axis of greatest resistance, by moving the pivot ends out of the geometric axis, a refinement which is entirely out of the question in practical construction.

This objection probably applies also to Professor Bauschinger's experiments on pivot-ended columns, but this point the writer has not been able to make out clearly from the records.

It may also be noted that M. Considère's test specimens were of very small sections, the heaviest being only 11 mm. × 23 mm. (rectangular) or 253 sq. mm. = about 0.4 sq. in., and the lightest only 77 sq. mm. (angle section) = about 0.12 sq. in.* This last objection does not apply to Bauschinger's pivot-ended tests, in which the sections ranged from a maximum of 63.8 sq. cm. (⊢-section, 25.2 cm. × 13.86 cm.), or about 9.9 sq. ins. sectional area, to a minimum of 5 sq. cm. (T-bar, 5.7 cm. × 2.82 cm.), or about 0.755 sq. in. sectional area.

Although it has been stated that Bauschinger's tests (pivot-ended) probably cannot be used as bases of reference in practical work, owing to the refinements probably adopted in carrying out the tests, yet it has seemed to the writer that a record of them is necessary to the completeness of this paper; and a still stronger reason for their presentation in diagram form lies in the fact that in spite of all the highly skilled care and accuracy bestowed on the tests, Bauschinger did not succeed in keeping the lower tests above the lower limit found in the tests of other experimenters, as will be seen on reference to Fig. 23.

Fig. 23.—Twenty-nine tests by Professor Bauschinger on pivot-ended columns of wrought iron.†

Fig. 24.—Nine results, representing eighteen experiments by Professor Tetmajer on pivot-ended wrought-iron columns under intentionally eccentric loads. These results are in each case the average of the tests of two specimens.‡ It is unfortunate that the record of each individual experiment has not been given by Professor Tetmajer.

* Considère's Report on "La Resistance au Flambement des Pièces Comprimées" (French Commission des Methodes D'Essai des Materiaux de Construction. Tome iii).

† Fünfzehntes Heft, *Mittheilung*, xviii.

‡ "Tetmajer's Communications."

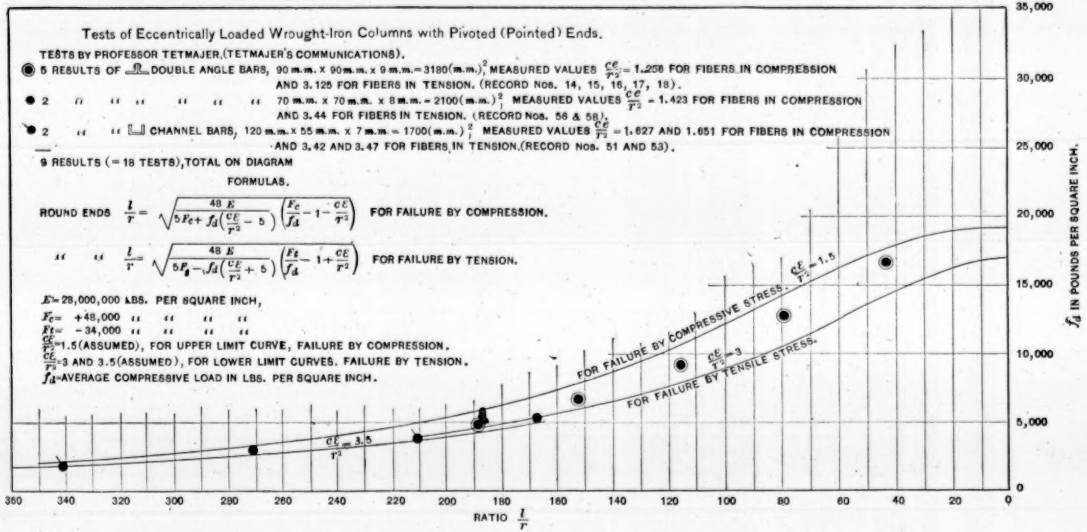
Tests of Eccentrically Loaded Wrought-Iron Columns with Pivoted (Pointed) Ends.
TESTS BY PROFESSOR TETMAJER. (TETMAJER'S COMMUNICATIONS).
5 RESULTS OF ⊏⊐ DOUBLE ANGLE BARS, 90 m.m. x 90 m.m. x 9 m.m. = 3180 (m.m.)², MEASURED VALUES cc/r² = 1.258 FOR FIBERS IN COMPRESSION AND 3.125 FOR FIBERS IN TENSION. (RECORD Nos. 14, 15, 16, 17, 18).
2 " " " " " " 70 m.m. x 70 m.m. x 8 m.m. = 2100 (m.m.)², MEASURED VALUES cc/r² = 1.423 FOR FIBERS IN COMPRESSION AND 3.44 FOR FIBERS IN TENSION. (RECORD Nos. 56 & 58).
2 " " " ⊏ CHANNEL BARS, 120 m.m. x 55 m.m. x 7 m.m. = 1700 (m.m.)², MEASURED VALUES cc/r² = 1.627 AND 1.651 FOR FIBERS IN COMPRESSION AND 3.42 AND 3.47 FOR FIBERS IN TENSION. (RECORD Nos. 51 AND 53).
9 RESULTS (= 18 TESTS), TOTAL ON DIAGRAM
FORMULAS.
ROUND ENDS l/r = √(48 E / 5 Fc + fd (cc/r² − 5) · (Fc/fd − 1 − cc/r²)) FOR FAILURE BY COMPRESSION.
" " l/r = √(48 E / 5 Ft − fd (cc/r² + 5) · (Ft/fd − 1 + cc/r²)) FOR FAILURE BY TENSION.
E = 28,000,000 LBS. PER SQUARE INCH,
Fc = +48,000 " " " "
Ft = −34,000 " " " "
cc/r² = 1.5 (ASSUMED), FOR UPPER LIMIT CURVE, FAILURE BY COMPRESSION.
cc/r² = 3 AND 3.5 (ASSUMED), FOR LOWER LIMIT CURVES. FAILURE BY TENSION.
fd = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
FOR FAILURE BY COMPRESSIVE STRESS.
FOR FAILURE BY TENSILE STRESS.
cc/r² = 1.5
cc/r² = 3
cc/r² = 3.5
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
fd IN POUNDS PER SQUARE INCH.
360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO l/r

FIG. 24

The record numbers of the results are noted on the diagram. Tetmajer's tests (non-axial), Nos. 27 to 34 inclusive, have not been plotted by the writer, as they were on T-bars, 100 mm. × 100 mm. × 10 mm., with the load imposed eccentrically in the line of the greatest radius of gyration, while the bars all failed in the direction of the least radius of gyration. It may be remarked that the amount of the intentional eccentricity in these tests (Nos. 27 to 34) was not sufficiently great in any case to ensure that failure would occur by flexure in the plane of the greatest radius of gyration, and the "accidental" equivalent eccentricity in the plane of the least radius of gyration was evidently the controlling factor.

The examples on the diagram have been selected for the sole reason that they were most nearly uniform in the character of section, method of loading, and amount of eccentricity.

Fourteen of the tests were on pairs of angle bars riveted together to form a T-section, and the remaining four were on channel bars.

In each case the eccentricity of loading was such that the tables of the T-sections or channels were subjected to the greatest compressive stress, and in consequence, the value of $\frac{c\,e}{r^2}$ was much less when referred to the table faces than when referred to the points of the legs of the T-sections or channels, rendering it necessary to use two different values of $\frac{c\,\varepsilon}{r^2}$, when plotting the curves by the writer's formula, one being for failure by compressive stress in the table faces, and the other for failure by tensile stress in the points of the legs.

The values of $\frac{c\,e}{r^2}$ deduced from the sections and the actual value of intentional eccentricity were as follows:

For results.	$\frac{c\,e}{r^2}$ for Compressive stress.	$\frac{c\,e}{r^2}$ for Tensile stress
Nos. 14, 15, 16, 17 and 18.....	1.256	3.125
Nos. 56 and 58.....	1.423	3.440
Nos. 51 and 52.....	1.627 and 1.651	3.420 and 3.470

The curves plotted on the diagram have been calculated from values:

	For Compressive stress.	For Tensile stress.
$\frac{c\,\varepsilon}{r^2}$	1.5	3 and 3.5

and the characteristic agreement is sufficiently satisfactory when it is

remembered that we are dealing with tests of ultimate strength, and that each result plotted on the diagram represents the average of two tests.

The other non-axial tests made by Tetmajer had values $\frac{c\,e}{r^2}$ of too little uniformity to allow of them being used to illustrate the influence

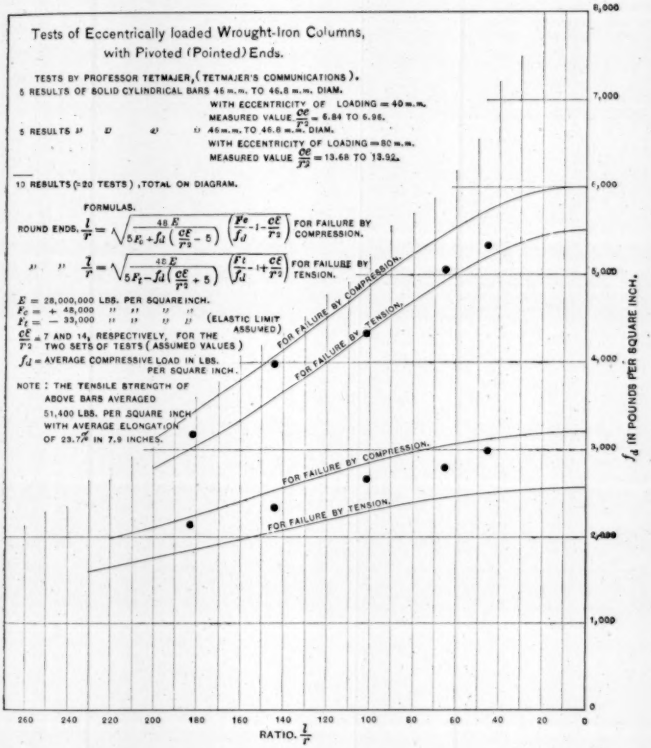

FIG. 25.

of eccentric loading combined with the influence of varying ratios of $\frac{l}{r}$. They do, however, most clearly show the serious loss of strength caused by eccentric loading.

Fig. 25.—Ten results, representing twenty tests by Professor Tetmajer on pivot-ended wrought-iron columns under intentionally

eccentric loads. Each result plotted is the average of two tests.* These results are especially interesting, notwithstanding their small number, as the tests were on solid, round bars of one make of iron throughout, and the observed and intentional value of eccentricity of loading was very large, and thus greatly overshadowed accidental conditions.

Ten of the experiments were made with a value of $\frac{c\,e}{r^2} = 6.84$ to 6.96, and the other ten with a value of $\frac{c\,e}{r^2} = 13.68$ to 13.92. The ultimate strength of the iron under direct tension is given by Tetmajer as 51 400 lbs. per square inch, with an ultimate elongation of 23.7% in 200 mm. ($7\frac{7}{8}$ ins. nearly).

In this diagram the vertical scale of the load has been made much larger than in the other diagrams in order to emphasize the difference in the results, and to show their characteristic agreement with the writer's calculated curves.

In each of the two sets of experiments plotted on this diagram the upper curve indicates the loads causing a maximum compressive stress of 48 000 lbs. per square inch, and the lower curve indicates the loads causing a maximum tensile fiber stress of 33 000 lbs. per square inch.

The curves for the upper set of tests have been plotted for a value of $\frac{c\,e}{r^2} = 7$, and those for the lower set for a value of 14.

Wrought Iron.—Fixed Ends.

Fig. 26.—Twenty-five tests by Mr. Christie on fixed-ended columns of angle bars.† Reference has already been made to the difficulty of realizing fixity of ends in columns, and Mr. Christie remarks in his description of his experiments, that the lengths of the fixed-ended struts were measured between the clamps, whereas the point of absolute fixing probably occurred at some place within the clamps, and the values given for the ratio $\frac{l}{r}$ would then be too low.

If Mr. Christie's suggestion were adopted and a somewhat higher value assumed for the value of $\frac{l}{r}$, the already fairly satisfactory agreement between the writer's lower-limit curve and the lower results of the experiments would be still more pronounced.

* "Tetmajer's Communications."
† *Transactions*, Am. Soc. C. E., Vol. xiii.

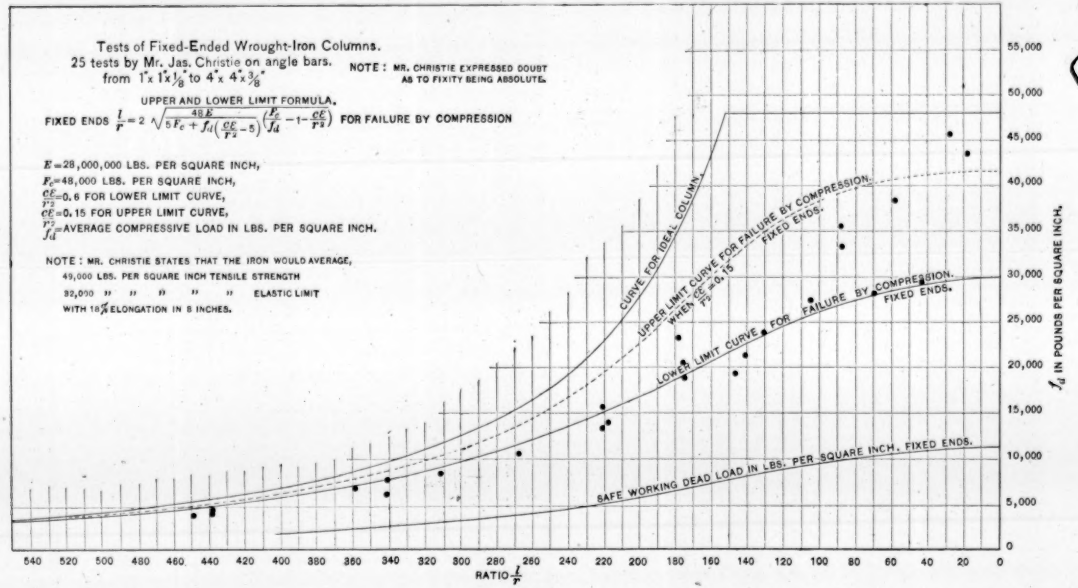
Tests of Fixed-Ended Wrought-Iron Columns.
25 tests by Mr. Jas. Christie on angle bars.
from 1" x 1" x 1/8" to 4" x 4" x 3/8"
NOTE: MR. CHRISTIE EXPRESSED DOUBT AS TO FIXITY BEING ABSOLUTE.
UPPER AND LOWER LIMIT FORMULA.
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{48E}{5F_c + f_d\left(\frac{cc}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{cc}{r^2}\right)}$ FOR FAILURE BY COMPRESSION
E=28,000,000 LBS. PER SQUARE INCH,
F_c=48,000 LBS. PER SQUARE INCH,
$\frac{cc}{r^2}$=0.6 FOR LOWER LIMIT CURVE,
$\frac{cc}{r^2}$=0.15 FOR UPPER LIMIT CURVE,
f_d=AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
NOTE: MR. CHRISTIE STATES THAT THE IRON WOULD AVERAGE,
49,000 LBS. PER SQUARE INCH TENSILE STRENGTH
32,000 " " " " " ELASTIC LIMIT
WITH 18% ELONGATION IN 8 INCHES.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS. WHEN $\frac{cc}{r^2}$=0.15
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
SAFE WORKING DEAD LOAD IN LBS. PER SQUARE INCH. FIXED ENDS.
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 26.

These experiments by Mr. Christie are the only series on fixed-ended columns of which the writer is aware. It is to be noted that Mr. Christie's Test No. 174, with ratio $\frac{l}{r} = 118$ (maximum load imposed being 24 050 lbs. per square inch), is not shown on the diagram, as failure did not take place.

Wrought Iron.—Flat Ends.

Figs. 27, 28, 29, 30 and 31.—Tests of 240 flat-ended wrought-iron columns. The diagrams are self-explanatory, as far as possible. Attention is directed to the low results evidenced in these tests of flat-ended columns when of considerable length, owing to their rotating on their ends. This mode of failure was found by Mr. Christie in his tests of flat-ended struts, as always occurring in the longest struts, and never in the shortest.

Fig. 32.—Results of seventy-nine tests of flat-ended wrought-iron columns of large size, of various sections, and by various experimenters. The writer has not been able to refer to the original records in every case, but the sources from which the information has been obtained are acknowledged on the diagram.

With regard to the Keystone columns tested by Mr. Bouscaren, only the four which were riveted through the projecting flanges, similarly to a Phœnix column, are recorded on the diagram.

Fig. 33.—Thirteen tests of wrought-iron flat-ended columns, by Professor Bauschinger.* These columns, which had flat ends, are not open to the objections raised against the use, as a basis of reference, of Bauschinger's pivot-ended columns.

Seven results, representing thirteen tests of flat-ended wrought-iron columns, by Professor Tetmajer†.

Twenty tests of flat-ended wrought-iron columns, by the late C. A. Marshall, M.Am. Soc. C. E ‡ Reference to Mr. Marshall's tests will be made subsequently.

Forty results = forty-six tests, total on diagram.

Fig. 34.—This diagram is compounded of Figs. 26 to 33, inclusive, and shows the results of 390 experiments (384 results), on wrought-iron columns, 25 of the tests being of fixed-ended columns, and 365 being of flat-ended columns. This diagram shows practi-

* Fünfzehntes Heft, *Mittheilung*, xviii.
† "Tetmajer's Communications."
‡ *Transactions*, Am. Soc. C. E., Vol. xvii

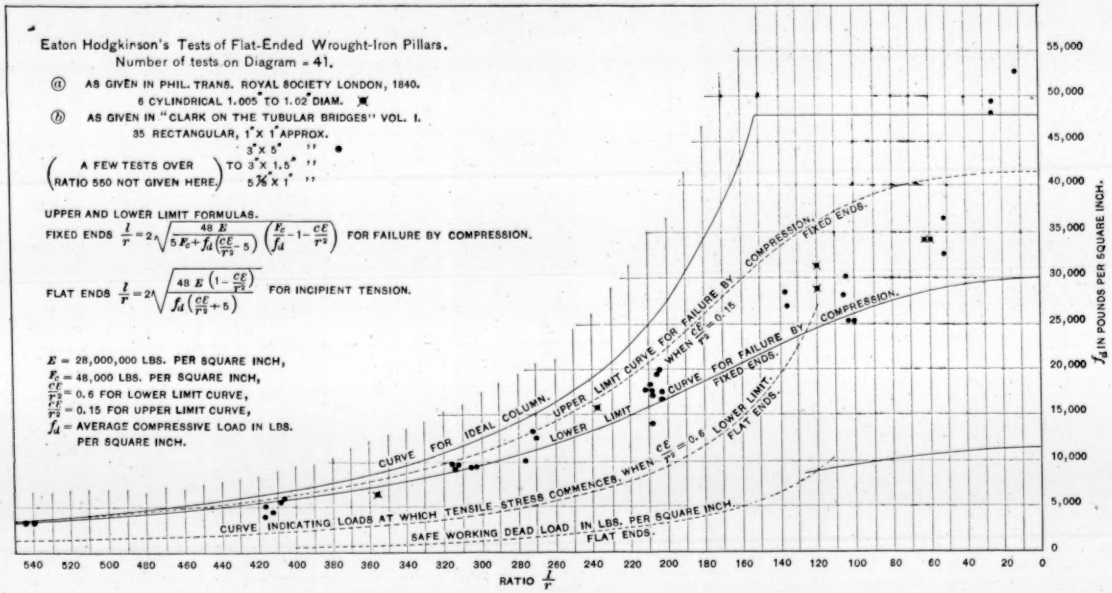
Eaton Hodgkinson's Tests of Flat-Ended Wrought-Iron Pillars.
Number of tests on Diagram = 41.
(a) AS GIVEN IN PHIL. TRANS. ROYAL SOCIETY LONDON, 1840.
6 CYLINDRICAL 1.005" TO 1.02" DIAM.
(b) AS GIVEN IN "CLARK ON THE TUBULAR BRIDGES" VOL. I.
35 RECTANGULAR, 1" X 1" APPROX.
3" X 5" "
(A FEW TESTS OVER RATIO 550 NOT GIVEN HERE.) TO 3" X 1.5" "
5⅞" X 1" "
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r}=2\sqrt{\frac{48\,E}{5F_c+f_d\left(\frac{c\epsilon}{r^2}-5\right)}\left(\frac{F_c}{f_d}-1-\frac{c\epsilon}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS $\frac{l}{r}=2\sqrt{\frac{48\,E\left(1-\frac{c\epsilon}{r^2}\right)}{f_d\left(\frac{c\epsilon}{r^2}+5\right)}}$ FOR INCIPIENT TENSION.
E = 28,000,000 LBS. PER SQUARE INCH,
F_c = 48,000 LBS. PER SQUARE INCH,
$\frac{c\epsilon}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE,
$\frac{c\epsilon}{r^2}$ = 0.15 FOR UPPER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. WHEN $\frac{c\epsilon}{r^2}$ = 0.15 FIXED ENDS.
CURVE INDICATING LOADS AT WHICH TENSILE STRESS COMMENCES. WHEN $\frac{c\epsilon}{r^2}$ = 0.6 LOWER LIMIT. FLAT ENDS.
SAFE WORKING DEAD LOAD IN LBS. PER SQUARE INCH. FLAT ENDS.
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 27

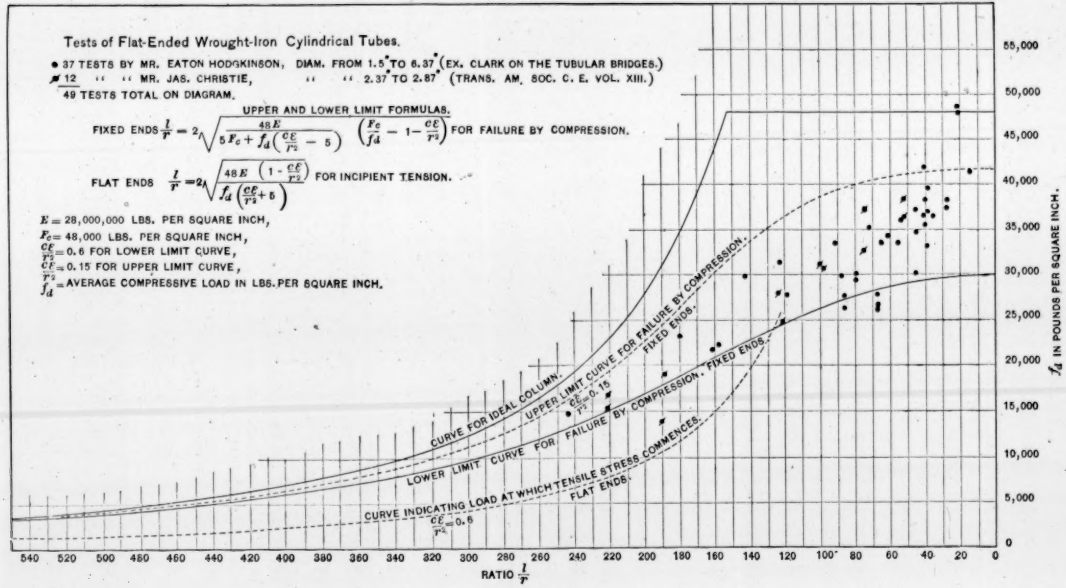
Tests of Flat-Ended Wrought-Iron Cylindrical Tubes.
• 37 TESTS BY MR. EATON HODGKINSON, DIAM. FROM 1.5" TO 6.37" (EX. CLARK ON THE TUBULAR BRIDGES.)
12 " " MR. JAS. CHRISTIE, " " 2.37" TO 2.87" (TRANS. AM. SOC. C. E. VOL. XIII.)
49 TESTS TOTAL ON DIAGRAM.
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{48E}{5F_c + f_d\left(\frac{c\epsilon}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{c\epsilon}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS $\frac{l}{r} = 2\sqrt{\frac{48E\left(1 - \frac{c\epsilon}{r^2}\right)}{f_d\left(\frac{c\epsilon}{r^2} + 5\right)}}$ FOR INCIPIENT TENSION.
E = 28,000,000 LBS. PER SQUARE INCH,
F_c = 48,000 LBS. PER SQUARE INCH,
$\frac{c\epsilon}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE,
$\frac{c\epsilon}{r^2}$ = 0.15 FOR UPPER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS. $\frac{c\epsilon}{r^2}$ = 0.15
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
CURVE INDICATING LOAD AT WHICH TENSILE STRESS COMMENCES. FLAT ENDS. $\frac{c\epsilon}{r^2}$ = 0.6
55,000 50,000 45,000 40,000 35,000 30,000 25,000 20,000 15,000 10,000 5,000 0
f_d IN POUNDS PER SQUARE INCH.
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 28.

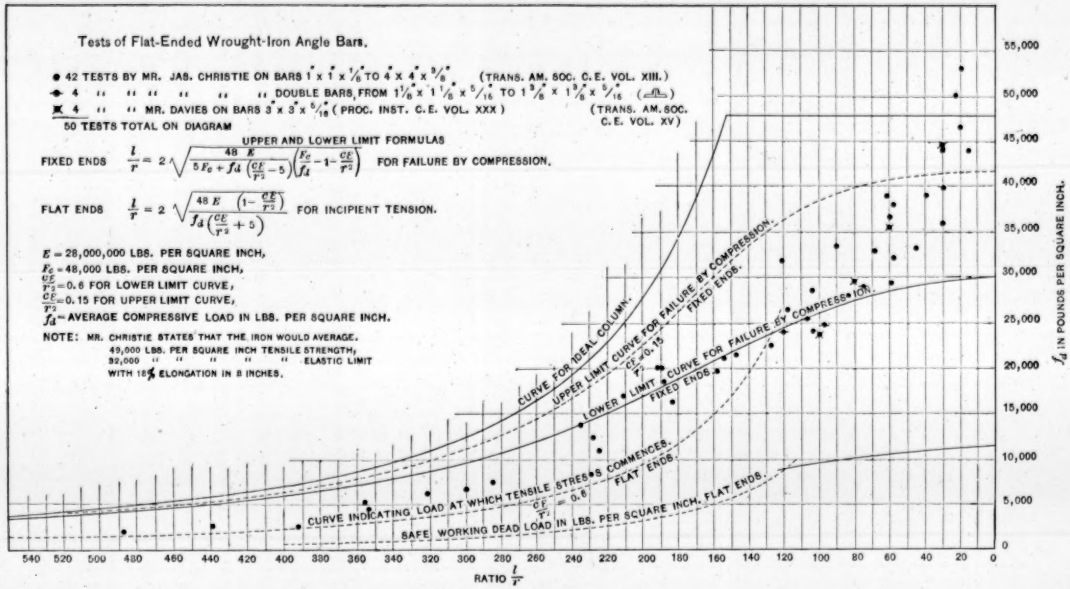
Tests of Flat-Ended Wrought-Iron Angle Bars.
● 42 TESTS BY MR. JAS. CHRISTIE ON BARS 1" x 1" x 1/8" TO 4" x 4" x 3/8" (TRANS. AM. SOC. C.E. VOL. XIII.)
4 " " " " " " DOUBLE BARS, FROM 1 1/8" x 1 1/8" x 5/16" TO 1 3/8" x 1 3/8" x 5/16" (TRANS. AM. SOC. C.E. VOL. XV)
4 " " MR. DAVIES ON BARS 3" x 3" x 6/16" (PROC. INST. C.E. VOL. XXX)
50 TESTS TOTAL ON DIAGRAM
UPPER AND LOWER LIMIT FORMULAS
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{48\ E}{5F_c + f_d\left(\frac{Cc}{r^2} - 5\right)\left(\frac{F_c}{f_d} - 1 - \frac{Cc}{r^2}\right)}}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS $\frac{l}{r} = 2\sqrt{\frac{48\ E\ \left(1 - \frac{Cc}{r^2}\right)}{f_d\left(\frac{Cc}{r^2} + 5\right)}}$ FOR INCIPIENT TENSION.
E = 28,000,000 LBS. PER SQUARE INCH,
F_c = 48,000 LBS. PER SQUARE INCH,
$\frac{Cc}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE,
$\frac{Cc}{r^2}$ = 0.15 FOR UPPER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
NOTE: MR. CHRISTIE STATES THAT THE IRON WOULD AVERAGE.
49,000 LBS. PER SQUARE INCH TENSILE STRENGTH,
32,000 " " " " " ELASTIC LIMIT
WITH 18% ELONGATION IN 8 INCHES.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
$\frac{Cc}{r^2}$ = 0.15
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
CURVE INDICATING LOAD AT WHICH TENSILE STRESS COMMENCES. FLAT ENDS.
$\frac{Cc}{r^2}$ = 0.6
SAFE WORKING DEAD LOAD IN LBS. PER SQUARE INCH. FLAT ENDS.
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 29.

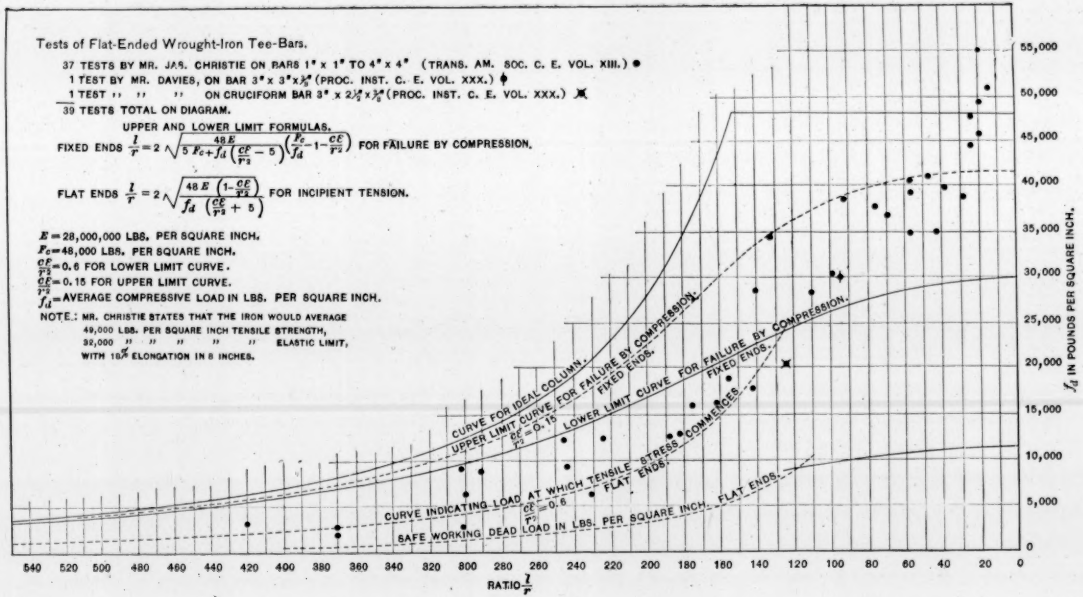
Tests of Flat-Ended Wrought-Iron Tee-Bars.
37 TESTS BY MR. JAS. CHRISTIE ON BARS 1" x 1" TO 4" x 4" (TRANS. AM. SOC. C. E. VOL. XIII.)
1 TEST BY MR. DAVIES, ON BAR 3" x 3" x 5/8" (PROC. INST. C. E. VOL. XXX.)
1 TEST " " " ON CRUCIFORM BAR 3" x 2 1/2" x 5/8" (PROC. INST. C. E. VOL. XXX.)
39 TESTS TOTAL ON DIAGRAM.
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{48E}{5F_c + f_d\left(\frac{cf}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{cf}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS $\frac{l}{r} = 2\sqrt{\frac{48E\left(1 - \frac{cf}{r^2}\right)}{f_d\left(\frac{cf}{r^2} + 5\right)}}$ FOR INCIPIENT TENSION.
E = 28,000,000 LBS. PER SQUARE INCH.
F_c = 48,000 LBS. PER SQUARE INCH.
$\frac{cf}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE.
$\frac{cf}{r^2}$ = 0.15 FOR UPPER LIMIT CURVE.
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
NOTE.: MR. CHRISTIE STATES THAT THE IRON WOULD AVERAGE
49,000 LBS. PER SQUARE INCH TENSILE STRENGTH,
32,000 " " " " " ELASTIC LIMIT,
WITH 18% ELONGATION IN 8 INCHES.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
$\frac{cf}{r^2}$ = 0.15
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
FLAT ENDS.
STRESS COMMENCES.
CURVE INDICATING LOAD AT WHICH TENSILE $\frac{cf}{r^2}$ = 0.6
SAFE WORKING DEAD LOAD IN LBS. PER SQUARE INCH. FLAT ENDS.
f_d IN POUNDS PER SQUARE INCH.
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 30.

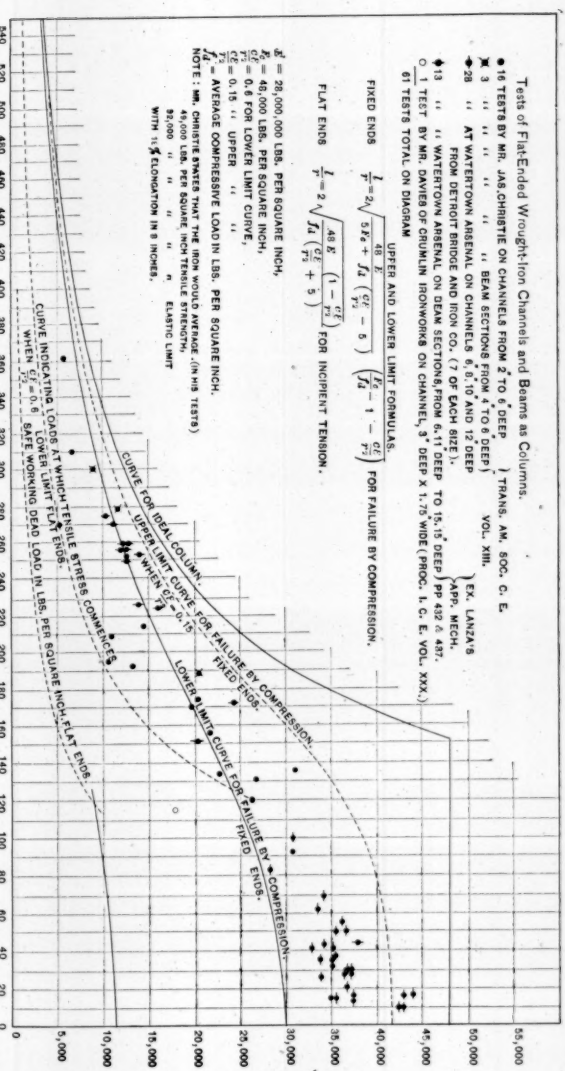
Tests of Flat-Ended Wrought-Iron Channels and Beams as Columns.
● 16 TESTS BY MR. JAS. CHRISTIE ON CHANNELS FROM 2" TO 6" DEEP } TRANS. AM. SOC. C. E.
✖ 3 " " " " " BEAM SECTIONS FROM 4" TO 6" DEEP } VOL. XIII.
◆ 28 " " AT WATERTOWN ARSENAL ON CHANNELS 6", 8", 10" AND 12" DEEP } EX. LANZA'S
FROM DETROIT BRIDGE AND IRON CO. (7 OF EACH SIZE). } APP. MECH.
◆ 13 " " WATERTOWN ARSENAL ON BEAM SECTIONS, FROM 6" 11" DEEP TO 15" 15" DEEP } PP 432 & 437.
○ 1 TEST BY MR. DAVIES OF CRUMLIN IRONWORKS ON CHANNEL, 3" DEEP X 1.75 WIDE (PROC. I. C. E. VOL. XXX.)
61 TESTS TOTAL ON DIAGRAM
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r}=2\sqrt{\frac{48\ E}{5f_E+f_A\left(\frac{c\ell}{r^2}-5\right)}\left(\frac{f_E}{f_A}-1-\frac{c\ell}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS $\frac{l}{r}=2\sqrt{f_A\left(\frac{48\ E}{f_E}\left(1-\frac{c\ell}{r^2}\right)+5\right)}$ FOR INCIPIENT TENSION.
f_E = 28,000,000 LBS. PER SQUARE INCH.
f_E = 40,000 LBS. PER SQUARE INCH.
$\frac{c\ell}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE.
$\frac{c\ell}{r^2}$ = 0.15 " " UPPER " "
f_A = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
NOTE: MR. CHRISTIE STATES THAT THE IRON WOULD AVERAGE (IN HIS TESTS)
49,000 LBS. PER SQUARE INCH TENSILE STRENGTH,
35,000 " " " " ELASTIC LIMIT
WITH 15% ELONGATION IN 8 INCHES.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
WHEN $\frac{c\ell}{r^2}$ = 0.15
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
CURVE INDICATING LOADS AT WHICH TENSILE STRESS COMMENCES.
LOWER LIMIT FLAT ENDS.
WHEN $\frac{c\ell}{r^2}$ = 0.6
SAFE WORKING DEAD LOAD IN LBS. PER SQUARE INCH FLAT ENDS.
f_A IN POUNDS PER SQUARE INCH.
RATIO $\frac{l}{r}$
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540
0 5,000 10,000 15,000 20,000 25,000 30,000 35,000 40,000 45,000 50,000 55,000

Fig. 81

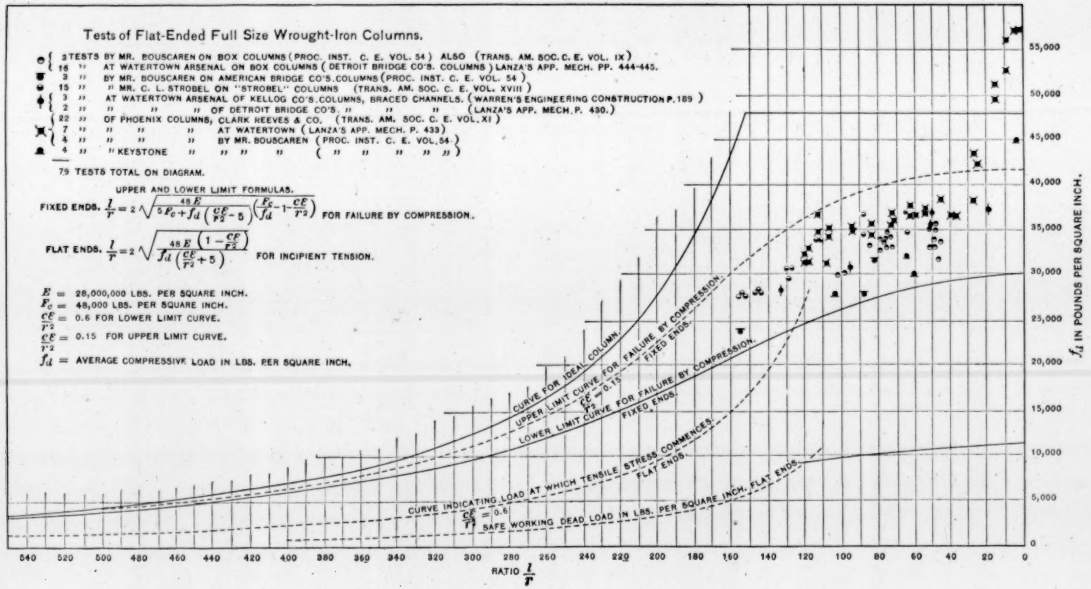
Tests of Flat-Ended Full Size Wrought-Iron Columns.
3 TESTS BY MR. BOUSCAREN ON BOX COLUMNS (PROC. INST. C. E. VOL. 54) ALSO (TRANS. AM. SOC. C. E. VOL. IX)
16 " AT WATERTOWN ARSENAL ON BOX COLUMNS (DETROIT BRIDGE CO'S. COLUMNS) LANZA'S APP. MECH. PP. 444-445.
3 " BY MR. BOUSCAREN ON AMERICAN BRIDGE CO'S. COLUMNS (PROC. INST. C. E. VOL. 54)
15 " " MR. C. L. STROBEL ON "STROBEL" COLUMNS (TRANS. AM. SOC. C. E. VOL. XVIII)
3 " AT WATERTOWN ARSENAL OF KELLOG CO'S. COLUMNS, BRACED CHANNELS. (WARREN'S ENGINEERING CONSTRUCTION P. 189)
2 " " " " OF DETROIT BRIDGE CO'S. " " " (LANZA'S APP. MECH. P. 430.)
22 " OF PHOENIX COLUMNS, CLARK REEVES & CO. (TRANS. AM. SOC. C. E. VOL. XI)
7 " " " " AT WATERTOWN (LANZA'S APP. MECH. P. 433)
4 " " " " BY MR. BOUSCAREN (PROC. INST. C. E. VOL. 54.)
4 " " KEYSTONE " " " " (" " " " " ")
79 TESTS TOTAL ON DIAGRAM.
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS. $\frac{l}{r} = 2\sqrt{\frac{48E}{5F_c + f_d\left(\frac{cf}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{cf}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS. $\frac{l}{r} = 2\sqrt{\frac{48E\left(1 - \frac{cf}{r^2}\right)}{f_d\left(\frac{cf}{r^2} + 5\right)}}$ FOR INCIPIENT TENSION.
E = 28,000,000 LBS. PER SQUARE INCH.
F_c = 48,000 LBS. PER SQUARE INCH.
$\frac{cf}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE.
$\frac{cf}{r^2}$ = 0.15 FOR UPPER LIMIT CURVE.
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS. $\frac{cf}{r^2} = 0.15$
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
CURVE INDICATING LOAD AT WHICH TENSILE STRESS COMMENCES. FLAT ENDS.
$\frac{cf}{r^2} = 0.6$ SAFE WORKING DEAD LOAD IN LBS. PER SQUARE INCH. FLAT ENDS.
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 32.

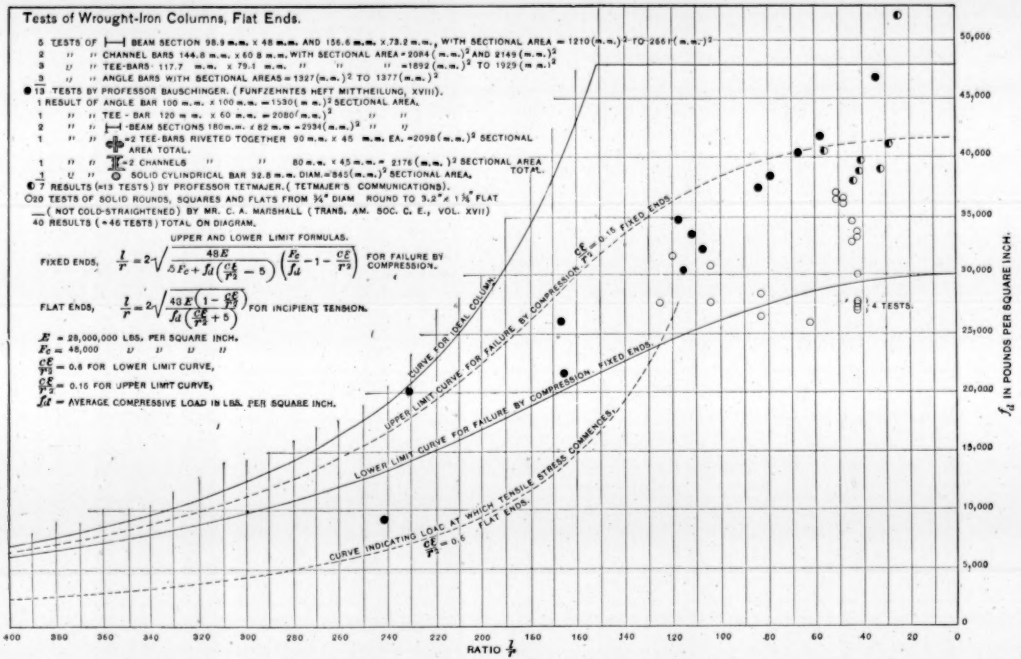
Tests of Wrought-Iron Columns, Flat Ends.
5 TESTS OF ⊢ BEAM SECTION 98.9 m.m. x 48 m.m. AND 156.6 m.m. x 73.2 m.m., WITH SECTIONAL AREA = 1210 (m.m.)² TO 2661 (m.m.)²
2 " " CHANNEL BARS 144.8 m.m. x 60.8 m.m. WITH SECTIONAL AREA = 2084 (m.m.)² AND 2149 (m.m.)²
3 " " TEE-BARS 117.7 m.m. x 79.1 m.m. " " " " =1892 (m.m.)² TO 1929 (m.m.)²
3 " " ANGLE BARS WITH SECTIONAL AREAS = 1327 (m.m.)² TO 1377 (m.m.)²
● 13 TESTS BY PROFESSOR BAUSCHINGER. (FUNFZEHNTES HEFT MITTHEILUNG, XVIII).
1 RESULT OF ANGLE BAR 100 m.m. x 100 m.m. =1530 (m.m.)² SECTIONAL AREA.
1 " " TEE - BAR 120 m.m. x 60 m.m. =2080 (m.m.)² " "
2 " " ⊢ BEAM SECTIONS 180 m.m. x 82 m.m. =2934 (m.m.)² " "
1 " " ✚ =2 TEE-BARS RIVETED TOGETHER 90 m.m. x 45 m.m. EA. =2098 (m.m.)² SECTIONAL AREA TOTAL.
1 " " ⊓⊔ =2 CHANNELS " " 80 m.m. x 45 m.m. = 2176 (m.m.)² SECTIONAL AREA TOTAL.
1 " " ○ SOLID CYLINDRICAL BAR 32.8 m.m. DIAM. =845 (m.m.)² SECTIONAL AREA.
◐ 7 RESULTS (=13 TESTS) BY PROFESSOR TETMAJER. (TETMAJER'S COMMUNICATIONS).
○20 TESTS OF SOLID ROUNDS, SQUARES AND FLATS FROM ¾" DIAM ROUND TO 3.2" x 1¾" FLAT (NOT COLD-STRAIGHTENED) BY MR. C. A. MARSHALL (TRANS. AM. SOC. C. E., VOL. XVII)
40 RESULTS (=46 TESTS) TOTAL ON DIAGRAM.
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS, $\frac{l}{r} = 2\sqrt{\frac{48E}{5F_c + f_d\left(\frac{cE}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{cE}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS, $\frac{l}{r} = 2\sqrt{\frac{48E\left(1 - \frac{cE}{r^2}\right)}{f_d\left(\frac{cE}{r^2} + 5\right)}}$ FOR INCIPIENT TENSION.
E = 28,000,000 LBS. PER SQUARE INCH.
F_c = 48,000 " " " "
cE/r² = 0.6 FOR LOWER LIMIT CURVE,
cE/r² = 0.15 FOR UPPER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
cE/r² = 0.15 FIXED ENDS.
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
CURVE INDICATING LOAD AT WHICH TENSILE STRESS COMMENCES.
cE/r² = 0.6 FLAT ENDS.
4 TESTS.
f_d IN POUNDS PER SQUARE INCH.
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO l/r

FIG. 33.

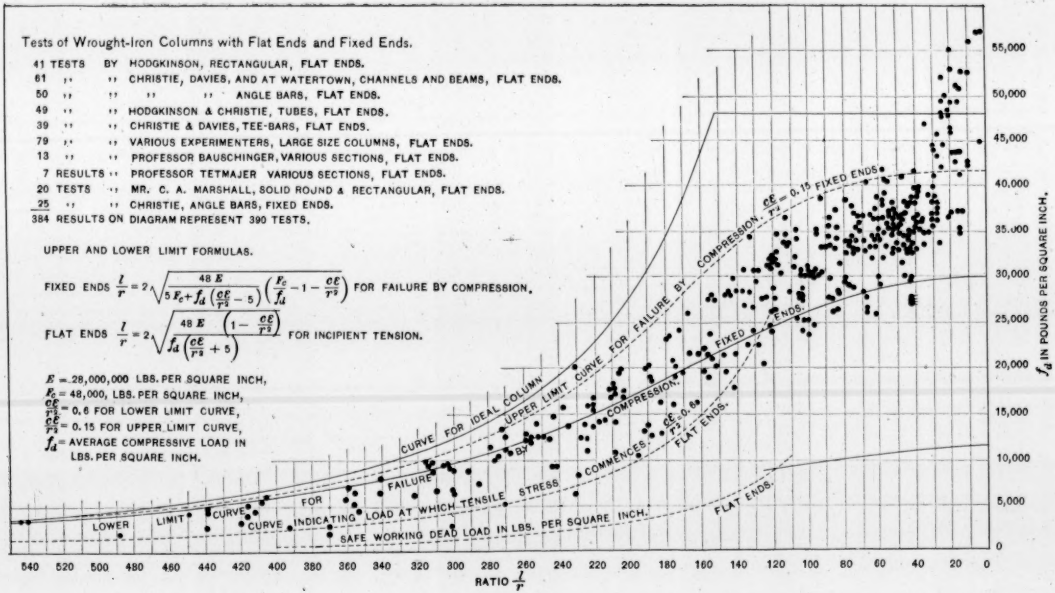
Tests of Wrought-Iron Columns with Flat Ends and Fixed Ends.
41 TESTS BY HODGKINSON, RECTANGULAR, FLAT ENDS.
61 " " CHRISTIE, DAVIES, AND AT WATERTOWN, CHANNELS AND BEAMS, FLAT ENDS.
50 " " " " ANGLE BARS, FLAT ENDS.
49 " " HODGKINSON & CHRISTIE, TUBES, FLAT ENDS.
39 " " CHRISTIE & DAVIES, TEE-BARS, FLAT ENDS.
79 " " VARIOUS EXPERIMENTERS, LARGE SIZE COLUMNS, FLAT ENDS.
13 " " PROFESSOR BAUSCHINGER, VARIOUS SECTIONS, FLAT ENDS.
7 RESULTS " PROFESSOR TETMAJER VARIOUS SECTIONS, FLAT ENDS.
20 TESTS " MR. C. A. MARSHALL, SOLID ROUND & RECTANGULAR, FLAT ENDS.
25 " " CHRISTIE, ANGLE BARS, FIXED ENDS.
384 RESULTS ON DIAGRAM REPRESENT 390 TESTS.
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{48E}{5F_c + f_d\left(\frac{cc}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{cc}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS $\frac{l}{r} = 2\sqrt{\frac{48E\left(1 - \frac{cc}{r^2}\right)}{f_d\left(\frac{cc}{r^2} + 5\right)}}$ FOR INCIPIENT TENSION.
E = 28,000,000 LBS. PER SQUARE INCH,
F_c = 48,000, LBS. PER SQUARE INCH,
$\frac{cc}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE,
$\frac{cc}{r^2}$ = 0.15 FOR UPPER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. $\frac{cc}{r^2}$ = 0.15 FIXED ENDS.
FIXED ENDS.
COMPRESSION. $\frac{cc}{r^2}$ = 0.6
FLAT ENDS.
LOWER LIMIT CURVE INDICATING FOR FAILURE LOAD AT WHICH TENSILE STRESS COMMENCES.
FLAT ENDS.
SAFE WORKING DEAD LOAD IN LBS. PER SQUARE INCH.
RATIO $\frac{l}{r}$
f_d IN POUNDS PER SQUARE INCH.
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0

FIG. 34.

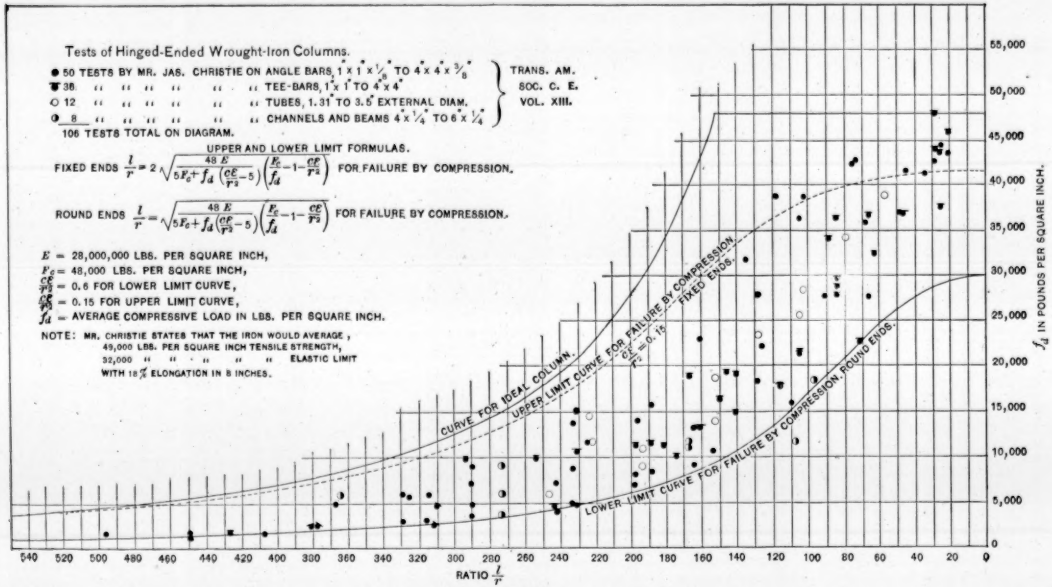
Tests of Hinged-Ended Wrought-Iron Columns.
50 TESTS BY MR. JAS. CHRISTIE ON ANGLE BARS, 1" x 1" x 1/8" TO 4" x 4" x 3/8"
38 " " " " " " TEE-BARS, 1" x 1" TO 4" x 4"
12 " " " " " " TUBES, 1.31" TO 3.5" EXTERNAL DIAM.
8 " " " " " " CHANNELS AND BEAMS 4" x 1/4" TO 6" x 1/4"
106 TESTS TOTAL ON DIAGRAM.
TRANS. AM. SOC. C. E. VOL. XIII.
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{48E}{5F_c + f_d\left(\frac{cE}{Fs}-5\right)}\left(\frac{F_c}{f_d}-1-\frac{cE}{Fs}\right)}$ FOR FAILURE BY COMPRESSION.
ROUND ENDS $\frac{l}{r} = \sqrt{\frac{48E}{5F_c + f_d\left(\frac{cE}{Fs}-5\right)}\left(\frac{F_c}{f_d}-1-\frac{cE}{Fs}\right)}$ FOR FAILURE BY COMPRESSION.
E = 28,000,000 LBS. PER SQUARE INCH,
F_c = 48,000 LBS. PER SQUARE INCH,
$\frac{cE}{Fs}$ = 0.6 FOR LOWER LIMIT CURVE,
$\frac{cE}{Fs}$ = 0.15 FOR UPPER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
NOTE: MR. CHRISTIE STATES THAT THE IRON WOULD AVERAGE,
49,000 LBS. PER SQUARE INCH TENSILE STRENGTH,
32,000 " " " " " ELASTIC LIMIT
WITH 18% ELONGATION IN 8 INCHES.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. $\frac{cE}{Fs}$ = 0.15 FIXED ENDS.
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. ROUND ENDS.
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 35

cally all the available experimental data on the strength of wrought-iron columns with flat ends.

The weakness of flat-ended columns in the higher ratios of $\frac{l}{r}$ is most clearly evident in this diagram. The high strength shown by a number of columns of ratio $\frac{l}{r} = 30$, and under, is also a striking feature, and it would appear that columns longer than 30 radii of gyration ($7\frac{1}{2}$ diameters in cylindrical solid bars) cannot be expected to develop higher strength due to plastic yielding under compression, and that column action begins to come into play more definitely at about ratios $\frac{l}{r} = 30$ to 40.

It is evident that in very short columns the useful ultimate strength must be measured by the elastic limit under compression, and this may not have been noted carefully in some of the experiments, but beyond the ratio 30, the influence of column length appears to become very strongly marked.

Wrought Iron.—Hinged or Pin Ends.

Fig. 35.—One hundred and six tests of hinged-ended angles, tees, tubes, channels and beams, by Mr. Christie.* In Mr. Christie's account of his tests, he remarks: "The hinged-ended tests varied all the way from the value of round-ended up to flat-ended."

The writer would extend this to, "the hinged-ended tests varied all the way from the lower values for round-ended up to the higher values for flat or fixed-ended."

The truth of this is apparent on referring to the diagram on which the writer has plotted the lower-limit curve for round-ended columns with $\frac{c\,\varepsilon}{r^2} = 0.6$, as on Figs. 20, 21 and 22, and the upper-limit dotted curve for fixed-ended columns with $\frac{c\,\varepsilon}{r^2} = 0.15$, as on Figs. 26 to 33.

The high results for ratios under 30 are again evident, and are similar to the results for flat-ended columns.

The tests on hinged-ended angles, given in Table No. 3 of Mr. Christie's paper, are all plotted on the diagram, excepting those which were not carried to the point of failure. In addition to these, the writer has also plotted five of Mr. Christie's extra tests on some of these angle bars, as given on pages 113 and 114 of Mr. Christie's paper.

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These are as follows:

1. First test, with 2-in. balls and sockets, apparently central, on the same bar as in Experiment No. 80, ratio $\frac{l}{r} = 164$, for which the result of the second test after moving the ends 0.06 in. from the original position was given in Mr. Christie's Table No. 3. The first test, apparently central, gave an ultimate strength of 19 400 lbs. (9 200 lbs. per square inch), and the second test, after moving the specimen 0.06 in., gave an ultimate strength of 27 850 lbs. (13 199 lbs. per square inch), Both of these results are on the diagram.

2 and 3. The second and third tests on the same bar as in Experiment No. 117, ratio $\frac{l}{r} = 290$. The first test (No. 117) on this bar was with 2-in. balls and sockets (7 020 lbs. per square inch), the second with 1-in. balls and sockets (3 525 lbs. per square inch), and the third with 2-in. pins. (4 790 lbs. per square inch).

4. The second test on the same bar as in Experiment No. 118, ratio $\frac{l}{r} = 233$. In the case of this bar, the first test result, with ends apparently central, is given in Mr. Christie's Table No. 3, the strength being 16 700 lbs. (8 650 lbs. per square inch), and in the second test the ends were moved 0.06 in., with the result that the strength was 26 450 lbs. (13 700 lbs. per square inch).

5. The first test on the same bar as in Experiment No. 119, ratio $\frac{l}{r} = 189$, giving a strength of 20 150 lbs. (8 340 lbs. per square inch). In the case of this bar, the second test, after moving the bar 0.07 in., with resulting ultimate strength of 38 175 lbs. (15 770 lbs. per square inch), is that given in Mr. Christie's Table No. 3.

It is not clear why Mr. Christie should have selected the second tests in the case of Experiments Nos. 80 and 119, for insertion in his Table No. 3, which otherwise referred to struts under apparently central loads.

Mr. Christie's extra tests, of considerable number, on angles, tees, tubes, and H-beams, in addition to the few above mentioned, made it most apparent that the physical axis did not always coincide with the geometric axis, and they also showed very markedly how very sensitive the columns were to apparently insignificant amounts of adjustment, and to variations in sizes of pin or socket.

Fig. 36.—One hundred and twenty-six tests of pin-ended wrought-iron columns of large size by various experimenters. The writer has not had the advantage of referring to the original published records of most of the tests, and he has therefore been compelled to take the records at second hand from the sources acknowledged on the diagram.

With regard to the Detroit Bridge Company's columns, the results have been taken from the paper by Theodore Cooper, M. Am. Soc. C. E.,* from Professor Lanza's "Applied Mechanics," and Professor W. H. Warren's "Engineering Construction."

The results, in many cases, if plotted to the values of $\frac{l}{r}$ given by Mr. Cooper, would differ considerably from the results as plotted on the diagram. This results from the writer's having found that the values of the ratio $\frac{l}{r}$, given by Mr. Cooper, do not agree with the dimensions of the columns as given by Professor Lanza and Professor Warren. The writer, accordingly, recalculated the values of $\frac{l}{r}$, using the lengths as given by Professors Lanza and Warren, and, calculating the radius of gyration from the detailed sections given on Plate XXVII of Mr. Cooper's paper, at the same time comparing carefully the sections as given by Mr. Cooper with the less fully detailed sections given by Professors Lanza and Warren.

It would appear that Mr. Cooper has taken the least radius of gyration in every case, but these columns were pin-ended, in which the radius of gyration at right angles to the pin was frequently greater than that parallel to the pin, and it is this greater radius of gyration which should be used, as the columns were practically flat-ended as regards failure in the direction of the least radius of gyration. The effect of this is that Mr. Cooper's values of $\frac{l}{r}$ are in many cases too great, and, consequently, too great strength has been credited to these high values of $\frac{l}{r}$.

It is worthy of note that a few of these columns did fail by deflecting in a direction parallel to the pins, and acting as flat-ended columns, and not as pin-ended. This draws attention to the necessity for care in proportioning pin-ended columns in which the radius of

* "American Railway Bridges," *Transactions*, Am. Soc. C. E., Vol. xxi.

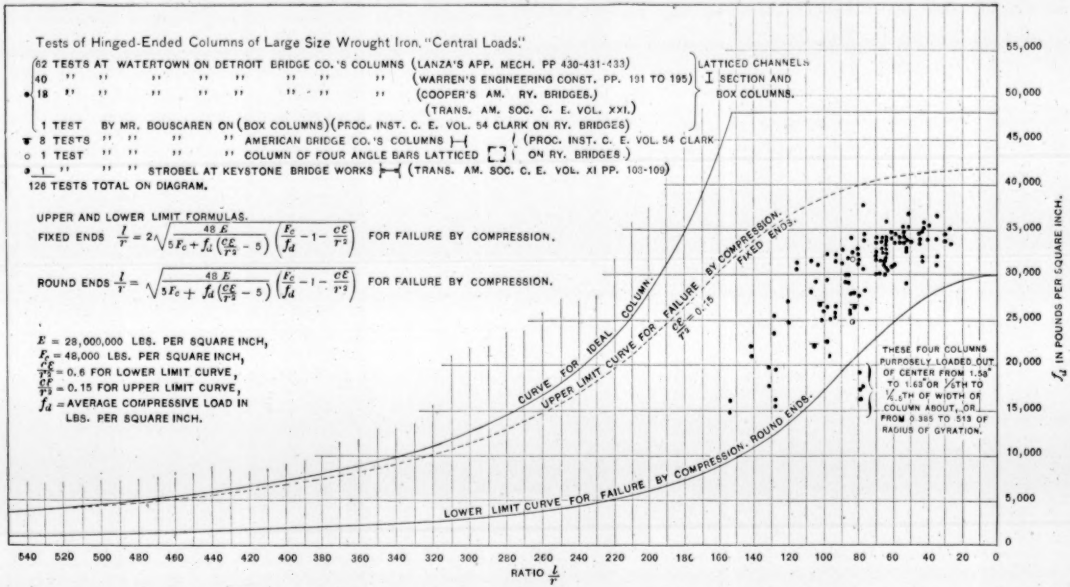
Tests of Hinged-Ended Columns of Large Size Wrought Iron. "Central Loads."
62 TESTS AT WATERTOWN ON DETROIT BRIDGE CO.'S COLUMNS (LANZA'S APP. MECH. PP 430-431-433)
40 " " " " " " " " (WARREN'S ENGINEERING CONST. PP. 191 TO 195)
18 " " " " " " " " (COOPER'S AM. RY. BRIDGES.)
(TRANS. AM. SOC. C. E. VOL. XXI.)
LATTICED CHANNELS I SECTION AND BOX COLUMNS.
1 TEST BY MR. BOUSCAREN ON (BOX COLUMNS) (PROC. INST. C. E. VOL. 54 CLARK ON RY. BRIDGES)
8 TESTS " " " " AMERICAN BRIDGE CO.'S COLUMNS (PROC. INST. C. E. VOL. 54 CLARK
1 TEST " " " " COLUMN OF FOUR ANGLE BARS LATTICED ON RY. BRIDGES)
1 " " " STROBEL AT KEYSTONE BRIDGE WORKS (TRANS. AM. SOC. C. E. VOL. XI PP. 108-109)
126 TESTS TOTAL ON DIAGRAM.
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{48\,E}{5F_c + f_d\left(\frac{c\xi}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{c\xi}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
ROUND ENDS $\frac{l}{r} = \sqrt{\frac{48\,E}{5F_c + f_d\left(\frac{c\xi}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{c\xi}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
E = 28,000,000 LBS. PER SQUARE INCH,
F_c = 48,000 LBS. PER SQUARE INCH,
$\frac{c\xi}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE,
$\frac{c\xi}{r^2}$ = 0.15 FOR UPPER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS. $\frac{c\xi}{r^2} = 0.15$
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. ROUND ENDS.
THESE FOUR COLUMNS PURPOSELY LOADED OUT OF CENTER FROM 1.58" TO 1.63" OR 1/4TH TO 1/4TH OF WIDTH OF COLUMN ABOUT, OR FROM 0.385 TO 513 OF RADIUS OF GYRATION.
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
540 520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 36.

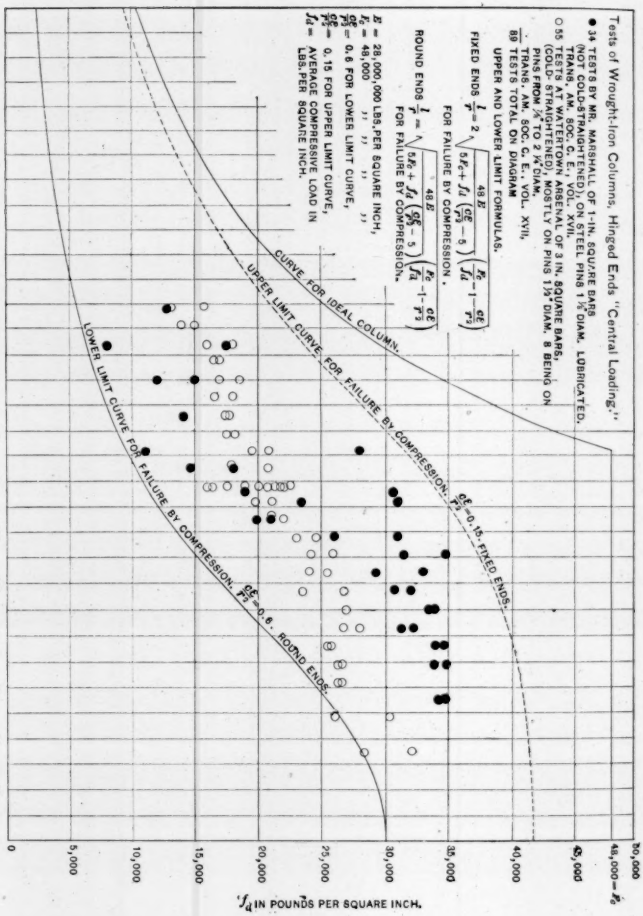
Tests of Wrought-Iron Columns, Hinged Ends. "Central Loading."
● 34 TESTS BY MR. MARSHALL OF 1-IN. SQU-RE BARS (NOT COLD-STRAIGHTENED), ON STEEL PINS 1 ¼" DIAM. LUBRICATED. TRANS. AM. SOC. C. E., VOL. XVII.
○ 55 TESTS BY MR. MARSHALL OF 2 IN. SQUARE BARS, (COLD- STRAIGHTENED), MOSTLY ON PINS 1¾ DIAM. & BEING ON PINS FROM ¾" TO 2 ¼" DIAM. TRANS. AM. SOC. C. E., VOL. XVII.
89 TESTS TOTAL ON DIAGRAM
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{49\,E}{5f_c + f_d\left(\frac{ce}{r^2}-5\right)}\left(\frac{f_c}{f_d}-1-\frac{ce}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
ROUND ENDS $\frac{l}{r} = \sqrt{\frac{49\,E}{5f_c + f_d\left(\frac{ce}{r^2}-5\right)}\left(\frac{f_c}{f_d}-1-\frac{ce}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
E = 28,000,000 LBS. PER SQUARE INCH,
f_c = 48,000 " "
$\frac{ce}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE,
$\frac{ce}{r^2}$ = 0.15 FOR UPPER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. $\frac{ce}{r^2}$ = 0.15. FIXED ENDS.
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. $\frac{ce}{r^2}$ = 0.6. ROUND ENDS.
50,000
45,000 = f_c
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320
RATIO $\frac{l}{r}$
f_d IN POUNDS PER SQUARE INCH.

Fig. 87.

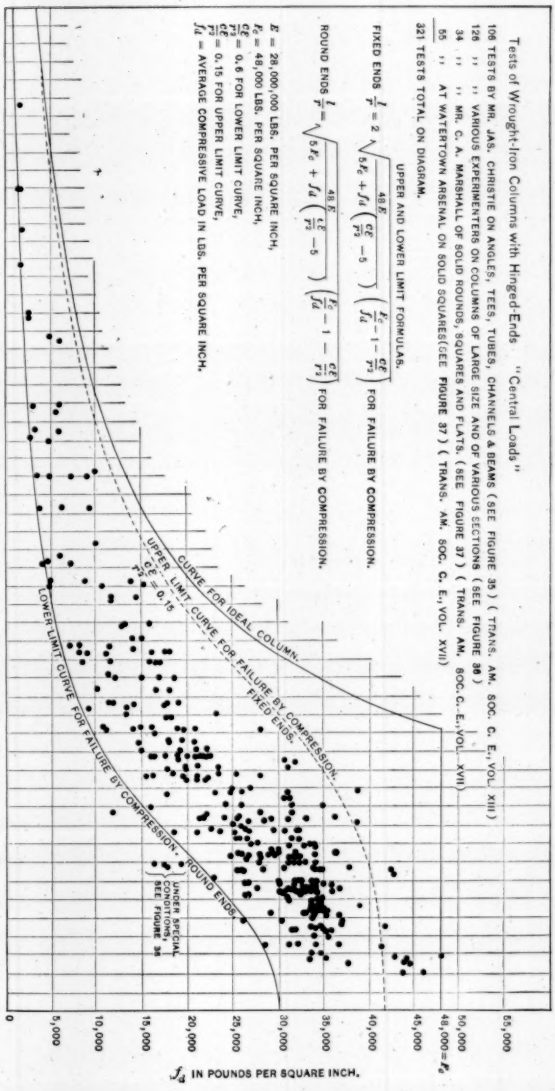
Tests of Wrought-Iron Columns with Hinged Ends "Central Loads"
106 TESTS BY MR. JAS. CHRISTIE ON ANGLES, TEES, TUBES, CHANNELS & BEAMS (SEE FIGURE 35) (TRANS. AM. SOC. C. E., VOL. XIII)
128 " " " VARIOUS EXPERIMENTERS ON COLUMNS OF LARGE SIZE AND OF VARIOUS SECTIONS (SEE FIGURE 36)
34 " " " MR. C. A. MARSHALL OF SOLID ROUNDS, SQUARES AND FLATS. (SEE FIGURE 37) (TRANS. AM. SOC. C. E., VOL. XVII)
55 " " AT WATERTOWN ARSENAL ON SOLID SQUARES (SEE FIGURE 37) (TRANS. AM. SOC. C. E., VOL. XVII)
321 TESTS TOTAL ON DIAGRAM.
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{48.E}{5f_c + f_d\left(\frac{cE}{Fs} - 5\right)}\left(\frac{F_s}{f_d} - 1 - \frac{cE}{Fs}\right)}$ FOR FAILURE BY COMPRESSION.
ROUND ENDS $\frac{l}{r} = \sqrt{\frac{48.E}{5f_c + f_d\left(\frac{cE}{Fs} - 5\right)}\left(\frac{F_s}{f_d} - 1 - \frac{cE}{Fs}\right)}$ FOR FAILURE BY COMPRESSION.
E = 28,000,000 LBS. PER SQUARE INCH.
F_s = 48,000 LBS. PER SQUARE INCH.
$\frac{cE}{Fs}$ = 0.6 FOR LOWER LIMIT CURVE.
$\frac{cE}{Fs}$ = 0.15 FOR UPPER LIMIT CURVE.
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
$\frac{cE}{Fs}$ = 0.15
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. ROUND ENDS.
UNDER SPECIAL CONDITIONS (SEE FIGURE 38)
RATIO $\frac{l}{r}$
f_d IN POUNDS PER SQUARE INCH.
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540
0 5,000 10,000 15,000 20,000 25,000 30,000 35,000 40,000 45,000 48,000=f_s 50,000 55,000

FIG. 38.

gyration, parallel to the axis of the pins, is of less value than the radius perpendicular to the pins.

If too great a difference exists, the column may be weaker as a flat-ended column deflecting parallel to the pins than as a pin-ended column deflecting at right angles to the pins.

Four of the Detroit Bridge Company's columns were purposely loaded out of axis, and these are plotted on the diagram with a note to that effect. Two of the columns were formed of a pair of 10-in.

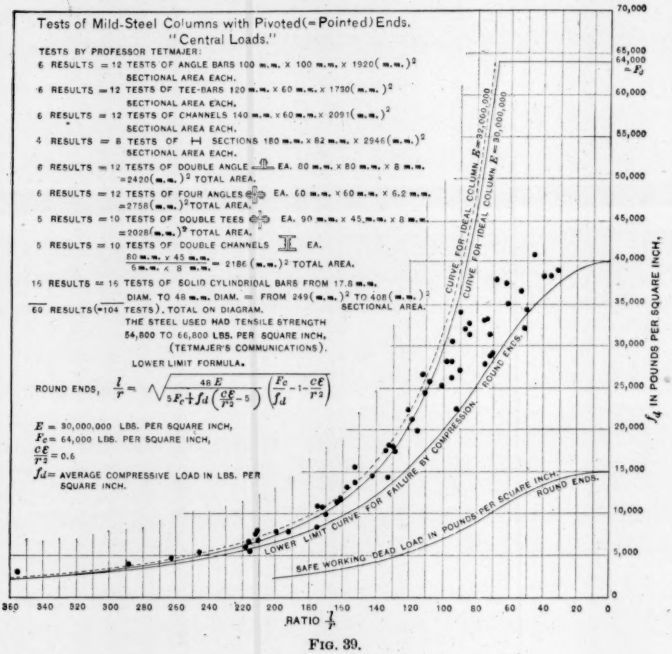

FIG. 39.

channels connected on one side by a plate, and on the other by latticing. The other two were of the same type, but with 8-in. channels.

The low strengths are worthy of careful attention, as the results were evidently not entirely due to these four columns being of unsymmetrical section, but to the pins not being placed at the center of area, since the highest result shown on the whole diagram was of the same type and proportions as these four columns, but with pins placed at the center of area.

Fig. 37.—This diagram represents:

Thirty-four tests of pin-ended wrought-iron columns by Mr. C. A. Marshall;* and fifty-five tests of pin-ended wrought-iron columns at Watertown;† a total of eighty-nine tests. A reference to these will be made when dealing with Mr. Marshall's tests on pin-ended steel columns.

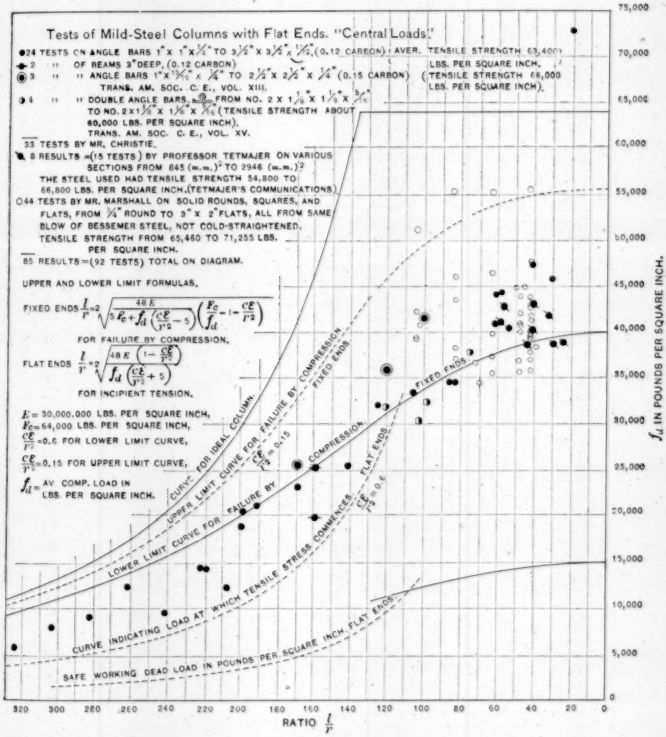

FIG. 40.

Fig. 38.—This is a combination of the diagrams on Figs. 35, 36 and 37, and shows the results of 321 tests of hinged-ended wrought-iron columns.

* *Transactions*, Am. Soc. C. E., Vol. xvii.

† Replotted from Plate XI of Mr. Marshall's paper, *Transactions*, Am. Soc. C. E., Vol. xvii.

Mild Steel.—Pivoted Ends.

Fig. 39.—Sixty results, representing 104 tests by Professor Tetmajer on mild-steel columns of various sections.* The remarks made with regard to Tetmajer's pivot-ended wrought-iron tests, shown on Fig. 21, apply also to these tests on mild steel.

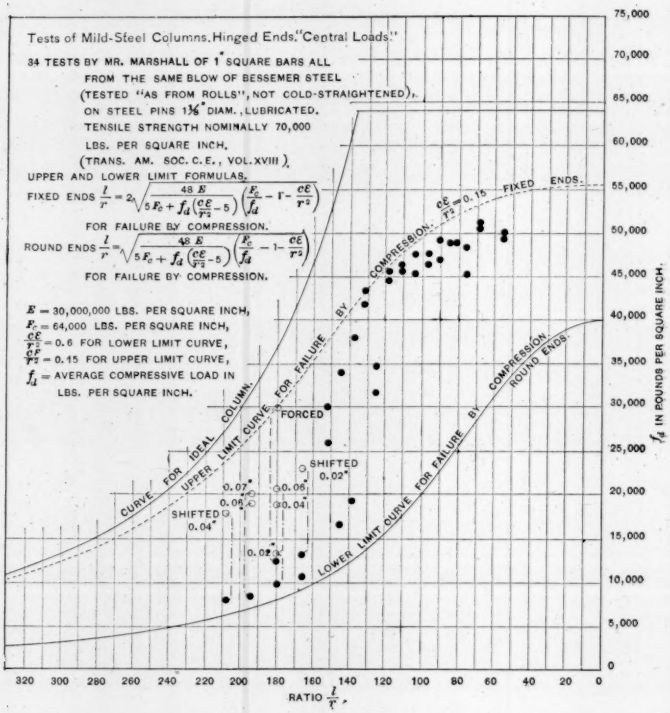

FIG. 41.

Mild Steel.—Flat Ends.

Fig. 40.—This diagram represents:

Twenty-nine tests by Mr. Christie;† four tests by Mr. Christie;‡ eight

* "Tetmajer's Communications."
† *Transactions*, Am. Soc. C. E., Vol. xiii.
‡ *Transactions*, Am. Soc. C. E., Vol. xv.

results by Professor Tetmajer,* representing fifteen tests; forty-four tests by Mr. Marshall;† a total of eighty-five results, ninety-two tests.

Mild Steel.—Hinged Ends.

Fig. 41.—Thirty-four tests of hinged-ended mild-steel columns by Mr. Marshall.‡ Mr. Marshall stated in his paper that his results indicated clearly the law that:

"The elastic limit of the material is the chief factor in determining the resistance of struts of ordinary length made out of wrought iron or steel, excepting the very hardest kinds, and that the two quantities elastic limit in compression and ultimate compressive strength are identical within a very considerable range of length of columns."

Mr. Marshall's conclusions were supported strongly by the results of his experiments, but the number of tests made was hardly sufficient to warrant the acceptance, in practice, of conclusions, which, if consistently applied, would allow the same stress upon a wrought-iron column having a length of 100 times the radius of gyration, as for one of only a third of that length.

The tests made by other experimenters show a considerable fall in strength with increase of length, and their evidence cannot be ignored in practice; and, judging from the results obtained by Hodgkinson, Christie, and Tetmajer, with pivot-ended columns in which the influence of frictional resistance or fixity of the column ends was eliminated, the range of length within which Mr. Marshall's law would apply, is very limited.

It will be noticed on referring to the diagrams Figs. 37 and 41, that Mr. Marshall's pin-ended tests exhibited a very rapid fall in strength in the longer column, *i. e.*, at the ratios 110 to 120 and over. This points to the probability that columns shorter than, but near, these ratios must have been in a very unstable condition, and were assisted greatly by the frictional resistance of the pin-ends.

It is also to be noted that the pin-ended tests made by Mr. Marshall on wrought iron and steel, were compared by him with the "compressive elastic limit" as determined from flat-ended and hinged-ended specimens 1 in. square by 12 ins. long,§ having a ratio $\frac{l}{r} = 41.5$, a

* "Tetmajer's Communications."
† Table No. 1 of Mr. Marshall's paper, *Transactions*, Am. Soc. C. E., Vol. xvii.
‡ Table No. 8 of Mr. Marshall's paper.
§ Table No. 7 of Mr. Marshall's paper.

column length quite sufficient to bring in influences which might, to a very considerable extent, affect the results, and all of these 12-in. bars deflected under their loads. In a few of the longer columns, Mr. Marshall showed how exceedingly sensitive they were to small adjustments in the testing machine, and some of these are indicated on Fig. 41. A very large increase of strength was obtained by merely moving the specimen a small fraction of an inch out of its original centering, and by the simple application of pressure at the start of a second test of a column having ratio $\frac{l}{r} = 180$, in order to cause failure by bending in the opposite direction to that in which it had first yielded, the strength was raised from 12 420 to 29 810 lbs. per square inch. This sensitiveness undoubtedly holds, in some degree, for shorter columns as well as for longer ones.

Mr. Marshall's comparison cannot therefore fairly be considered as referred to "compressive elastic limit" of the material, but only as one between the strength of certain columns, 41.5 radii of gyration long, and others of greater length.

The flat-ended tests of steel columns made by Mr. Marshall (Fig. 40) were, however, in part compared with short specimens, two sides long, from the same bars tested as columns, and it is this comparison which lends the greatest strength to Mr. Marshall's conclusions.

It is necessary to keep in view that even in a short compression specimen, eccentricity of loading will have more or less effect, and an ordinary test will only give the average stress on the material, while the important point to be determined as regards column strength is the maximum elastic fiber stress.

Mr. Marshall also made comparison between the ultimate strength of his column and the tensile elastic limit of the same bars as the columns, and the result of the comparison in most cases showed a very remarkable agreement. It is difficult, however, to see how any fixed relation of equality can exist between the elastic limit of a tension specimen and the compressive strength of a column. In a tension test, any inaccuracy of loading or lack of straightness does not tend to increase in influence, while in a compression specimen, even of moderate length, the reverse is the case.

In the curves plotted on Fig. 40 it will be noticed that no attempt is made to differentiate between the soft steel of Professor Tetmajer's

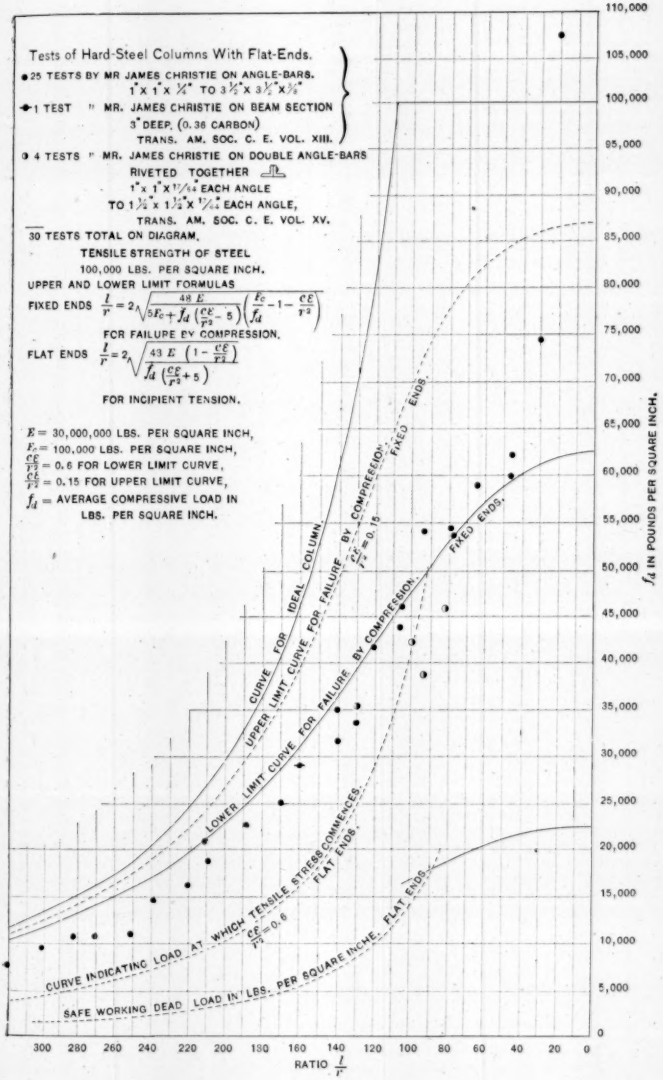
Tests of Hard-Steel Columns With Flat-Ends.
25 TESTS BY MR JAMES CHRISTIE ON ANGLE-BARS. 1" x 1" x ¼" TO 3½" x 3½" x ⅜"
1 TEST " MR. JAMES CHRISTIE ON BEAM SECTION 3" DEEP. (0.36 CARBON) TRANS. AM. SOC. C. E. VOL. XIII.
4 TESTS " MR. JAMES CHRISTIE ON DOUBLE ANGLE-BARS RIVETED TOGETHER 1" x 1" x 11/64 EACH ANGLE TO 1½" x 1½" x 1¼" EACH ANGLE, TRANS. AM. SOC. C. E. VOL. XV.
30 TESTS TOTAL ON DIAGRAM.
TENSILE STRENGTH OF STEEL 100,000 LBS. PER SQUARE INCH.
UPPER AND LOWER LIMIT FORMULAS
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{48\,E}{5F_c + f_d\left(\frac{cc'}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{cc'}{r^2}\right)}$
FOR FAILURE BY COMPRESSION.
FLAT ENDS $\frac{l}{r} = 2\sqrt{\frac{43\,E\,\left(1 - \frac{cc'}{r^2}\right)}{f_d\left(\frac{cc'}{r^2} + 5\right)}}$
FOR INCIPIENT TENSION.
E = 30,000,000 LBS. PER SQUARE INCH,
F_c = 100,000 LBS. PER SQUARE INCH,
$\frac{cc'}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE,
$\frac{cc'}{r^2}$ = 0.15 FOR UPPER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS. $\frac{cc'}{r^2} = 0.15$
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
CURVE INDICATING LOAD AT WHICH TENSILE STRESS COMMENCES. FLAT ENDS. $\frac{cc'}{r^2} = 0.6$
SAFE WORKING DEAD LOAD IN LBS. PER SQUARE INCH. FLAT ENDS.
110,000
105,000
100,000
95,000
90,000
85,000
80,000
75,000
70,000
65,000
60,000
55,000
50,000
45,000
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
f_d IN POUNDS PER SQUARE INCH.
300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 42.

tests and the somewhat stronger steel used by Mr. Marshall, with Mr. Christie's standing intermediate.

The number of tests in any of the three sets is not sufficient to warrant any nicety of distinction between the materials used in each, and it may be remarked that the complete range of strength, from Professor Tetmajer's minimum tensile strength of 54 800 lbs. per square inch to Mr. Marshall's tensile maximum of 71 255 lbs. per square inch, is very little greater in proportion than the range of tensile strength found in Professor Tetmajer's tensile tests of one class of wrought iron, in which the minimum and maximum tensile strengths were respectively 46 000 and 59 000 lbs. per square inch.

The curves in Figs. 39, 40 and 41, therefore, have been plotted with a value of $F_c = 64\,000$ lbs., and may be taken as applying to steel of tensile strength of from 60 000 to 70 000 lbs. per square inch.

Hard Steel.—Flat Ends.

Fig. 42.—This diagram represents:

Twenty-six tests by Mr. Christie,* and four tests by Mr. Christie,† a total of thirty tests. It may be urged with regard to this diagram that the lower-limit curve for failure by incipient tension is too low, and is not justified by the experiments.

This curve is not dependent in any way on the ultimate or elastic strength of the material, but solely on the modulus of elasticity and on the value of $\frac{c\,\varepsilon}{r^2}$, and the latter quantity has been taken the same as for all the other diagrams. The evidence of the more numerous tests on wrought iron is so strong that it does not appear advisable to count on higher loads than those indicated.

There is no reason why hard-steel struts should be assumed to be loaded with greater accuracy, or as having greater immunity from injury by cold-straightening, or as being less likely to have initial bends, than those of wrought iron or mild steel.

Timber.—Flat Ends.

Fig. 43. Tests of Yellow Pine or Pitch Pine.—The diagram represents:

Sixty-nine tests, with flat ends, made at Watertown Arsenal.‡ The

* *Transactions*, Am. Soc. C. E., Vol. xiii.

† *Transactions*, Am. Soc. C. E., Vol. xv.

‡ Exec. Doc. 12, 47th Congress, first session. These have been plotted from the figures given in Professor Lanza's "Applied Mechanics," pp. 664 to 668, 5th edition.

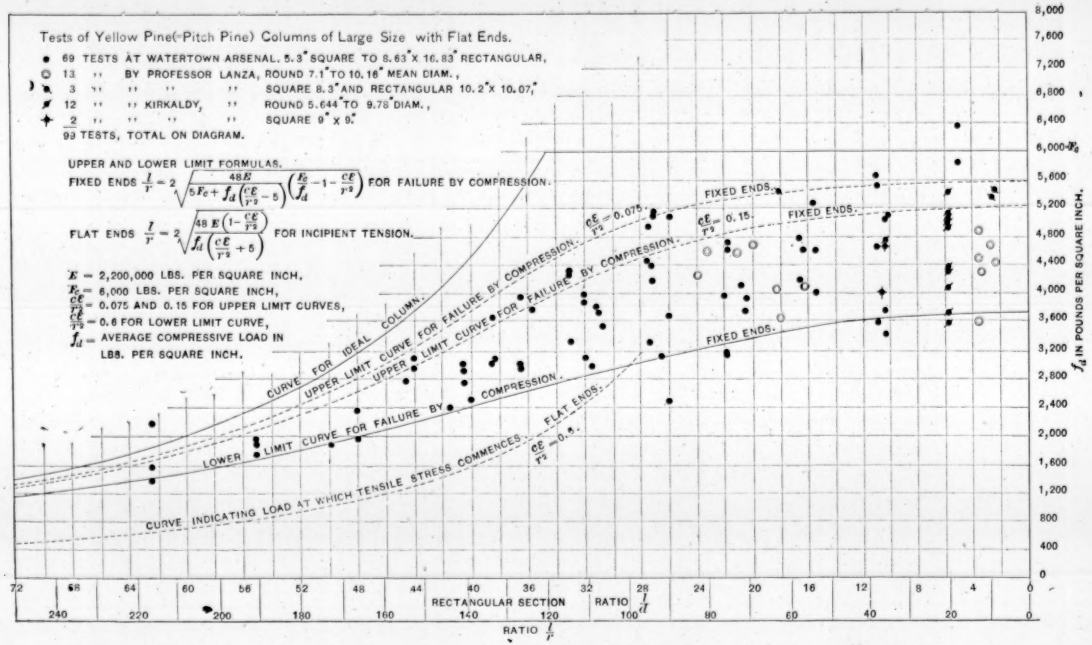
Tests of Yellow Pine(-Pitch Pine) Columns of Large Size with Flat Ends.
69 TESTS AT WATERTOWN ARSENAL, 5.3" SQUARE TO 8.63" X 16.83" RECTANGULAR,
13 " BY PROFESSOR LANZA, ROUND 7.1" TO 10.16" MEAN DIAM.,
3 " " " " SQUARE 8.3" AND RECTANGULAR 10.2" X 10.07",
12 " " KIRKALDY, " ROUND 5.644" TO 9.78" DIAM.,
2 " " " " SQUARE 9" X 9".
99 TESTS, TOTAL ON DIAGRAM.
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r} = 2\sqrt{\frac{48E}{5F_c + f_d\left(\frac{c\epsilon}{r^2} - 5\right)}\left(\frac{F_c}{f_d} - 1 - \frac{c\epsilon}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS $\frac{l}{r} = 2\sqrt{\frac{48E\left(1 - \frac{c\epsilon}{r^2}\right)}{f_d\left(\frac{c\epsilon}{r^2} + 5\right)}}$ FOR INCIPIENT TENSION.
E = 2,200,000 LBS. PER SQUARE INCH,
F_c = 6,000 LBS. PER SQUARE INCH,
$\frac{c\epsilon}{r^2}$ = 0.075 AND 0.15 FOR UPPER LIMIT CURVES,
$\frac{c\epsilon}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH,
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. $\frac{c\epsilon}{r^2}$ = 0.075
FIXED ENDS.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. $\frac{c\epsilon}{r^2}$ = 0.15.
FIXED ENDS.
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION.
FIXED ENDS.
CURVE INDICATING LOAD AT WHICH TENSILE STRESS COMMENCES. FLAT ENDS. $\frac{c\epsilon}{r^2}$ = 0.6.
8,000
7,600
7,200
6,800
6,400
6,000 F_c
5,600
5,200
4,800
4,400
4,000
3,600
3,200
2,800
2,400
2,000
1,600
1,200
800
400
0
f_d IN POUNDS PER SQUARE INCH.
72 68 64 60 56 52 48 44 40 36 32 28 24 20 16 12 8 4 0
RECTANGULAR SECTION RATIO $\frac{l}{d}$
240 220 200 180 160 140 120 100 80 60 40 20 0
RATIO $\frac{l}{r}$

FIG. 43.

single sticks only are shown on this diagram. The built posts tested have been omitted. Three second tests of the same sticks are not plotted. Sixteen tests by Professor Lanza made at Watertown for the Boston Manufacturers' Mutual Fire Insurance Company. The flat-ended tests only are plotted.* Fourteen tests by Kirkaldy, London, England, on pitch-pine blocks with flat ends.†

Total number of tests on diagram = 99.

The inclusion of Kirkaldy's tests of pitch pine in the same diagram as the Watertown tests and Professor Lanza's tests of yellow pine is justified by a comparison of the results obtained by Kirkaldy and Lanza on short blocks. The comparison is shown in Table No. 2.

TABLE No. 2.

KIRKALDY.		LANZA.	
Round, 9.78 ins. diameter, 50 ins. long.		Round, 7.70 to 10.46 ins. diameter Rectangular, 8.98 × 9.02 ins... And 10.20 × 10.07 ins.	Length, 23.8 to 24.33 ins
COMPRESSIVE STRENGTH, in pounds per square inch.			
Minimum	3 586	Minimum	3 604
Maximum	5 438	Maximum	5 452
Mean of 5 tests	4 689	Mean of 8 tests	4 658
MODULUS OF ELASTICITY.			
Average up to load of 2 400 lbs. per square inch	2 000 000 lbs.	Mean of 4 tests	1 911 385 lbs
(*Minutes of Proceedings*, Institution of Civil Engineers, Vol. liii. p. 158.)		(Lanza's "Applied Mechanics," p. 351.)	

Fig. 44.—Sixty-six tests of rectangular flat-ended white pine columns, made at Watertown Arsenal.‡ The single sticks only are plotted on the diagram. The built posts tested have been omitted.

Fig. 45.—This diagram represents:

Thirty-six tests of square flat-ended seasoned French oak columns, by Lamandé;§ and thirteen tests of square flat-ended Dantzic oak columns, by Hodgkinson,‖ a total of forty-nine tests.

* Lanza's "Applied Mechanics," pp. 651 and 665, 5th edition.

† From Kirkaldy's Reports (Kirkaldy's Life) and *Minutes of Proceedings*, Institution of Civil Engineers, Vol. liii, p. 158.

‡ Exec. Doc. 12, 47th Congress, 1st Session. Plotted from the figures given in Lanza's "Applied Mechanics," pages 659 to 662, 5th edition.

§ Plotted from the figures given in Hurst's "Tredgold's Carpentry," page 84, 6th Edition, 1888.

‖ *Philosophical Transactions*, Royal Society, London, 1840.

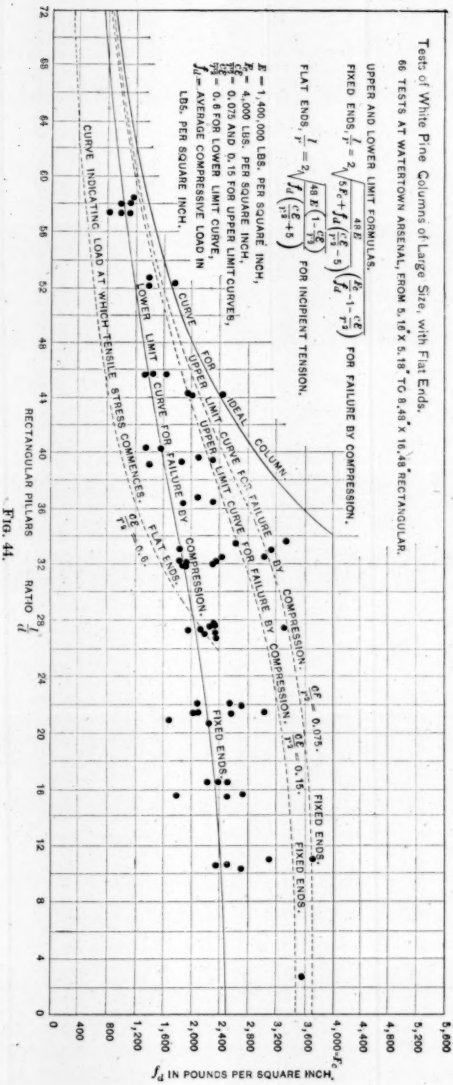
Tests of White Pine Columns of Large Size, with Flat Ends.
66 TESTS AT WATERTOWN ARSENAL, FROM 5.16" X 5.18" TO 8.48" X 16.48" RECTANGULAR.
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS, $\frac{l}{r} = 2\sqrt{\frac{48E}{5F_c + f_d\left(\frac{cE}{r^2}-5\right)}\left(\frac{F_c}{f_d}-1-\frac{cE}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS, $\frac{l}{r} = 2\sqrt{\frac{48E}{f_d\left(\frac{cE}{r^2}+5\right)}}$ FOR INCIPIENT TENSION.
E = 1,400,000 LBS. PER SQUARE INCH.
F_c = 4,000 LBS. PER SQUARE INCH.
$\frac{cE}{r^2}$ = 0.075 AND 0.15 FOR UPPER LIMIT CURVES,
$\frac{cE}{r^2}$ = 0.6 FOR LOWER LIMIT CURVE,
f_d = AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
IDEAL COLUMN.
CURVE FOR UPPER LIMIT FOR FAILURE BY COMPRESSION.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. $\frac{cE}{r^2}$ = 0.075.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. $\frac{cE}{r^2}$ = 0.15.
FIXED ENDS.
FIXED ENDS.
FIXED ENDS.
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. FLAT ENDS. $\frac{cE}{r^2}$ = 0.6.
CURVE INDICATING LOAD AT WHICH TENSILE STRESS COMMENCES.
RECTANGULAR PILLARS
RATIO $\frac{2l}{d}$
f_d IN POUNDS PER SQUARE INCH.
0 4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68 72
0 400 800 1,200 1,600 2,000 2,400 2,800 3,200 3,600 4,000 4,400 4,800 5,200 5,600
4,000=F_c

Fig. 44.

In attempting to fit the curves to these three diagrams of tests of timber columns, the writer has not lost sight of the great variation known to exist in the character of specimens of timber, even from the same tree, due to a number of causes and conditions to which it is not necessary to allude. The tests plotted, however, are in each case the most important series of which the writer is aware, and they comprise the most reliable data available for the design of compression members of timber.

There have been other tests made on large-sized timbers, but they are too few in number to be of use in any endeavor to determine the influence of length on column strength. Notwithstanding the difficulties attaching to the question, it will be seen from the diagrams that the tests do, in a considerable measure, exhibit the general characteristics accompanying the numerous tests on the more uniform materials previously dealt with.

These diagrams complete the comparison attempted to be made between the results of tests of ultimate strength and the writer's rational formulas.

It may be remarked that reference has been made in the diagrams to no less than 1 620 results, representing 1 789 tests, of ultimate strength of columns of cast iron, wrought iron, steel, and timber, exclusive of the references to tests of deflection.

Application of the Formulas in Practice.

The opinion has already been expressed in the paper that the design and proportioning of columns to carry given loads should be based on the maximum fiber stresses allowable, and on the stiffness of the material, while making reference to experimental results in order to ensure having sufficient margin against ultimate failure.

In order to put this opinion into practical operation, it is necessary to adopt suitable values for the three fundamental factors in the formulas, viz., ε, F_c, and E.

It will not be necessary, in the case of "centrally" loaded columns, in practice to deal with the value of the allowable tensile stress, since tensile stress will not be developed under ordinary working loads.

The value of ε is that concerning which our knowledge must be drawn entirely from the comparisons just made with experimental re-

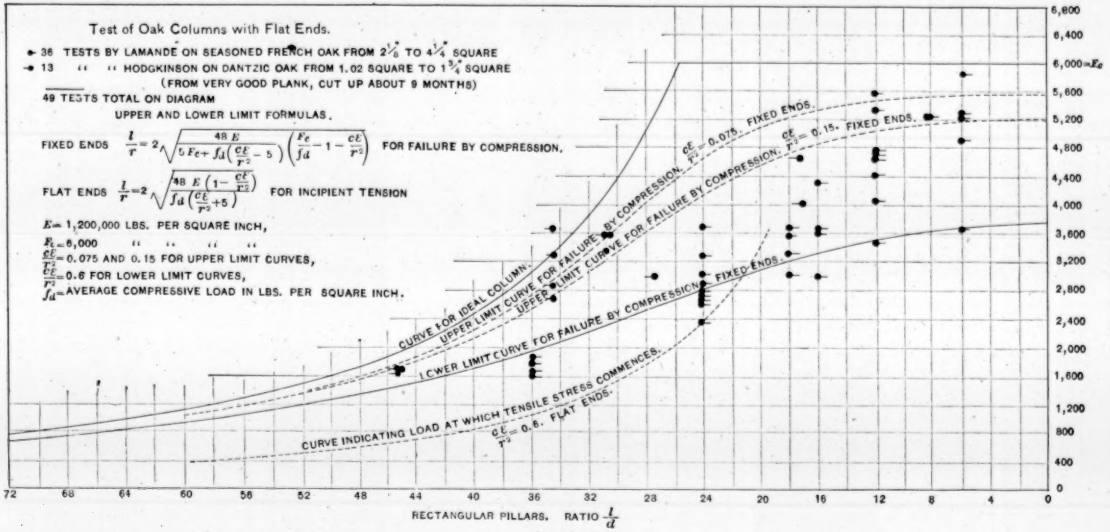
Test of Oak Columns with Flat Ends.
36 TESTS BY LAMANDÉ ON SEASONED FRENCH OAK FROM 2 1/8 TO 4 1/4 SQUARE
13 " " HODGKINSON ON DANTZIC OAK FROM 1.02 SQUARE TO 1 3/4 SQUARE
(FROM VERY GOOD PLANK, CUT UP ABOUT 9 MONTHS)
49 TESTS TOTAL ON DIAGRAM
UPPER AND LOWER LIMIT FORMULAS.
FIXED ENDS $\frac{l}{r}=2\sqrt{\frac{48\,E}{5\,Fe+f_d\left(\frac{c\ell}{r^2}-5\right)}\left(\frac{Fe}{f_d}-1-\frac{c\ell}{r^2}\right)}$ FOR FAILURE BY COMPRESSION.
FLAT ENDS $\frac{l}{r}=2\sqrt{\frac{48\,E\left(1-\frac{c\ell}{r^2}\right)}{f_d\left(\frac{c\ell}{r^2}+5\right)}}$ FOR INCIPIENT TENSION
E= 1,200,000 LBS. PER SQUARE INCH,
F_e=6,000 " " " "
$\frac{c\ell}{r^2}$= 0.075 AND 0.15 FOR UPPER LIMIT CURVES,
$\frac{c\ell}{r^2}$=0.6 FOR LOWER LIMIT CURVES,
f_d=AVERAGE COMPRESSIVE LOAD IN LBS. PER SQUARE INCH.
CURVE FOR IDEAL COLUMN.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. $\frac{c\ell}{r^2}$ = 0.075. FIXED ENDS.
UPPER LIMIT CURVE FOR FAILURE BY COMPRESSION. $\frac{c\ell}{r^2}$ = 0.15. FIXED ENDS.
LOWER LIMIT CURVE FOR FAILURE BY COMPRESSION. FIXED ENDS.
CURVE INDICATING LOAD AT WHICH TENSILE STRESS COMMENCES.
$\frac{c\ell}{r^2}$ = 0.6. FLAT ENDS.
6,800
6,400
6,000=F_e
5,600
5,200
4,800
4,400
4,000
3,600
3,200
2,800
2,400
2,000
1,600
1,200
800
400
0
f_d IN POUNDS PER SQUARE INCH.
72 68 64 60 56 52 48 44 40 36 32 28 24 20 16 12 8 4 0
RECTANGULAR PILLARS. RATIO $\frac{l}{d}$

FIG. 45.

sults, and, although the value of $\frac{c}{r^2}$ can be at once calculated from the section of column proposed to be used, yet it appears desirable in the case of "centrally" loaded columns that one constant value should be adopted for the quantity $\frac{c\,\varepsilon}{r^2}$; and judging from experimental evidence, the value to be assigned to it should not be taken at less than 0.6, in the present state of our knowledge.

In the case of a solid cylindrical column, this value of $\frac{c\,\varepsilon}{r^2}$ corresponds to an equivalent eccentricity of only 0.3 of the radius of gyration.

The values to be given to F_c for different materials may properly be approximately those in common use for maximum working fiber stresses in solid beams under transverse stress.

With regard to the value to be given to E, it must be noted that in every column it is necessary, not only to provide that the maximum fiber stress shall be confined within proper limits, but also that the stability of the column, as governed by the stiffness of the material of which it is made, shall also have a proper and sufficient margin of safety under the loads to be imposed.

The necessity for this is at once seen by a reference to the calculations of deflection and resulting maximum fiber stresses of a long column, given on page 351, and in connection with which it was shown that the column would be perfectly safe as regards the intensity of maximum fiber stress under a load amounting to over 90% of the ultimate supporting power.

Now, on reference to Formula (1), the general expression for the deflection of a column:

$$\Delta = \frac{P\,l^2\,e}{8\,E\,I - 2\,P\,l^2\,y\,x} \qquad (1)$$

or its practical modification

$$\Delta = \frac{P\,l^2\,e}{8\,E\,I - \frac{5}{6}\,P\,l^2} \qquad (2)$$

it will be seen that the deflection of any given column under any given load increases directly as the eccentricity e, and further, that if we had a material to deal with, such that we need give no thought to the maximum stress developed, we would have the curious result that the ulti-

mate load would be precisely the same whatever the amount of eccentricity might be.

The practical conclusion to be drawn from the foregoing is that the eccentricity of loading has nothing to do with the stability or instability of the column, if the condition of instability be defined as that which occurs when the deflection under load becomes infinite.

From this we see that in designing a column to carry a given load we have two totally independent modes of failure to guard against:

(1) Against failure by excessive intensity of fiber stress.

(2) Against failure by instability.

In the first we are dependent on strength, and in the second on stiffness.

We will proceed, therefore, on perfectly correct rational lines if we proportion our "centrally" loaded columns in practice to meet these two conditions, which are satisfied by using the two formulas:

$$\text{For columns with both ends round} \left\{ \frac{l}{r} = \sqrt{\frac{48\,E}{5\,F_c + f_a\left(\frac{c\,\varepsilon}{r^2} - 5\right)}\left(\frac{F_c}{f_a} - 1 - \frac{c\,\varepsilon}{r^2}\right)} \right. \quad ..(7)$$

where F_c is the maximum allowable working intensity of fiber stress,

$$P = K\frac{9.6\,E\,I}{l^2} \text{ for stability} \quad .....(2)$$

where K is a suitable fractional coefficient of safety,

or, since $$f_a = \frac{P}{a},\ f_a = K\frac{9.6\,Er^2}{l^2} = \frac{9.6\,KE}{\frac{l^2}{r^2}}.$$

Applying these formulas to the material which is in most general use in constructional work at the present day, *i. e.*, mild steel, and adopting as suitable values for the various factors,

E = 30 000 000 lbs.;

F_c = 24 000 lbs. per square inch, working dead stress;

$\frac{c\,\varepsilon}{r^2}$ = 0.6 for columns in which $\frac{c}{r} = 2$, and $\frac{\varepsilon}{r} = 0.3$;

K = coefficient of safety against instability, say $\frac{1}{3}$;

we will have the working loads per square inch on solid, cylindrical, mild-steel, round-ended columns indicated by the curves in Fig. 46, in which, for every ratio $\frac{l}{r}$, the conditions are satisfied that the maximum fiber stress shall not be more than 24 000 lbs. per square inch,

and the working load shall not be more than one-third of that which will cause instability.

It will be noticed that, at about ratio $\frac{l}{r} = 88$, the two curves intersect, and at this point the factor against ultimate failure, as compared with experimental results, is somewhat less than is given either for very short or very long columns, and however rational the basis upon which the results depend, the writer thinks that, like himself, other engineers would hesitate to adopt heavier loads, in relation to ultimate

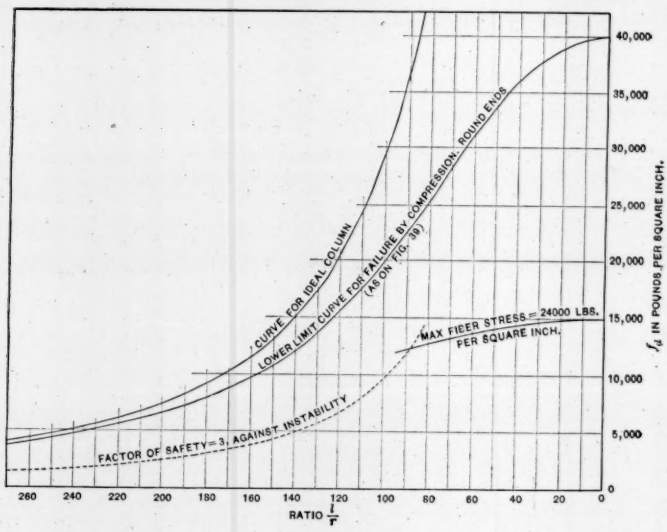

FIG. 46.

strength for a column 89 or 90 radii long, than would be considered safe for columns only one-quarter of the length, or four times the length.

Again, it may reasonably be objected that if we require the factor of safety against failure by instability of a long column to be at least 3 (assuming this as a fair value), it should follow that we would not have a much smaller value against ultimate strength as shown by experiment, in the case of columns of ratio $\frac{l}{r} = 80$ to 90.

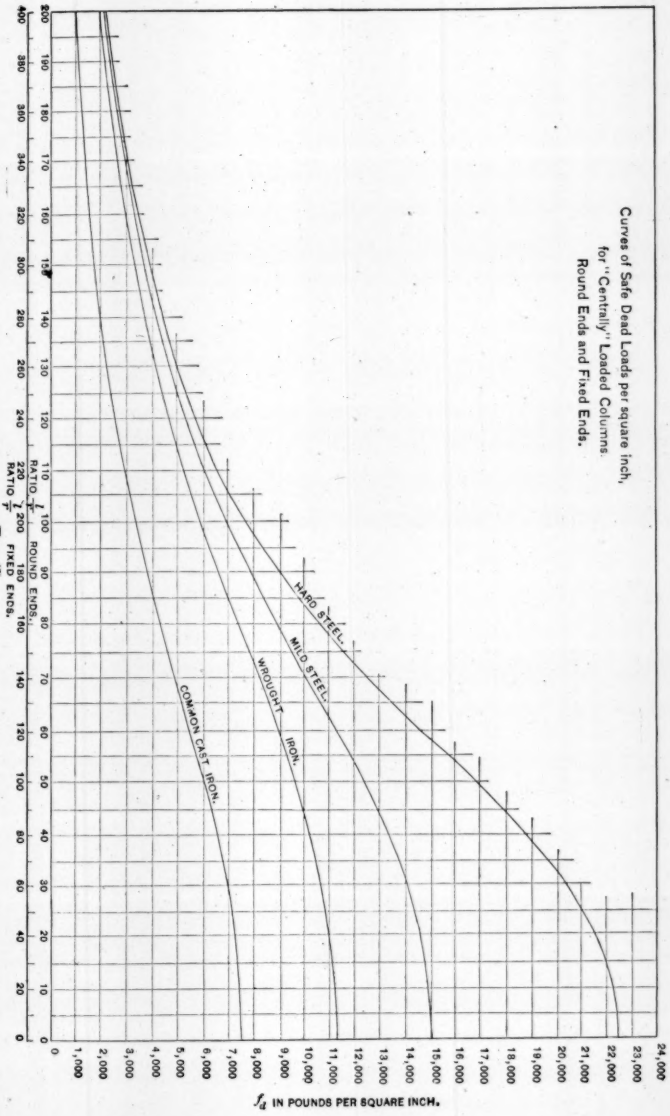
Curves of Safe Dead Loads per square inch,
for "Centrally" Loaded Columns
Round Ends and Fixed Ends.
HARD STEEL.
MILD STEEL.
WROUGHT IRON.
COMMON CAST IRON.
RATIO $\frac{l}{r}$ ROUND ENDS.
RATIO $\frac{l}{r}$ FIXED ENDS.
f_d IN POUNDS PER SQUARE INCH.

FIG. 47.

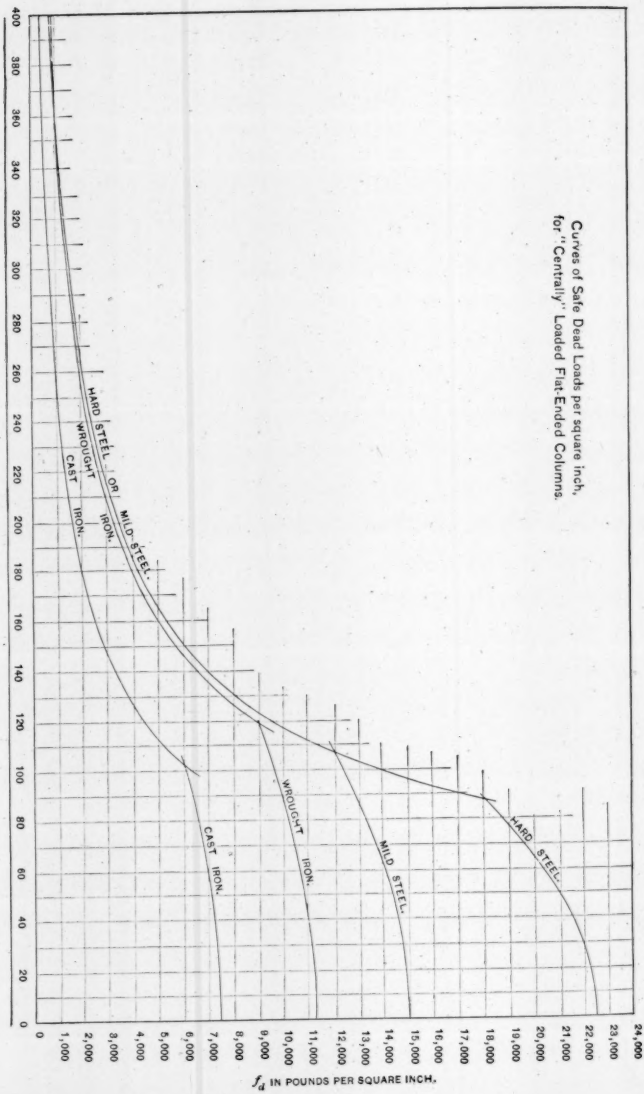
Curves of Safe Dead Loads per square inch, for "Centrally" Loaded Flat-Ended Columns.
HARD STEEL.
MILD STEEL.
WROUGHT IRON.
CAST IRON.
HARD STEEL OR MILD STEEL.
WROUGHT IRON.
CAST IRON.
f_d IN POUNDS PER SQUARE INCH.
RATIO: LENGTH ÷ LEAST RADIUS OF GYRATION. FLAT ENDS.

FIG. 48.

TABLE No. 3.—Working Formulas for Columns under "Central Loading."

(1) Material of Column.	(2) F_c = Maximum compressive fiber stress. Pounds per square inch dead load.	(3) $K E$ = Mod. Elas. / 3 Pounds per square inch.	(4) Value of $\frac{c\,\epsilon}{r^2}$	(5) Working formulas for round-ended or pivot-ended columns. $\frac{l}{r} = \sqrt{\frac{48\,K\,E}{5\,F_c + f_d\left(\frac{c\,\epsilon}{r^2} - 5\right)}\left[\frac{F_c}{f_d} - 1 - \frac{c\,\epsilon}{r^2}\right]}$
Common cast iron	12 000 = 1.2 units.	$\frac{14\,000\,000}{3} = 466.6$ units.	0.6	$\frac{l}{r} = 10\sqrt{\frac{224}{6 - 4.4 f_d}\left(\frac{1.2}{f_d} - 1.6\right)}$
Wrought iron. Tensile strength, 45 000 to 50 000 lbs. per square inch.	18 000 = 1.8 units.	$\frac{28\,000\,000}{3} = 933.3$ units.	0.6	$\frac{l}{r} = 10\sqrt{\frac{448}{9 - 4.4 f_d}\left(\frac{1.8}{f_d} - 1.6\right)}$
Mild steel. Tensile strength, 60 000 to 70 000 lbs. per square inch.	24 000 = 2.4 units.	$\frac{30\,000\,000}{3} = 1\,000$ units.	0.6	$\frac{l}{r} = 10\sqrt{\frac{480}{12 - 4.4 f_d}\left(\frac{2.4}{f_d} - 1.6\right)}$
Hard steel. Tensile strength, about 100 000 lbs. per square inch.	36 000 = 3.6 units.	$\frac{30\,000\,000}{3} = 1\,000$ units.	0.6	$\frac{l}{r} = 10\sqrt{\frac{480}{18 - 4.4 f_d}\left(\frac{3.6}{f_d} - 1.6\right)}$
Timber (Unit for F_c, f_d and E = 1 000 lbs.)				
Yellow pine or pitch pine	2 000 = 2 units.	$\frac{2\,200\,000}{3} = 733.3$ units.	0.6	$\frac{l}{r} = 10\sqrt{\frac{352}{10 - 4.4 f_d}\left(\frac{2}{f_d} - 1.6\right)}$
White pine	1 300 = 1.3 units.	$\frac{1\,400\,000}{3} = 466.6$ units.	0.6	$\frac{l}{r} = 10\sqrt{\frac{224}{6.5 - 4.4 f_d}\left(\frac{1.3}{f_d} - 1.6\right)}$
French oak or Dantzic oak	2 000 = 2 units	$\frac{1\,200\,000}{3} = 400$ units.	0.6	$\frac{l}{r} = 10\sqrt{\frac{192}{10 - 4.4 f_d}\left(\frac{2}{f_d} - 1.6\right)}$

In Column (5) a unit of 10 000 lbs. is used for F_c, f_d and E for metals, and 1 000 lbs. for timber.

This objection may be met very easily and simply by using only Formula (7), with the modification of applying the factor of safety against instability to the value of E in that formula, which then becomes:

$$\frac{l}{r} = \sqrt{\frac{48\ K\ E}{5\ F_c + f_d\left(\frac{c\ \varepsilon}{r^2} - 5\right)}\left[\frac{F_c}{f_d} - 1 - \frac{c\ \varepsilon}{r^2}\right]},$$

$$\text{where } K = \frac{1}{\text{factor of safety.}}$$

It will be seen that the actual result of this formula is to give the ratio $\frac{l}{r}$ corresponding to a given average load per square inch for material in which the maximum compressive fiber stress is limited to F_c, and for which the value of the modulus of elasticity is taken to be $K\,E$, instead of the actual value E pertaining to the material.

The use of the coefficient K in this manner has an inappreciable influence on the results of the formulas when applied to short columns, while its effect gradually increases with the length, ultimately affording a factor of safety of 3, against failure by instability, in the case of exceedingly long columns, and the factor of safety against ultimate strength is fairly even between these extremes.

Table No. 3 gives the resulting formulas reduced by the insertion of suitable values of $\frac{c\ \varepsilon}{r^2}$, F_c and $K\,E$.

For fixed-ended columns the values of $\frac{l}{r}$ obtained by the formulas in Table No. 3 are to be doubled, or in other words, for a given strength, a fixed-ended column is twice as long as a round-ended or pivot-ended one.

For flat-ended columns, the relation between the strength and the ratio $\frac{l}{r}$ is the same as for fixed-ended columns, up to the point when incipient tension is the controlling factor, and it is then necessary to use the modified formulas as previously described for that condition, except that, as we are now dealing with safe working loads, the coefficient of safety K must be inserted in Formula (9), which then becomes

$$\frac{l}{r} = 2\sqrt{\frac{48\ K\ E\left(1 - \frac{c\ \varepsilon}{r^2}\right)}{f_d\left(\frac{c\ \varepsilon}{r^2} + 5\right)}}$$

or inserting

$$K = \tfrac{1}{3} \text{ and } \frac{c\,\varepsilon}{r^2} = 0.6,$$

$$\frac{l}{r} = 2.138\sqrt{\frac{E}{f_d}}$$

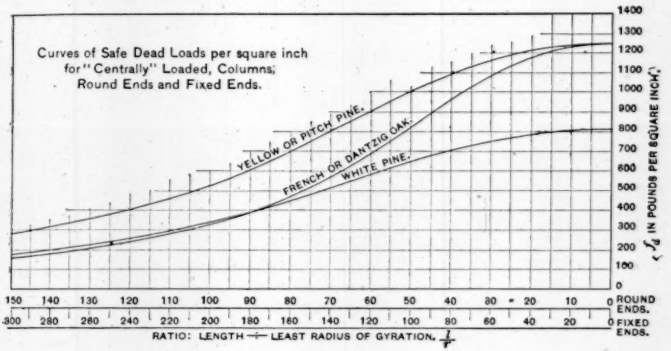

FIG. 49.

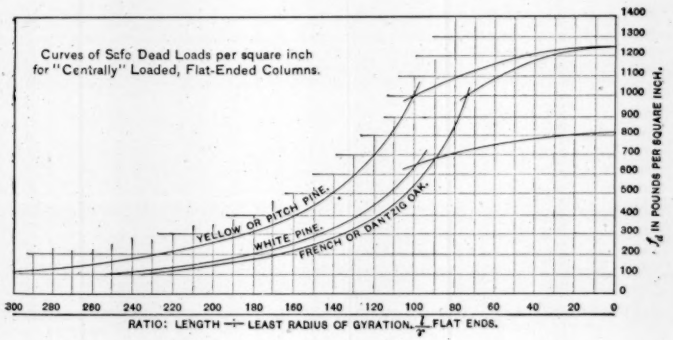

FIG. 50.

The results of the various reduced formulas for working dead loads in Table No. 3 for round-ended columns, and of the modifications above stated for fixed and flat-ended columns, are shown on various diagrams accompanying the paper, and are also shown in Figs. 47, 48, 49 and 50, drawn to a larger vertical scale for convenience in use.

With regard to the curves for working dead-loads on fixed-ended and flat-ended columns, it must be remembered that in a laboratory or test-room experiment on a fixed or flat-ended column, the conditions of end fixing are totally different from those met in ordinary practice. The rigidity of the testing machine bearing faces, as compared with the column under test, is far greater than is afforded by any end bearing or connection in practice, and for this reason it is well not to count, in actual work, upon such high strengths as are indicated by experiment.

The only modification necessary in dealing with live loads or moving loads instead of the dead loads for which the stresses adopted in Table No. 3 are suitable, consists obviously of a reduction in the value of F_c or F_t, and an increase in the value of K, although in the writer's practice he adopts the method of increasing the moving load by a suitable percentage, dependent on the character of the load, and then treats the result as a dead-load equivalent.

Columns Under Intentionally Eccentric Loads.

The experimental data, to which reference can be made in regard to columns on which the load is imposed with large eccentricity as compared with the size of column, are exceedingly few, and as far as the writer is aware, are limited to the tests by Tetmajer (Figs. 24 and 25); but as the theoretical principles upon which the formulas are based are those in common use and acceptance in all cases involving transverse bending simply combined with the influence of direct loading, there can be no more hesitation in applying them to columns than there is in any case of simple bending.

In making use of the formulas in practice, it is, however, not only sufficient to take into account the actual measured eccentricity and the value of $\frac{c}{r^2}$ as fixed by the section, but allowance should be made for the "accidental" value of $\frac{c\,\varepsilon}{r^2}$, as dealt with in the case of presumably central loading.

This being the case, it will be necessary to add the value of $\frac{c\,\varepsilon}{r^2} = 0.6$, as determined for centrally-loaded columns, to the measured

value of $\frac{c\,e}{r^2}$, as obtained from the intended eccentricity e, and the dimensions and form of the sections which fix the value of $\frac{c}{r^2}$.

In this way a perfectly rational recognition is given to the form of section, in precisely the same manner as in the case of beams under simple flexure, and the formulas for the relation between the ratio $\frac{l}{r}$ and the average load per square inch for a free-ended column carrying an eccentric load thus become, for safe working loads,

$$\frac{l}{r} = \sqrt{\frac{48\,K\,E}{5\,F_c + f_d\left\{\left(\frac{c\,e}{r^2}+0.6\right)-5\right\}\left[\frac{F_c}{f_d}-1-\left(\frac{c\,e}{r^2}+0.6\right)\right]}}$$

where F_c is the maximum allowable working fiber stress in compression, or

$$\frac{l}{r} = \sqrt{\frac{48\,K\,E}{5\,F_t - f_d\left\{\left(\frac{c\,e}{r^2}+0.6\right)+5\right\}\left[\frac{F_t}{f_d}-1+\left(\frac{c\,e}{r^2}+0.6\right)\right]}}$$

where F_t is the maximum allowable tensile fiber stress (and being tensile it must be accorded the minus sign independently of the fixed signs in the formula, as has been pointed out previously).

These formulas again reduce to very simple expressions when the values of $K\,E$, F_c or F_t and $\frac{c\,e}{r^2}$ are inserted, as will be seen by the following application to mild steel, using the same values for the various quantities as adopted for "centrally" loaded columns.

Since the maximum compressive stress actually developed in a column of symmetrical section always exceeds the maximum tensile stress, it will not be necessary, in the case of mild steel, to use the formula containing the value of maximum allowable tensile stress F_t, as this may be taken, for this material under working loads, as having the same value accorded to F_c, the allowable compressive fiber stress.

In the case of cast iron it will, of course, be necessary to use both expressions, and to adopt whichever gives the lower value to the ratio $\frac{l}{r}$ for any given load f_d.

The formula, then, for the following values of $\left(\frac{c\,e}{r^2} + 0.6\right)$ inserted, will be for mild-steel columns of symmetrical section, carrying dead loads, using 10 000 lbs. as a unit for E, F_c and f_d:

When $\left(\frac{c\,e}{r^2} + 0.6\right) = 2$, $\frac{l}{r} = 10\sqrt{\frac{480}{12 - 3f_d}\left(\frac{2.4}{f_d} - 3\right)}$

" " $= 4$, $\frac{l}{r} = 10\sqrt{\frac{480}{12 - f_d}\left(\frac{2.4}{f_d} - 5\right)}$

" " $= 6$, $\frac{l}{r} = 10\sqrt{\frac{480}{12 + f_d}\left(\frac{2.4}{f_d} - 7\right)}$

" " $= 8$, $\frac{l}{r} = 10\sqrt{\frac{480}{12 + 3f_d}\left(\frac{2.4}{f_d} - 9\right)}$

The working out of these values of the formula is shown by the curves on Fig. 51.

A reference to the remarks previously made in connection with Fig. 24 will show that in dealing with columns of unsymmetrical section it may be necessary to consider the question of tensile stress, and to use the modification of the formula suitable to such conditions.

Columns Subjected to Side Loads.

The effect of a side load in addition to the end load on a column is easily traced by the aid of the principles and formulas in the foregoing pages. To illustrate this it will be convenient to take a direct example.

Assume a mild-steel column with pivot ends, and having the following particulars:

$l = 100$ ins. = length;

$I = 10$ ins.^{4};

$r = 1$ in.;

$c = 2.5$ ins., from axis to extreme fibers;

Sectional area $= 10$ sq. ins.;

$E = 30\,000\,000$ lbs.;

and assume the side load to be 4 000 lbs., distributed along the column's length.

This will produce a bending moment at the center,

$$M = \frac{4\,000 \text{ lbs.} \times 100 \text{ ins.}}{8} = 50\,000 \text{ inch-pounds},$$

and the resulting maximum fiber stress will be

$$f = \frac{Mc}{I} = \frac{50\,000 \times 2.5}{10} = \pm\ 12\,500 \text{ lbs. per square inch.}$$

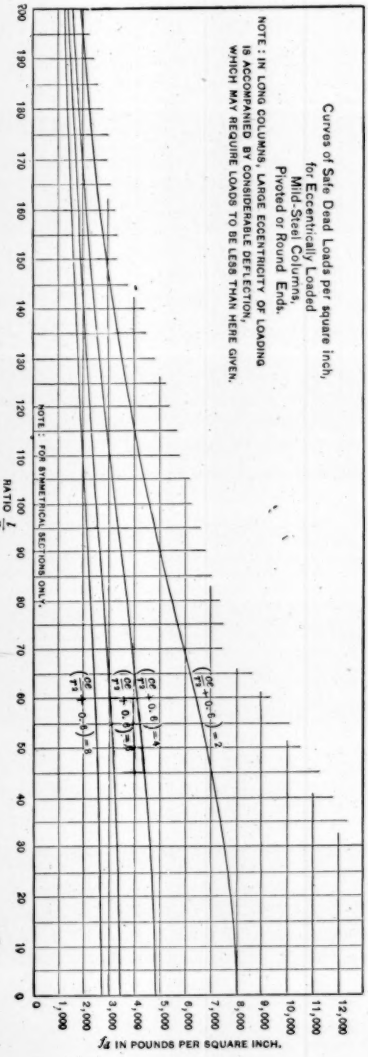
Curves of Safe Dead Loads per square inch,
for Eccentrically Loaded
Mild-Steel Columns,
Pivoted or Round Ends.
NOTE: IN LONG COLUMNS, LARGE ECCENTRICITY OF LOADING
IS ACCOMPANIED BY CONSIDERABLE DEFLECTION,
WHICH MAY REQUIRE LOADS TO BE LESS THAN HERE GIVEN.
NOTE: FOR SYMMETRICAL SECTIONS ONLY.
RATIO $\frac{l}{r}$
f_a IN POUNDS PER SQUARE INCH.
$\left(\frac{cc}{r^2}+0.6\right)=2$
$\left(\frac{cc}{r^2}+0.6\right)=4$
$\left(\frac{cc}{r^2}+0.6\right)=6$
$\left(\frac{cc}{r^2}+0.6\right)=8$

Fig. 51.

The side-load bending moment diagram will have a parabolic outline, as shown in Fig. 52, the central depth being 50 000 inch-pounds to any convenient scale.

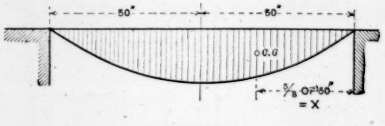

Fig. 52.

Treating each half of the column (or beam) as a cantilever, with respect to its central section, the deflection at the center will be

$$v = \frac{A\,X}{E\,I},$$

where $A = 50\,000 \times 50$ ins. $\times \frac{2}{3}$, and $X = \frac{3}{8}$ of 50 ins., and therefore,

$$v = \frac{(50\,000 \times 50 \times \frac{2}{3}) \times (\frac{3}{8} \text{ of } 50)}{30\,000\,000 \times 10} = 0.1736 \text{ in.}$$

In order to determine the deflection of the column under end loads it is now only necessary to treat it in precisely the same way as already indicated for a column having an initial curvature, with the value of central versed sine, $v = 0.1736$ in., keeping in view, when considering its strength as against end loads, that we have already absorbed 12 500 lbs. of the allowable maximum fiber stress.

In the present case it is not difficult to determine the precise theoretic form of the curve assumed by the bent column under its side load alone, this curve depending on the disposition of the load.

In practical work, however, it is not necessary to go to this degree of accuracy, and we may assume the curve to be a parabola, without involving any serious error, and the justification is found in the fact that, had we assumed the side load to be concentrated at the center, the assumption of a parabola instead of the precise curve taken by the bent column would only involve an excess, as regards its influence on the deflection, of a little over 4 per cent. As compared with the curve produced by a distributed load, the parabola would have a much less difference than even that named.

In order to make the comparison between the column with side load, and one of ordinary character with presumably central loading, it will be necessary to treat both on the same lines as far as possible, and in calculating the safe dead load on the centrally-loaded column, we have limited the maximum fiber stress to 24 000 lbs. per square inch, and have at the same time used a value of

$$K\,E = \frac{30\,000\,000}{3} = 10\,000\,000 \text{ lbs.},$$

and also a value of $\frac{c\,\varepsilon}{r^2} = 0.6$.

These same values must be used also in the side-loaded columns as regards the influence of the end loads at least.

The value of $\frac{c\,\varepsilon}{r^2} = 0.6$, in the case of this column with a radius of gyration = 1 in., corresponds to an equivalent eccentricity of 0.24 in., and we thus have a column with an initial curvature measured by a versed sine of $v = 0.174$ in. and an eccentricity of loading $\varepsilon = 0.24$ in., and making use of Formula (3) (with K inserted) to determine the deflection caused by different values of the end load, P, we have

$$\Delta = \frac{P\,l^2\,(\varepsilon + 2\,y_1\,x_1\,v)}{8\,(K\,E)\;I - 2\,P\,l^2\,y\,x}.$$

We have already assumed the curve of the bent column, from both side and end loads, to be a parabola, and the values of $2\,y_1\,x_1$ and $2\,y\,x$ will each be $\left(2 \times \frac{2}{3} \times \frac{5}{8}\right) = \frac{5}{6}$, and, inserting these and the values of the other factors in the formula, we have:

$$\Delta = \frac{P \times 10\,000\,\left(0.24 + \frac{5}{6} \times 0.174\right)}{(8 \times 10\,000\,000 \times 10) - \left(\frac{5}{6}\,P \times 10\,000\right)} = \frac{0.462\,P}{96\,000 - P} \text{ ins.},$$

for the elastic deflection of this column under any end load P.

TABLE No. 4.

P, in pounds.	$\Delta = \frac{0.462\,P}{96\,000 - P}$	Bending moment from end loads. $M = P\,(0.414 + \Delta)$ inch-pounds.	Maximum fiber stress from end-load bending moments. Pounds per square inch.	Direct stress. $f_d = \frac{P}{a}$. Pounds per square inch.	Maximum fiber stress from side-load bending moments. Pounds per square inch.	Total maximum compressive fiber stress. Pounds per square inch.
30 000	0.21	18 720	4 680	3 000	12 500	20 180
32 000	0.231	20 640	5 160	3 200	12 500	20 860
34 000	0.253	22 680	5 670	3 400	12 500	21 570
36 000	0.277	24 880	6 220	3 600	12 500	22 320
38 000	0.303	27 240	6 810	3 800	12 500	23 110
40 000	0.33	29 760	7 440	4 000	12 500	23 940

The bending moment at the center of the column, from end loads alone, will be $M = P\,(\varepsilon + v + \Delta) = P\,(0.24 + 0.174 + \Delta)$

$= P\,(0.414 + \Delta)$ ins.;

and now calculating the deflections, bending moments, and resulting maximum fiber stresses caused by the end loads, and combining them with the maximum fiber stress of 12 500 lbs. caused by the end loads alone, we have the results given in Table No. 4.

From these results we see that with an end load of 4 000 lbs. per square inch, the maximum fiber stress reaches the limiting value of 24 000 lbs. per square inch, and, on referring to Fig. 47, we find that with the presumably centrally-loaded column, under the same basis of calculation, the safe load f_d would be (for the ratio $\frac{l}{r} = 100$) 7 000 lbs. per square inch, or in other words, the side load has reduced the safe end load by nearly 43 per cent.

The foregoing calculations will serve to illustrate the proper mode of treating the conditions met with in side-loaded columns, although they have been applied to a column of such proportions as would rarely be trusted to carry any appreciable amount of side load other than its own weight.

The values of I, r and c, and the sectional area assumed, are roughly those of a rolled girder section 12 × 5 ins. × 33 lbs. per foot.

The formulas were deduced by the writer in 1896 and 1897, for use in his own practice, and he hopes that the results given in this paper may be found to be fairly in accordance with the leading principles set forth in the opening sentences as being those with which a column theory and its resulting formulas should comply, at least from the point of view of the practicing engineer rather than that of the pure mathematician.

The question of the proper construction of columns so as to ensure the full development of the strength which may rightfully be expected from any given section, is one which has not been dealt with herein, although it is certainly of the greatest importance, in view of the inferior types sometimes adopted in present-day practice.

Of these inferior types the writer would especially draw attention to one which appears to be in more common use, and that is, the column with batten-plate connections between the main members forming the column, as a substitute for a properly constructed web,

and a reference to the illustrations of the failure of a bridge in Servia,* will give ample evidence of the inefficiency of the type. Its inferiority cannot be questioned in the case of columns carrying eccentric loads, or having to resist shearing stresses from side wind-loads as in some of the lofty buildings constructed in America.

The writer is conscious that in these pages there will be found (and necessarily so) many points of resemblance and identity, both in views and in their expression, with the contributions of others, and he desires to acknowledge his indebtedness to the numerous writers from whose papers he has drawn information and assistance.

While, from a strictly mathematical point of view, objections may possibly be raised to the reasoning and results in this paper, yet if a close comparison be made with more highly analytical investigations, it is believed that the differences from the latter will be found to be of little importance in any practical sense.

The writer was impelled to attempt the deduction of new formulas by having to design a number of columns to carry heavy loads with exceptionally large eccentricity, for which condition he could find no theory or formulas sufficiently simple and easy of application under varied circumstances.

The concentration of such a large mass of scattered experimental evidence will, it is hoped, be of sufficient value to justify the labor involved in its collection, examination and arrangement in diagram form.

* *Engineering*, February 3d, 1893.

DISCUSSION.

Mr. Jonson. ERNST F. JONSON, Assoc. M. Am. Soc. C. E. (by letter).—The author states that the curve of flexure of a column of uniform cross-section and elasticity is a curve of sines only when the eccentricity of loading is infinitely small.

The writer, therefore, would call attention to the fact that the nature of the curve is in nowise changed by increasing the eccentricity. Considering the line in which the external forces act, as the axis of abscissas, the bending moment is still proportional to the ordinate. Hence, no matter how great the eccentricity may be, the curve is a curve of sines, the equation of which is

$$y = D \cos. \frac{x}{r}\sqrt{\frac{w}{E}} \quad \text{I}$$

where x is abscissa, y ordinate, D maximum ordinate, r radius of gyration, w load per unit of cross-section area, and E modulus of elasticity.

For maximum w

$$D = \frac{r^2\, s}{a\, w} \quad \text{II}$$

where a is distance to extreme fiber and $s = p - w$ when $p - w \gtreqless t + w$, or $= t + w$ when $p - w > t + w$, p being the proportional limit in compression and t the proportional limit in tension.

Hence,

$$y = \frac{r^2\, s}{a\, w} \cos. \frac{x}{r}\sqrt{\frac{w}{E}} \quad \text{III}$$

If y is given, the value of $e = c + b$, c being the equivalent eccentricity, and b the intentional eccentricity, x becomes the length of the elementary column, *i. e.*, the column fixed at one end and free at the other. We have then

$$c + b = \frac{r^2\, s}{a\, w} \cos. \frac{l}{r}\sqrt{\frac{w}{E}} \quad \text{IV}$$

or

$$l = r\sqrt{\frac{E}{w}} \cos.^{-1} \frac{a\, w}{r^2\, s}(c + b) \quad \text{V}$$

Apart from its mathematical form, the foregoing formula differs from that of the author in that it makes the maximum load a function, not of the ultimate strength of the material, but of the proportional limit. When this limit has been passed at the point of greatest bending the deflection increases very rapidly, owing to the decrease of the modulus of elasticity. If the eccentricity is very small a very small addition to the load will produce an infinite deflection. If the eccentricity is larger, the extent to which the column may be strained beyond the propor-

tional limit also becomes larger. Until this point has been determined we must, however, regard the proportional limit as the determining factor in calculating the strength of columns. These facts are mentioned by the author, but he fails to draw the conclusion to which they point. Mr. Jonson.

The author argues that in designing a column it is necessary to guard against two totally independent modes of failure, viz., failure by excessive intensity of fiber stress, and failure by instability. He accordingly proposes the use of two formulas (7 and 2), or rather two forms of one formula, the second being nothing but the special form which the first assumes when the equivalent eccentricity is reduced to zero. This is clearly unnecessary. If a column is strong enough to sustain a certain load eccentrically applied, it is also strong enough when this load is applied exactly in the axis of the column.

In Fig. 46, the curve of Formula (2) falls below that of Formula (7) when $\frac{l}{r} > 88$. But this is due to the application of a factor of safety to E in the former formula. If the same factor of safety had been applied to E in both formulas, Curve 2 would always have been above Curve 7.

The formula here given was, as far as the writer knows, first applied to experimental results by A. Marston, Assoc. M. Am. Soc. C. E. Mr. Marston, the same as the author, proposes a constant value for the relation $\frac{a\,c}{r}$. This implies that a short column is more likely to be imperfect than a long one.

It seems to the writer that the amount of imperfection in a column ought to be proportional to the length, hence, since the effect of imperfection is also as the length, the equivalent eccentricity ought to be proportional to the square of the length of the column.

Now, if l^2 were introduced into the cyclometrical function of the formula, it could not be solved conveniently. We will, therefore, substitute the value $\frac{1}{w}$, thus making $c = \frac{r^2\,k}{a\,w}$, where k is the empirical constant, the coefficient of imperfection, and consequently Equation V becomes

$$l = r\sqrt{\frac{E}{w}}\ \cos.^{-1}\left(\frac{k}{s} + \frac{a\,b\,w}{r^2\,s}\right) \ldots\ldots\ldots\ldots \text{VI}$$

How this conclusion agrees with experience, may be seen from Table No. 5, which shows the coefficient of imperfection of some of the round-ended wrought-iron columns tested by Mr. James Christie. This table seems to show that we come much nearer the truth by assuming k to be a constant than when we make it a variable, proportional to w. A very close agreement, of course, cannot be expected.

Mr. Jonson.

TABLE No. 5.

No.	$\frac{l}{r}$	k.	Averages of k.	No.	$\frac{l}{r}$	k	Averages of k.
221	44	562		202	229	2 159	
222	44	4 430		213	239	9 164	2 963
218	48	7 403	4 507	208	251	1 667	
220	63	2 170		225	294	−1 137	
214	80	8 209		227	304	−32	
219	81	4 270		228	305	2 244	
215	103	1 905		203	306	4 627	3 205
210	113	7 640		209	310	9 111	
205	129	3 772		204	378	−76	
224	130	634		226	424	7 873	4 478
216	138	1 901		229	456	1 083	
211	145	5 542					3 694
206	146	834	3 318				
200	158	−2 166					
217	170	10 882					
201	179	2 160					
207	186	2 193					
212	195	−183					

$$k = (p - w) \cos. \frac{l}{2\,t}\sqrt{\frac{w}{E}},\ p = 38\,000,\ E = 28\,000\,000.$$

It seems to the writer that a fractional factor of safety should be applied to each of the three empirical constants in the formula, viz., E, k and p or t. Designating this factor by K, the least of the two values $K_3 p - w$ and $K_3 t + w$ by s_1, we have the formulas for columns of uniform cross-section and elasticity in their final form.

One end fixed and one end free:

"Central" load;

$$\frac{l}{r} = \sqrt{\frac{K_1 E}{w}} \cos.^{-1} \frac{k}{K_2 s_1}$$

Eccentric load;

$$l = r\sqrt{\frac{K_1 E}{w}} \cos.^{-1}\left(\frac{k}{K_2 s_1} + \frac{a\,b\,w}{r^2 s_1}\right)$$

Both ends hinged:

"Central" load;

$$\frac{l}{r} = 2\sqrt{\frac{K_1 E}{w}} \cos.^{-1} \frac{k}{K_2 s_1}$$

Eccentric load;

$$l = r\sqrt{\frac{K_1 E}{w}}\left[\cos.^{-1}\frac{k}{K_2 s_1} + \cos.^{-1}\left(\frac{k}{K_2 s_1} + \frac{a\,b\,w}{r_2 s_1}\right)\right]$$

One end fixed and one end hinged:

"Central" load;

$$\frac{l}{r} = 3\sqrt{\frac{K_1 E}{w}} \cos.^{-1} \frac{k}{K_2 s_1}$$

Eccentric load;

$$l = r\sqrt{\frac{K_1 E}{w}}\left[2\cos.^{-1}\frac{k}{K_2 s_1} + \cos.^{-1}\left(\frac{k}{K_2 s_1} + \frac{a\,b\,w}{r^2 s_1}\right)\right]$$

Mr. Jonson.

Both ends fixed:

$$\frac{l}{r} = 4\sqrt{\frac{K_1 E}{w}}\cos.^{-1}\frac{k}{K_2 s_1}$$

Both ends flat:

For tension: $$\frac{l}{r} = 4\sqrt{\frac{K_1 E}{w}}\cos.^{-1}\frac{k}{K_2 w}.$$

For compression: Same as both ends fixed.

C. T. PURDY, M. Am. Soc. C. E.—This is an extremely practical subject for engineers who have many columns to dimension or design. In the Astoria Hotel there are about 3 600 separate columns, each of which had to be especially considered. Ten minutes given to each would require 600 hours for all of them. In the speaker's practice every column is analyzed; if there are eccentric loads the exact effects must be determined, and it is therefore important that the methods should be such as will secure the greatest amount of work in the least possible time. Mr. Purdy.

Very few who write regarding column designing realize or appreciate the importance of eccentricity of loading. The speaker does not refer to bridge designing so much as to building work. Most of the columns in buildings are short, and therefore the element of eccentric loading becomes the more important one. The effect of the direct load in such columns is mostly a crushing strain, while the element of bending is of least importance. The effect of eccentricity is rarely lacking, for an even loading is exceptional. From the practical point of view, therefore, the speaker believes that the author's first statement is the most notable one in the paper.

There is great need for a thorough series of experiments on eccentrically loaded columns, but the task is too great to be undertaken by a private engineer. So far as the speaker knows, only two practical tests of eccentrically loaded columns have been made.

WILLIAM CAIN, M. Am. Soc. C. E. (by letter).—This is a valuable paper, not only for a very clear presentation of the theory affecting the practical column, but for the numerous diagrams comparing the theory, supposed to apply to crippling loads approximately, with the results of 1 789 tests of columns by various experimenters. Mr. Cain.

Assuming the neutral line of the column under eccentric loading to be a parabola, the author's equations (5), (7) and (8), follow logically. The writer has called attention* to the propriety of using the parabola approximately, for the neutral line, by comparing results with the exact formula and finding them identical when $\frac{l}{r}$ was

* "Theory of the Ideal Column," *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 120.

Mr. Cain. small. The agreement is still more plainly seen when the author's equation

$$\Delta = \frac{P l^2 e}{8 E I - \frac{5}{6} P l^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

for the deflection at the center of the column pivoted at the ends, from the primitive axis, is compared with the exact result for a prismatic homogeneous column,*

$$\Delta = e \left[\text{sec.} \left(\frac{1}{2} \frac{l}{r} \sqrt{\frac{P}{A E}} \right) - 1 \right]$$

Putting $\theta = \frac{1}{2} \frac{l}{r} \sqrt{\frac{P}{A E}}$, the development of sec. θ for those values of $\frac{l}{r}$ that give a converging series, is found by the differential calculus to be

$$\text{Sec. } \theta = 1 + \frac{\theta^2}{2} + \frac{5}{24} \theta^4 + \frac{61}{720} \theta^6 + \ldots\ldots$$

On neglecting θ^6 and higher powers of θ, the last formula gives approximately,

$$\Delta = \frac{e}{8} \frac{l^2}{r^2} \frac{P}{A E} \left[1 + \frac{5}{48} \frac{l^2}{r^2} \frac{P}{A E} \right]$$

which is sensibly correct when θ is a comparatively small fraction, and incorrect otherwise.

But this is exactly equal to Equation (2), if the numerator is divided by the denominator in the right member and the division carried out to two terms.

Hence, the assumption that the neutral line is approximately a parabola for small values of $\frac{l}{r}$ is again proved. The result is a closer approximation, likewise, than that given in "Theory of the Ideal Column" where the writer neglected terms containing θ^4 and higher powers of θ.

Of course, the approximate development for Δ could not be used for most of the values of $\frac{l}{r}$ given on the author's diagrams, and it cannot be utilized, therefore, in deducing the equivalents of (7) and (8); in fact, there would be no gain in doing so, as the author's formulas are briefer than the results corresponding.

The most compact formula is the exact one given by Ernst F. Jonson, Assoc. M. Am. Soc. C. E., and which, adopting the author's notation, is,

$$\frac{l}{r} = \sqrt{\frac{E}{f_a}} \cos.^{-1} \left(\frac{c e}{r^2} \frac{f_a}{f_b} \right)$$

* See Mr. Marston's Formula (1); *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 113.

where $f_b = (F_c - f_d)$ for the extreme fiber on the concave side and $f_b = (-F_t + f_d)$ for the extreme fiber on the convex side, F_t being + when compressive, — when tensile (page 350). Mr. Cain.

Recurring now to "the application of the formulas in practice" (page 414), there is room for difference of opinion as to basing the factor of safety on the lower-limit curves of the diagrams, especially where the lower-limit curves pass below all the plotted positions as given by experiment.

If very small factors of safety are used, then, undoubtedly, the lower-limit curves should give the basis for the formulas to use in practice, or disaster might follow on increasing the loads, as the column in question might have a crippling strength near that given by the lower-limit curve. Thus, if the factor of safety was 2 and the loads were eventually doubled (as has happened on a number of railways in this country in the past), the column would certainly be regarded as dangerous if the factor 2 was based on the average curve passing somewhat centrally through the points on the diagram given by experiment.

But if the factor is 4 or 5, it seems safe to take the average curve, even if the loads are to be doubled in twenty or thirty years. Some of the columns will thus have factors of safety above that given and others below; so it would seem that the greatest economy should be secured by basing the safe curve upon the average curve.

If there is any merit in this contention, then the objection is more marked when for flat-ended columns "the curve indicating the load at which tensile stress commences" falls far below the results for experiments, as in some of the diagrams. In fact, failure does not begin for such a load for any given column. The case is similar in one respect to the voussoir arch without mortar joints, when the line of the centers of pressure falls outside the middle third. No tension is exerted on a joint where this occurs, but the compressive stresses are distributed according to a uniformly increasing law, and the material may be strong enough to withstand the maximum stress induced. Similarly, in the column, where the resultant force is not too near the edges of the end sections, the maximum stresses in the extreme fibers may not be too great for strength or stability. Some allowance, however, should be made here for such cases, and the results of experiments should be the main guide.

JOSEPH MAYER, M. Am. Soc. C. E. (by letter).—This paper contains a valuable collection of tests of columns, and is an interesting effort to derive, by means of mathematical reasoning, formulas giving the fiber stresses in actual columns. Mr. Mayer.

The author appreciates the necessity of introducing empirical coefficients in his formulas, for the purpose of allowing for eccentricity

Mr. Mayer. of loading, initial curvature, etc., to make them agree with the tests. One of the purposes of the paper is to find formulas or curves for the ultimate strengths of columns. The curves obtained deviate very far from the ultimate strengths of short columns shown by the tests. The engineer cares most for the strength of columns of a length of less than 150 radii of gyration, and the curves of ultimate strength ought to be approximately correct for these columns. For all kinds of steel, wrought-iron and wooden columns it is possible to find straight lines which agree better with the actual ultimate strength shown by the tests of columns of less than 150 radii of gyration than the author's complicated curves. These straight lines are more convenient for use and more easily remembered.

The engineer desires to use dead loads per square inch equal to half of those producing stresses equal to the elastic limit. For very short steel or wrought-iron columns the elastic limit is about 0.6 of the ultimate strength. The proper dead load is, therefore, 0.3 of the ultimate strength, and the ultimate strength is about equal for tension and compression. For longer columns, the elastic limit and ultimate strength are nearly identical. A curve which starts with 0.3 of the ultimate strength for columns of no length and approaches half the ultimate strength of long columns answers the purposes of the engineer. A straight line, of the equation $u = a - b\,\frac{l}{r}$, where $a = 0.3$ of the ultimate strength of the steel or wrought iron of which the column is made, $b = \frac{a}{250}$, will give the proper unit strain u for the dead load of columns of less than 150 radii of gyration, with as much accuracy as the uncertainty of the case allows. More complicated formulas would only be justified if they agreed more closely with the tests than do these simple formulas, which is not the case for columns of moderate length, which are alone used in practice.

The formulas become, therefore, for iron $14\,000 - 56\,\frac{l}{r}$; for medium steel $18\,000 - 72\,\frac{l}{r}$.

The usual straight-line formulas for timber are as accurate as the author's formulas. While the writer, therefore, cannot agree with the author in his conclusions, he highly appreciates the services rendered by the latter in collecting the tests and presenting his views in such a lucid and simple manner.

Mr. Frye. ALBERT I. FRYE, M. Am. Soc. C. E. (by letter).—The author's assumption that a deflected column will assume a parabolic curve when subjected to an eccentric loading is fairly correct and allowable. It will be, as he states, somewhere between a curve of sines and a circle, depending on the amount of eccentricity.

Mr. Frye.

The writer found, several years ago, that for a beam loaded with a parabolic load W, in Fig. 53, the bending moments and the deflections will be nearly proportional to the sines of the respective angles; the ends of the beam corresponding to 0°, and the center of the beam to 90°. From this he obtained practically Euler's formula,

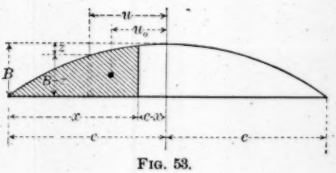

FIG. 53.

$$P = \frac{\pi^2 E I}{l^2}.$$

This demonstration is as follows:

Let W = total weight on the beam;
W_o = weight to left of section considered;
B = maximum intensity of load;
c = half span $= \frac{l}{2}$;
x = distance from end to section considered;
u_o = distance to center of gravity of load to left of section considered.

$$\text{then,} \quad u_o = \frac{\int_{c-x}^{c} u\,(B-z)\,du}{\int_{c-x}^{c} (B-z)\,du} = \frac{12\,c^2 - 12\,c\,x + 3\,x^2}{12\,c - 4\,x}, \quad \ldots\ldots\ldots (1)$$

and when $$x = c,\ u_o = \frac{3}{8}c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

$$W_o = \int_{c-x}^{c} (B-z)\,du = \frac{B\,(3\,c\,x^2 - x^3)}{3\,c^2} \ldots\ldots\ldots\ldots\ldots (2)$$

and when $$x = c,\ W_o = \frac{2}{3}\,B\,c$$

or $$W = \frac{4}{3}\,B\,c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2a)$$

The moment at any section distant x from the end of the beam equals:

$$M_x = \frac{2}{3}B\,c\,x - \frac{B\,(3\,c\,x^2 - x^3)}{3\,c^2}\,(u_o + x - c).$$

Substituting the value of u_o in (1) and reducing,

$$M_x = \frac{B\,(8\,c^3\,x - 4\,c\,x^3 + x^4)}{12\,c^2} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

and when $$x = c,\ M = \frac{5}{12}\,B\,c^2 \ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

Mr. Frye. The angle of slope of the deflected beam at any point distant x from the end equals:

$$i = \frac{1}{EI}\int M\,dx$$

$$= \frac{B}{12\,c^2\,EI}\left(4\,c^3\,x^2 - c\,x^4 + \frac{x^5}{5}\right) + k.$$

To find the value of the constant k,

when $x = c, \quad i = o,$

therefore, $$k = -\frac{B}{12\,c^2\,EI}\left(\frac{16\,c^5}{5}\right)$$

and $$i = \frac{B}{12\,c^2\,EI}\left(4\,c^3\,x^2 - c\,x^4 + \frac{x^5}{5} - \frac{16\,c^5}{5}\right) \ldots\ldots\ldots (4)$$

when $$x = 0, \quad i = \frac{4}{15}\,\frac{B\,c^3}{EI} \ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

The deflection of the beam at any point distant x from the end equals:

$$v = \int i\,dx = \frac{B}{12\,c^2\,EI}\int\left(4\,c^3\,x^2 - c\,x^4 + \frac{x^5}{5} - \frac{16\,c^5}{5}\right)dx$$

$$= \frac{B}{12\,c^2\,EI}\left(\frac{4\,c^3\,x^3}{3} - \frac{c\,x^5}{5} + \frac{x^6}{30} - \frac{16\,c^5\,x}{5}\right) + k_1$$

when $x = 0, \quad v = 0$; therefore $k_1 = 0,$

and $$v = \frac{B}{12\,c^2\,EI}\left(\frac{4\,c^3\,x^3}{3} - \frac{c\,x^5}{5} + \frac{x^6}{30} - \frac{16\,c^5\,x}{5}\right) \ldots\ldots\ldots (5)$$

when $$x = c, \quad v = -\frac{61}{360}\,\frac{B\,c^4}{EI} \ldots\ldots\ldots\ldots\ldots (5a)$$

Dividing the half span c corresponding to 90° into six parts of 15° each, and calculating from (3) the bending moments at each section; calling the bending moment at the center of the beam unity, and likewise, using (5) for deflections; calling the deflection at the center of the beam unity, and comparing these results with the curve of sines, and the parabolic curve, gives the results shown in Table No. 6.

TABLE No. 6.

	0°	15°	30°	45°	60°	75°	90°
Bending moments	0	0.263	0.506	0.712	0.869	0.966	1
Deflections	0	0.262	0.501	0.707	0.866	0.966	1
Curve of sines	0	0.259	0.500	0.707	0.866	0.966	1
Parabolic curve	0	0.306	0.556	0.750	0.889	0.972	1

Mr. Frye.

It will be seen on examination of Table No. 6 that the bending moments and the deflections correspond very nearly with the curve of sines of the theoretical deduction of Euler. As stated by the author, the actual curve will depart from the above ideal upon the application of an eccentric load on the column, and tend to conform more closely to the circle. By calculation, the writer has found that, for a deflection of unity at the center, in a column 480 units in length, the circle and the parabola are practically coincident. This would tend to show that, so far as the curve of the column is concerned, the assumption is on the basis of a preponderance of eccentric loading, since the latter, taken abstractly, would curve the column to the arc of a circle, or practically a parabola. The assumption, therefore, is on the side of safety.

The bending moment at the center section of the column—Equations (3*a*) and (2*a*)—equals:

$$M = \frac{5}{12} B c^2 = \frac{5}{8} W_o c$$

$$= \frac{5}{32} W l \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

If $M = \frac{f I}{y}$, then $f = \frac{5}{32} \frac{W l y}{I}$ ..... (6*a*)

The deflection at the center section of the column—Equations (5*a*) and (2*a*)—equals:

$$v = -\frac{61}{360} \frac{B c^4}{E I} = -\frac{61}{240} \frac{W_o c^3}{E I}$$

$$= -\frac{61}{3840} \frac{W l^3}{E I} \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

From (6*a*), $W = \frac{32 f I}{5 l y}$, therefore $v = -\frac{61}{600} \frac{f l^2}{E y}$ ..... (7*a*)

Substituting this value of v in the general equation $M = P v$, and, omitting the minus sign, there is obtained:

$$P = \frac{M}{v} = \frac{\frac{f I}{y}}{\frac{61}{600} \frac{f l^2}{E y}} = \frac{600}{61} \frac{E I}{l^2} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$\frac{600}{61} = 9.836, \text{ whereas } \pi^2 = 9.87.$$

This proves that a lateral parabolic load W will produce practically the same curvature in a column as a direct concentric load P.

The paper is very valuable in presenting such a collection of tests, and so closely interpreting the causes of failure.

Mr. Moncrieff. J. M. MONCRIEFF, M. Am. Soc. C. E. (by letter).—The precise character of the curve assumed by a bent column is stated in the paper to be a matter of small importance, and the writer will go further than this, it is of no practical importance. This is shown clearly in the paper in the comparison between the formula (Euler's) for an ideal column and the writer's formula based on the assumption that the curve is parabolic.

As a matter of fact, the irregularities in the condition of the material, and in the conditions usually met with in practice, will cause the curve to be neither a curve of sines nor a parabola.

Mr. Ernst F. Jonson's modification or transformation of Mr. Marston's formula does not remove it from the category of those formulas which are awkward and inconvenient to use in practice.

A comparison, in actual working out, between Mr. Jonson's formula and that of the writer, will certainly convince anyone that the simplicity of the latter, in practical work, is very much greater, and also that it is, in every practical sense, quite as correct. There is nothing to be gained by introducing unnecessary mathematical refinements which have little or no influence on the results.

Mr. Jonson states that:

"Apart from its mathematical form, the foregoing formula (Mr. Jonson's) differs from that of the author in that it makes the maximum load a function, not of the ultimate strength of the material, but of the proportional limit."

There is no difference between the formulas in this respect, as each is based on the primary assumption that the material dealt with is perfectly elastic.

Mr. Jonson probably means that he, personally, does not approve of taking the maximum load to be a function of the ultimate strength of the material.

In the paper, the values given to F_c, in making the comparison between the results of calculation and the records of tests carried to ultimate column failure, are not values of the ultimate compressive strength of the material, but are only moduli of column rupture (see lines 18 to 28 of page 369).

The writer is very glad to have Professor Cain's appreciation of the theory of the paper and of the formulas deduced therefrom. His suggestion, that the greatest economy should be secured by basing the safe load upon the average tests, however, does not appear to the writer to be a rational one.

Reference to the diagram (Fig. 34) showing the tests of wrought-iron flat-ended columns, will show that, between the proportions of $\frac{l}{r} = 40$ to 490, a considerable number of results falls close to the writer's lower-limit curves, some being below the curves; and, unless

doubts can be cast upon the care and accuracy of the experimenters who carried out these tests, it must be recognized that the lesson which these results teach is, that, in columns of these proportions, greater ultimate strength cannot be safely counted upon. Mr. Moncrieff.

Why, then, should engineers soothe their minds with the hope that the strength may happen to be considerably higher? No economy can result from such a procedure.

The writer is much obliged to Mr. O'Hanly for the trouble he has taken in checking over some of the computed deflections. The results caused the writer to have an entirely independent check made of the working out of the calculated deflections, and, while it was found that Mr. O'Hanly's figures could not be entirely accepted, his criticism has resulted in several errors being discovered and corrected.*

In reply to Mr. Joseph Mayer's remarks, the writer desires to disclaim that "one of the purposes of the paper is to find formulas or curves for the ultimate strengths of columns." The writer's purpose was to develop a practical and simple theory of column strength under central or eccentric loads, on the lines stated in the beginning of the paper, and, in order to justify the theory and its resulting formulas, they were compared with the results of practical experiment. The results of the comparison scarcely warrant Mr. Mayer's criticism that:

"The curves obtained deviate very far from the ultimate strengths of short columns shown by the tests. The engineer cares most for the strength of columns of a length of less than 150 radii of gyration, and the curves of ultimate strength ought to be approximately correct for these columns."

Attention is called to the diagrams of tests of the materials which are in most common use to-day, *i. e.*, wrought iron and mild steel.

Fig. 22; Containing all the Chief Tests of Wrought-Iron Columns with Round and Pivoted Ends.

At $\frac{l}{r}$ = about 46, lower-limit curve is 1 600 lbs. per square inch below lowest tests.

" 76, " " 1 100 " per square inch below lowest tests.

" 112, " " 900 " per square inch above lowest tests.

" 131, " " 1 000 " per square inch below lowest tests.

" 145, " " 200 " per square inch above lowest tests.

" 170, " " 2 000 " per square inch above lowest tests.

* This refers to the discussion by J. L. Power O'Hanly, M. Am. Soc. C. E., which was published in *Proceedings* for August, 1900, but is not reproduced here, the revision of the paper making it unnecessary.

Mr. Moncrieff. *Fig. 24; Eccentric Loading of Wrought-Iron Bars.*

From ratio $\frac{l}{r} = 44$ to ratio $\frac{l}{r} = 342$ } The tests are included between two curves which, at their point of maximum deviation, are 3 700 lbs. per square inch apart.

Fig. 25; Eccentric Loading of Wrought-Iron Bars.

From ratio $\frac{l}{r} = 46$ to ratio $\frac{l}{r} = 183$ } The tests lie between pairs of curves which are only 600 lbs. per square inch apart at the maximum point.

Fig. 26; Tests of Fixed-Ended Wrought-Iron Columns.

For $\frac{l}{r} =$ 43, lower-limit curve is 400 lbs. per square inch below lowest test.

70, " " 200 " per square inch below lowest test.

105, " " 1 600 " per square inch below lowest test.

131, " " 400 " per square inch below lowest test.

141, " " 1 200 " per square inch above lowest test.

147, " " 2 500 " per square inch above lowest test.

175, " " 400 " per square inch above lowest test.

217, " " 1 300 " per square inch above lowest test.

221, " " 1 400 " per square inch above lowest test.

Fig. 34; Containing all the Tests of Flat-Ended Wrought-Iron Columns. In this diagram the lower-limit curve from $\frac{l}{r} = 40$ to $\frac{l}{r} = 490$ gives a very fair line for the lowest tests for the great range just mentioned.

Between $\frac{l}{r} = 40$ and $\frac{l}{r} = 150$ there is one test, *i. e.*, at 125 about, which is 3 600 lbs. per square inch below the writer's curve, and there are several others below the curve between 40 and 150, but the maximum of these is 2 500 lbs. per square inch below the curve.

Does Mr. Mayer mean that the writer has overestimated the ultimate strength of these columns up to 150 radii in length? If so, how would he regard the estimate of strength recommended by Professor Cain, *i. e.*, the average of all tests, which would be very much higher than the writer's curve?

Fig. 38; Containing all the Tests of Hinged-Ended Wrought-Iron Columns.—It is unnecessary to take figures for this diagram. The lower-

limit curve drawn by the writer runs almost through the whole of the lowest tests, with one exception, *i. e.*, that at $\frac{l}{r} = 107$, and the agreement is not only for tests up to 150 radii in length, but extends from 35 up to 500. Mr. Moncrieff.

Fig. 39; Containing all Tetmajer's Tests of Mild-Steel Columns with Pivoted Ends.—From $\frac{l}{r} = 30$ to $\frac{l}{r_1} = 355$, the writer fails to find the great divergence of the lower-limit curve from the tests, referred to by Mr. Mayer.

Fig. 40; Flat-Ended Mild-Steel Columns.—From $\frac{l}{r} = 25$ to $\frac{l}{r} = 325$, the lower-limit curve drawn cannot truly be said to deviate, to any great extent, from the tests.

Fig. 41; Mild-steel; Hinged Ends.—Can Mr. Mayer, or any other engineer, draw through these tests a straight line which will be a fair guide in practice? If a column at $\frac{l}{r} = 118$ has a strength of 45 000 lbs. per square inch, why should another column at $\frac{l}{r} = 125$ have a strength of only about 32 000 lbs. per square inch?

Why should columns of from 55 to 110 radii long be credited with the strengths shown by the tests on this diagram, when columns very little longer, of the same material and by the same experimenter, exhibit such a serious and sudden drop in the test results?

What straight and single-line formula can give a rational idea of such vagaries as these?

Are Mr. Mayer's straight-line formulas to be applied to columns with pivoted ends, hinged ends, flat ends, or fixed ends, indiscriminately?

The idea of the paper was to give a general theory, with simple resulting formulas, which "should have a wide range of application to cover the conditions met in engineering practice."

No straight-line formula can fulfil this condition, or be applied to the varying cases of eccentric loading, or to cases where there are side loads or exceptional forms of cross-section.

In conclusion, the writer heartily thanks those who have discussed the paper for their very kindly expressions of approval.

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TRANSACTIONS.

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No. 892.

SOME PECULIAR RAILROAD BRIDGE ACCIDENTS.

BY J. I. BOGGS, ASSOC. M. AM. SOC. C. E.

PRESENTED DECEMBER 19TH, 1900.

WITH DISCUSSION.

Several years ago, while the writer was Division Engineer of Maintenance of Way on the Houston and Texas Central Railway, two or three accidents happened to the bridging under his charge, which may possibly be of some historical interest.

The first bridge, Fig. 1, a pin-connected Pratt truss over Richland Creek, of 128 ft. 7½ ins. span, was built by the Phœnix Bridge Company, in the early part of 1887, and, as in all similar work by that company at that time, the posts and upper chords consisted of the old closed Phœnix columns, set into castings at the panel points.

The structure was at the foot of a stiff grade, and while a heavily loaded freight train, moving at a high rate of speed, was passing, a bale of cotton became dislodged and was hurled against the first post, marked "A," Fig. 1, with sufficient force to bend it 13 ins. out of line and knock it clear of the structure, thus leaving the truss with one of its main members missing.

At the time of the accident the structure was entirely covered by

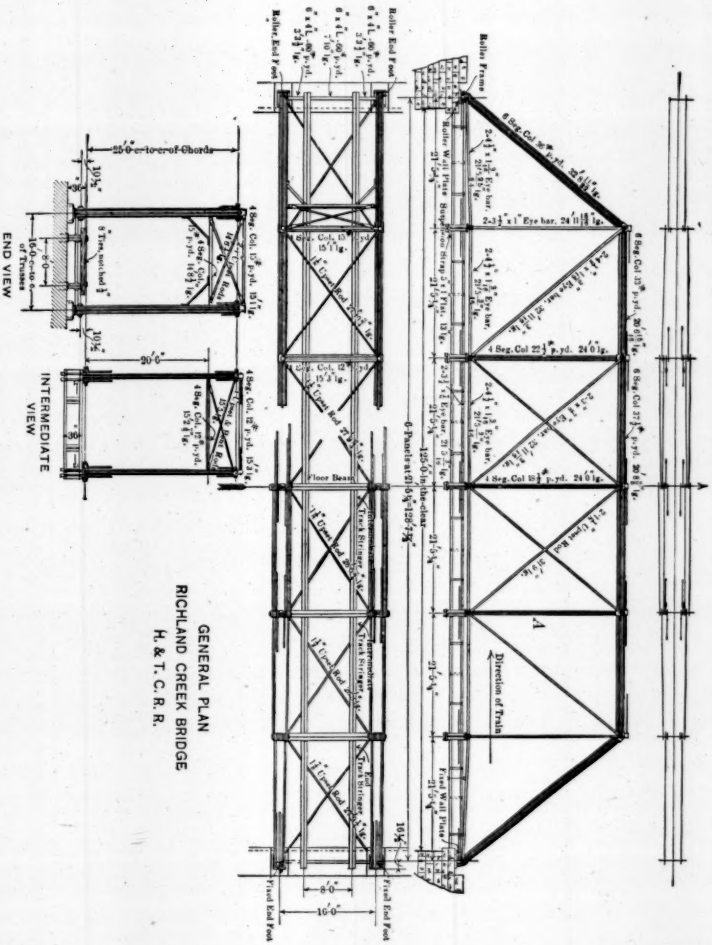

GENERAL PLAN
RICHLAND CREEK BRIDGE
H. & T. C. R. R.
END VIEW
INTERMEDIATE VIEW
Direction of Train
Roller Frame
Roller End Feet
Floor Beam
Final End Feet
25'0" c-to-c of Chords
16'0" c-to-c of Trusses
20'6"
6 Panels at 21'0" = 126'0" in the clear

Fig. 1.

the train, weighing approximately 2 500 lbs. per lineal foot. Contrary to all expectations and calculations the bridge carried the remainder of the train, consisting of about twenty loaded cars, across in safety, although the main post was gone.

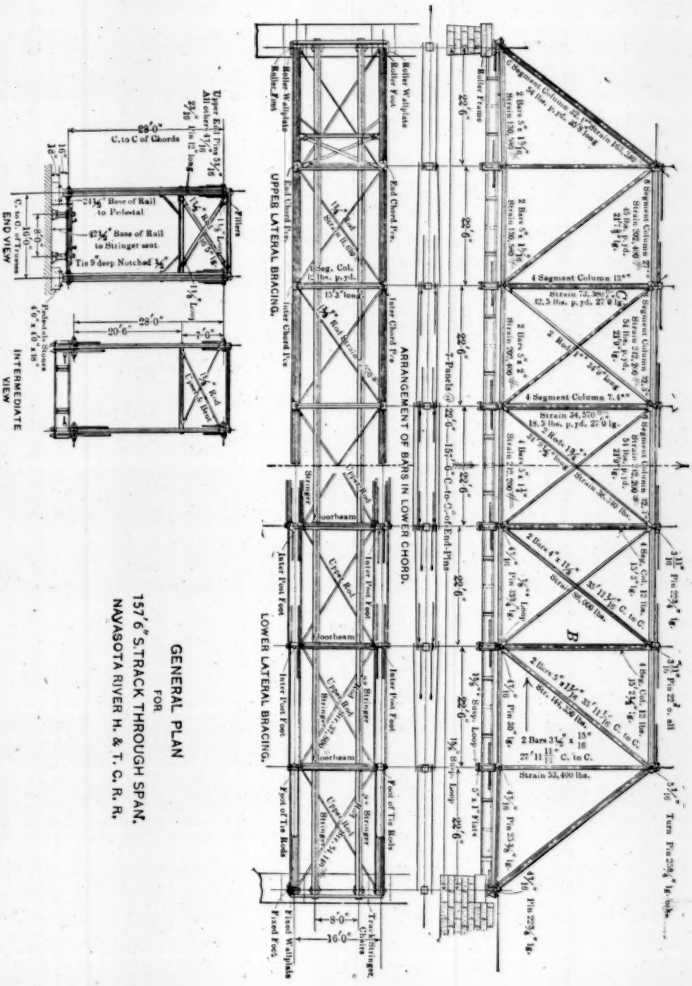
That there was a tremendous bending moment in the upper chord was shown by the following: As quickly as possible a 12 x 12-in. stick of timber, cut to accurate height, was fitted between the upper and lower castings, and a train allowed to pass; as soon as this train had crossed, it was found that the ends of the timber were so badly crushed that it was unfit for further service. A piece of ½-in. plate iron was then inserted between the castings and the timber. This did good service and continued in use until the original member had been taken to the shops, thoroughly repaired and replaced.

In the case of the Navasota River Bridge, Fig. 2, a similar structure, of 157 ft. 6 ins. span, the two posts marked "B" and "C," the former in the left, and the latter in the right-hand trusses, were likewise destroyed by two bales of cotton falling off the same train in opposite directions. Again the train crossed safely, the same steps were taken, and the posts were repaired and replaced.

In both cases the bridges were floored with 8 x 10-in. ties, spaced at 16-in. centers, and had 56-lb. rails.

There was another interesting occurrence at the Trinity River Bridge, a very light Whipple truss, of 130 ft. span, with panel lengths of 12 ft. 6 ins., built in 1877 by the same company. The writer regrets that he has no diagram of this structure. The only idea he can give of its frailty being the statement that the floor beams were only 20 ins. in depth, and the track stringers 7 x 14-in. wooden beams, packed three under each rail. In this instance part of a carload of lumber fell off and struck the first post, permanently bending it about 10 ins. out of the vertical. Several very heavy trains, both freight and passenger, with engines weighing 105 tons, passed over the bridge before the accident to the post was discovered.

The Richland Creek and Navasota River cases are interesting from the fact that it has sometimes been argued that no Pratt truss bridge would stand up under these conditions, and the writer very much doubts if it would if the connections were all rigid. If it did stand it is quite likely that the structure would be so disturbed that a good portion, if not all, would have to be replaced, and this would also



GENERAL PLAN
FOR
157'6" S. TRACK THROUGH SPAN.
NAVASOTA RIVER H. & T. C. R. R.

FIG. 2.

probably apply in the case of the floor beam riveted to the post above the chord.

The American pin-connected truss has become famous, the world over, and why American engineers should now incline to the rigid connections of their foreign brethren is, to say the least, remarkable. In the case of the pin-connected truss, where the load to be transmitted is 50 000 lbs., it is known that it must enter the pin, and the pin be designed accordingly, but in the rigid connection, with, say, a shear of 5 000 lbs., we find 10 rivets, two are thrown in for good measure, making twelve all told; these are distributed two, three, two, and so on. Now, how is the 5 000 lbs. distributed? Does each rivet carry an equal portion, or do the first five rivets take the greater part of the stress, producing secondary stresses which will eventually destroy the usefulness of the structure at a much earlier date than would the old pin-connected? At the hazard of being considered ancient, the writer has ventured on the above remarks, and, while not decrying modern methods, he is inclined to the opinion, that there was considerable inherent merit in the designs of our great predecessors, and that the design of later bridges can be seriously questioned; and as the former has stood the test of time, he thinks that it might be as well to hesitate somewhat before venturing too far into new fields.

The writer submits the foregoing as a matter of history and to show the remarkable performances of some of the old designs. He regrets that he has not the time to enter into a mathematical demonstration of cause and effect.

DISCUSSION.

Mr. Mayer. JOSEPH MAYER, M. Am. Soc. C. E.—The pin-connected bridges described in the paper had a remarkable experience, and behaved certainly much better than their most enthusiastic advocate would venture to hope.

It is a curious fact that, while rigid connections are becoming the fashion in this country, they are abandoning them in Germany. The Rhein Bridge, in Worms, is an instance where, for the purpose of avoiding the secondary strains due to the loading of one track only, and the consequent larger deflection of one truss, the floor is freely suspended from the trusses above. The top struts are practically hinged, and there are no intermediate vibration bracings. The stringers have sliding connections to the floor beams to prevent the bending of the latter.

It cannot well be doubted that the secondary strains, which are very large in some members, like end suspenders and vertical posts with floor beams riveted between, could be largely reduced or altogether avoided by such connections. The speaker believes that the movement against hinges which is now in vogue in this country will be abandoned after trial and experience of the large secondary strains caused thereby.

Mr. Aylett. PHILIP AYLETT, Assoc. M. Am. Soc. C. E. (by letter).—A few weeks ago there occurred, on an important railway system traversing the Southern States, one of the most remarkable bridge accidents of which the writer has knowledge.

This disaster occurred at the crossing of the railway over a small stream about 150 ft. in width, and with an average depth of 8 ft.

That portion of the structure over the channel consisted of a 50-ft. half-through girder span, while on the east and west approaches there were 1 232 ft. and 118 ft., respectively, of cypress pile trestling.

The spacing of the girders was 12 ft. from center to center of the webs, and the cross-ties or floor beams were of 12 × 12-in. × 12-ft. pine, resting on shelf angles riveted to the webs of the girders. There is a heavy ascending grade in both directions from the river, and it is presumed that trains attained a high rate of speed in crossing the stream in order to facilitate the climbing of the opposite grade.

The girders were supported by three pile-bents (cypress) under each end, each bent having five piles, these being sawed off and capped with two (criss-cross) layers of 12 × 12-in. timbers, drift-bolted to the piling, upon which the girders rested. The wooden portion of the structure had recently been rebuilt, and the girders were reported to have been in first-class condition and amply sufficient for any traffic to which they would be subjected.

On December 14th, 1901, while a local east-bound freight train, containing 38 cars loaded with cross-ties, lumber and naval stores, was

Mr. Aylett. crossing this stream, and after the engine and 8 cars had crossed the girder span, the girders were suddenly knocked from their seats by the derailed trucks or body of a box car, the ninth car from the engine, this car striking the end of the down-stream girder. The fall of the span precipitated 27 of the following cars into the river, breaking down the pile-bents of the west portion of the trestle until the wreckage had completely filled this space and was piled up over 50 ft. above the base of rails. A flat car stood vertically at the summit of this pile of wreckage.

The box car which struck the girder became derailed some 250 ft. before reaching the trestle, as was indicated by the flange marks on the cross-ties, and the car was leaning well over to the right when it eeached the girder span.

Upon striking the girder, the car knocked off its front cross-beam and lost both trucks, and, although loaded with cross-ties, passed over the opening and slid 60 ft. beyond, upon the undamaged portion of the trestle. The caboose and one car stopped at the foot of this mountain of wreckage.

The engine with eight cars passed over the girders safely before these were struck by the ninth car, which, although truckless, followed closely behind.

Five days were consumed in reopening the line for traffic, and it would have been impossible to have accomplished it in that time had not the wreckage been burned to the water level, while fire engines were kept constantly playing upon the adjacent undamaged trestle. Even after the wreckage had been burned to the water level, great difficulty was encountered in pile-driving on account of the sunken timber, barrels of resin, etc., the bed of the river being filled almost solidly with débris and naval stores.

The writer has always been of the opinion that "through girders" were subject to damage and accidents to a greater extent than "through truss spans" in the case of derailments, and has always been opposed to that abominable practice, so much resorted to in cheap through-girder construction, of resting the ties (or floor beams) upon bottom flanges, or angles riveted to the webs of the girders above the bottom flanges, in lieu of a complete floor system of metal floor beams and stringers with riveted connections.

In this case, however, the cause cannot be attributed, in any direct way, to the shelf-angle construction. The writer believes, however, that possibly this disaster would have been avoided had these girders been protected by inside bridge guards, and also probably avoided had the girders been spaced further apart.

It is the writer's practice to space "through girders" 14 ft. and over between centers of flanges of girders, using a steel floor system with riveted connections, all through girders having inside guards.

The direct cause of the derailment was a broken flange in one of the wheels of the ninth box car, portions of this flange being found 500 ft. from the west end of trestle approach.

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No. 893.

THE ADJUSTMENT OF A TRANSIT SURVEY AS COMPARED WITH THAT OF A COMPASS SURVEY.

By CHARLES L. CRANDALL, M. Am. Soc. C. E.

TO BE PRESENTED FEBRUARY 6TH, 1901.

WITH DISCUSSION.

COMPASS SURVEY.

The method of adjustment of the latitudes and departures of a compass survey by Dr. Bowditch's rule, distributing the error in latitude proportionately to the lengths of the sides, and similarly for the departures, is familiar to all civil engineers and surveyors.

The fundamental principles upon which it is based, and the assumption which was made to render the rule simple, are not so generally known.

The main steps are readily traced. The latitude equation is

$$L = l \cos. B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where L = latitude, l = length of side, and B = bearing.

If this equation be differentiated first with respect to l and then with respect to B, it will give the effect upon the latitude of a small change in distance, or of a small change in bearing.

Thus, $d\,L' = d\,l \cos.\ B.$

$$d\,L'' = -\,l \sin.\ B\,d\,B.$$

The uncertainty, or error as it is usually called, in latitude, due to the combined effect of an uncertainty in distance and also in bearing, is equal to the square root of the sum of the squares of the effects due to each separately, where positive and negative errors are equally probable. Thus, if r_l = the error in distance, replacing $d\,l$, r_B = the error in bearing, replacing $d\,B$, the corresponding error in latitude squared,

$$r_L^2 = r_l^2 \cos.^2 B + l^2\, r_B^2 \sin.^2 B \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For the departure,

$$D = l \sin.\ B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Differentiating and substituting as above,

$$r_D^2 = r_l^2 \sin.^2 B + l^2\, r_B^2 \cos.^2 B \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Dr. Bowditch assumed,

$$r_l^2 = r_B^2\, l^2 = l \times \text{constant} = l\,C \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

which reduces equations (2) and (4) to

$$r_L^2 = r_D^2 = l\,C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

i. e., the squares of the uncertainties, or probable errors, in latitude and in departure, are proportional to the lengths of the sides.

In the Method of Least Squares it is shown that the most probable corrections will be those the sum of whose squares is a minimum after each correction has been divided by its probable error; *i. e.*, if the corrections in latitude for the different sides be denoted by v_1, v_2, $v_3 \ldots$, we shall have

$$\frac{v_1^2}{l_1} + \frac{v_2^2}{l_2} + \frac{v_3^2}{l_3} + \ldots = \text{minimum.}$$

Differentiating,

$$\frac{v_1}{l_1} d\,v_1 + \frac{v_2}{l_2} d\,v_2 + \frac{v_3}{l_3} d\,v_3 + \ldots = 0 \ldots\ldots\ldots\ldots (7)$$

Since the sum of the corrections must equal the total error in latitude with its sign changed, a constant,

$$v_1 + v_2 + v_3 + \ldots = k.$$

Differentiating,

$$d\,v_1 + d\,v_2 + d\,v_3 + \ldots = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Comparing (7) and (8), and remembering that each must hold whatever the number of sides, or v's, we have

$$\frac{v_1}{l_1} = \frac{v_2}{l_2} = \frac{v_3}{l_3} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

i. e., the corrections in latitude are proportional to the lengths of the sides, according to the old rule.

The same demonstration will hold for the corrections in departure.

The interesting point, in this connection, is the assumption—Equation (5)—made by Dr. Bowditch that the probable error in chaining equals the probable error in bearing (in π-measure) multiplied by the distance, and increases as the square root of the distance.

The inaccuracy of chaining is usually spoken of as proportional to the distance, but in comparing the different lines of a survey measured with the same chain, the square root of the distance as assumed above is more nearly correct. Assuming an uncertainty which would amount to 1 ft. in 500, which is not unreasonable for compass work, we would have from (5)

$$r_{500} = 1 = \sqrt{500\,C} \quad ; \text{ or } C = 0.0447,$$

from which $r_l = 0.0447\sqrt{l}$; r_B (in minutes) $= \dfrac{0.0447}{\text{sin. } 1'\sqrt{l}}$

Substituting for different distances gives the results shown in Table No. 1.

TABLE No. 1.

Distance, in feet.	Uncertainty in chaining, in feet.	Uncertainty in bearing.
10	0.14	0° 48′
50	0.32	0 21
100	0.45	0 15
500	1.00	0 07
1 000	1.41	0 05
2 000	2.00	0 03

An examination of Table No. 1 will show that the assumption is fairly reasonable, although it gives too great weight to the bearings of long lines, and perhaps too small to those of very short lines.

The method is thus satisfactory as to accuracy and simplicity when applied to the adjustment of a compass survey.

Transit Survey.

In a transit survey proper, no bearings are taken except as a check, but the horizontal angles between the lines are measured. In computing latitudes and departures, or co-ordinates, a meridian is assumed, and the bearings or azimuths are computed from the

horizontal angles. The bearing of the first line from the meridian involves only one angle; that of the second, two angles; etc., one-half way around the figure; the bearings for the other half being more accurately computed by working backward from the meridian.

Thus, with the following notes:

Sta. 1.	Deflection, $M_1 = 92° 0' R.$	Distance, $l_1 = 236$ ft.
2.	$M_2 = 46 \ 0 \ R.$	$l_2 = 244$
3.	$M_3 = 82 \ 0 \ R.$	$l_3 = 324$
4.	$M_4 = 51 \ 57 \ R.$	$l_4 = 183$
5.	$M_5 = 88 \ 0 \ R.$	$l_5 = 437$

FIG. 1.

Assume 5 — 1 as a meridian (Fig. 1).

Bearing, $B_1 = 180 - M_1 = \text{S.}88° 00' \text{E.}$

$B_2 = B_1 - M_2 = 180 - (M_1 + M_2) = \text{S.}42 \ 00 \ \text{E.}$

$B_3 = M_3 - B_2 = M_1 + M_2 + M_3 - 180° = \text{S.}40 \ 00 \ \text{W.}$

$= 180° - (M_5 + M_4) = \text{S.}40 \ 03 \ \text{W.}$

$B_4 = M_4 = \text{N.}88 \ 00 \ \text{W.}$

The discrepancy in angle work is thus thrown into the difference between the bearings of the two lines meeting at 3, and would not be properly distributed by the compass method of adjustment. If the deflections were first adjusted to close the figure, the compass survey method would allow the inaccuracy of the chaining to change the adjusted angles without giving proper weight to the accuracy of angle measurement.

There is thus no reason to assume, as appears to have been almost universally done, that because the Dr. Bowditch rule gives a close adjustment for a compass survey, it will do so for a transit survey.

Theoretically, there are three equations which must be satisfied in adjusting the observed quantities:

(*a*) The sum of the interior angles must equal twice as many right angles as the figure has sides, less four.

(*b*) The algebraic sum of the latitudes must equal zero.

(*c*) The algebraic sum of the departures must equal zero.

The Method of Least Squares requires that the corrections must be such as to satisfy the above conditions, and make the sum of their squares, each square being divided by the square of its probable error, a minimum. Since each bearing must be expressed in terms of the measured angles, and the number of measured angles in each

varies from one to one-half the number of sides, the labor involved is too great for the adjustment of an ordinary transit survey.

In ordinary work the probable error of an angle need not exceed 1 minute, if care is taken in setting over the points and in plumbing the flag poles, using tacks on the stakes for all lines of less than 300 ft., swinging without delay from the back sight to the front sight, and lining in a "range" point to swing from for all lines of less than 50 ft. With these precautions, the probable error need not increase with the shortness of the line, as with the compass.

On very rough ground, or in going through brush, where the flag pole is partly hidden, it may be difficult to keep the probable error below 2 minutes; while, for careful work, the probable error can be readily kept within $\frac{1}{2}$ minute.

For good work the length of sight should be limited to about 1 200 ft.

Ordinarily, it will be more difficult to measure distances to 1:500 than angles to minutes, while an accuracy of 1 : 1 000 is seldom reached except on level ground or for city work.

The accuracy of angle work is thus considerably greater than that of chaining, 1 minute in angle giving 0.15 ft. in 500 as compared with 1 ft. in chaining; or, 0.5 minute, 0.15 ft. in 1 000, as compared with the 1 ft. due to the more accurate chaining.

On this account, it will be admissible to first adjust the angles to close the figure, then compute the bearings or azimuths and assume them to be correct in the final adjustment.

This leaves only the two conditions:

(*b*) Sum of latitudes equal zero.

(*c*) Sum of departures equal zero.

That is,

$$\left.\begin{aligned} l_1 \cos. B_1 + l_2 \cos. B_2 + l_3 \cos. B_3 + = 0 \\ l_1 \sin. B_1 + l_2 \sin. B_2 + l_3 \sin. B_3 + = 0 \end{aligned}\right\} \quad \ldots\ldots\ldots (10)$$

where the total corrections are to be applied on the basis of inaccuracy in chaining.

In order to derive these corrections on the basis of least squares, denote the observed distances by M_1, M_2, M_3, . . . , and the required corrections by v_1, v_2, v_3, . . . The corrected distances will be

$$l_1 = M_1 + v_1, \quad l_2 = M_2 + v_2, \quad l_3 = M_3 + v_3 \ldots$$

Substituting in (10),

$$\left.\begin{aligned}(M_1 + v_1)\cos. B_1 + (M_2 + v_2)\cos. B_2 + \ldots &= 0\\ (M_1 + v_1)\sin. B_1 + (M_2 + v_2)\sin. B_2 + \ldots &= 0, \text{ or}\\ v_1\cos. B_1 + v_2\cos. B_2 + v_3\cos. B_3 + \ldots + q_1 &= 0\\ v_1\sin. B_1 + v_2\sin. B_2 + v_3\sin. B_3 + \ldots + q_2 &= 0\end{aligned}\right\} \ldots (11)$$

where

$q_1 = M_1\cos. B_1 + M_2\cos. B_2 + M_3\cos. B_3 + \ldots =$ error in latitude,

$q_2 = M_1\sin. B_1 + M_2\sin. B_2 + M_3\sin. B_3 + \ldots =$ error in departure.

For convenience, change (11) to

$$\left.\begin{aligned}v_1\frac{L_1}{l_1} + v_2\frac{L_2}{l_2} + v_3\frac{L_3}{l_3} + \ldots + q_1 &= 0\\ v_1\frac{D_1}{l_1} + v_2\frac{D_2}{l_2} + v_3\frac{D_3}{l_3} + \ldots + q_2 &= 0\end{aligned}\right\} \ldots\ldots\ldots (12)$$

where $L_1, L_2, \ldots, D_1, D_2, \ldots,$ denote latitudes and departures.

Aside from (12) we must also have $v_1^2 + v_2^2 + v_3^2 + \ldots$ a minimum, for equal accuracy in chaining the different lines; or, if the accuracy in chaining be assumed to decrease as the square root of the distance,

$$\frac{v_1^2}{l_1} + \frac{v_2^2}{l_2} + \frac{v_3^2}{l_3} + \ldots = \text{minimum}.$$

Differentiating,

$$\frac{v_1}{l_1}dv_1 + \frac{v_2}{l_2}dv_2 + \frac{v_3}{l_3}dv_3 + \ldots = 0 \ldots\ldots\ldots\ldots (13)$$

If the dv's were independent, their coefficients would have to separately equal zero for (13) to hold true. But the v's are connected by (12). Differentiating (12),

$$\left.\begin{aligned}\frac{L_1}{l_1}dv_1 + \frac{L_2}{l_2}dv_2 + \frac{L_3}{l_3}dv_3 + \ldots &= 0\\ \frac{D_1}{l_1}dv_1 + \frac{D_2}{l_2}dv_2 + \frac{D_2}{l_2}dv_2 + \ldots &= 0\end{aligned}\right\} \ldots\ldots\ldots\ldots (14)$$

If we eliminate a dv in (13) for each of (14), the remaining dv's will be independent, and hence their coefficients will separately equal zero. This elimination is best done by the method of indeterminate multipliers, *i. e.*, multiply the first of (14) by x, the second by y, and (13) by -1, and add,

$$\left(x\frac{L_1}{l_1} + y\frac{D_1}{l_1} - \frac{v_1}{l_1}\right)dv_1 + \left(x\frac{L_2}{l_2} + y\frac{D_2}{l_2} - \frac{v_2}{l_2}\right)dv_2 + \ldots = 0 \ldots (15)$$

Now give x and y such values that the coefficients of two of the dv's will be zero, thus eliminating them. The remaining dv's will be inde-

pendent, and hence their coefficients will be zero. This makes the coefficients of all the dv's in (15) equal zero, giving

$$\left.\begin{aligned} x\frac{L_1}{l_1} + y\frac{D_1}{l_1} - \frac{v_1}{l_1} &= 0 \\ x\frac{L_2}{l_2} + y\frac{D_2}{l_2} - \frac{v_2}{l_2} &= 0 \\ x\frac{L_3}{l_3} + y\frac{D_3}{l_3} - \frac{v_3}{l_3} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (16)$$

To find x and y, multiply the first of (16) by L_1, the second by L_2, . . . , and add; then by D_1, D_2, . . . , and add; then by (12)

$$\left.\begin{aligned} \Sigma\left(\frac{L^2}{l}\right)x + \Sigma\left(\frac{L\,D}{l}\right)y + q_1 &= 0 \\ \Sigma\left(\frac{L\,D}{l}\right)x + \Sigma\left(\frac{D^2}{l}\right)y + q_2 &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (17)$$

Σ, as usual, denoting the summation of a number of similar terms,

$$i.\,e.,\ \Sigma\left(\frac{L^2}{l}\right) = \frac{L_1^2}{l_1} + \frac{L_2^2}{l_2} + \frac{L_3^2}{l_3} + .$$

Solving,

$$\left.\begin{aligned} x &= \frac{q_2\,\Sigma\left(\frac{L\,D}{l}\right) - q_1\,\Sigma\left(\frac{D^2}{l}\right)}{\Sigma\left(\frac{D^2}{l}\right)\Sigma\left(\frac{L^2}{l}\right) - \Sigma\left(\frac{L\,D}{l}\right)^2} \\ y &= \frac{q_1\,\Sigma\left(\frac{L\,D}{l}\right) - q_2\,\Sigma\left(\frac{L^2}{l}\right)}{\Sigma\left(\frac{D^2}{l}\right)\Sigma\left(\frac{L^2}{l}\right) - \Sigma\left(\frac{L\,D}{l}\right)^2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (18)$$

Substituting in (16),

$$\left.\begin{aligned} v_1 &= L_1 x + D_1 y \\ v_2 &= L_2 x + D_2 y \\ &\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Adding, $\Sigma(v) = x\,\Sigma(L) + y\,\Sigma(D), = 0$, nearly. . . . . (20)

Also,

$$\left.\begin{aligned} v_{L_1} = \frac{L_1}{l_1}v_1 &= \frac{L_1^2}{l_1}x + \frac{L_1 D_1}{l_1}y \\ v_{L_2} = \frac{L_2}{l_2}v_2 &= \frac{L_2^2}{l_2}x + \frac{L_2 D_2}{l_2}y \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (21)$$

$$\left.\begin{aligned} v_{D_1} = \frac{D_1}{l_1}v_1 &= \frac{L_1 D_1}{l_1}x + \frac{D_1^2}{l_1}y \\ v_{D_2} = \frac{D_2}{l_2}v_2 &= \frac{L_2 D_2}{l_2}x + \frac{D_2^2}{l_2}y \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (22)$$

with, by (17), $\Sigma(v_L) = -q_1$ and $\Sigma(v_D) = -q_2$ . . . . . (23)

Practical Applications.

Having adjusted the transit work, or the angles, to geometrical consistency, by distributing the small error of closure equally among the angles, and having computed the co-ordinates, Formulas (18) to (23) give the method of adjustment of the co-ordinates and of the measured sides by least squares. In deriving these formulas the assumption made by Dr. Bowditch, that the inaccuracy in chaining increases as the square root of the distance, has been retained. The application is quite simple, as all the terms are made up from the latitudes, departures and distances already entered upon the co-ordinate sheet, while the multiplications and divisions can be made with the ordinary slide rule, retaining only three figures.

In order to equalize numbers, so as to retain the same number of decimal places throughout, $100\,l$ is used in place of l in (18), and the change carried through, as follows:

$$\left.\begin{aligned} x' &= \frac{q_2 \Sigma\left(\frac{L\,D}{100\,l}\right) - q_1 \Sigma\left(\frac{D^2}{100\,l}\right)}{\Sigma\left(\frac{D^2}{100\,l}\right)\Sigma\left(\frac{L^2}{100\,l}\right) - \Sigma\left(\frac{L\,D}{100\,l}\right)^2} \\ y' &= \frac{q_1 \Sigma\left(\frac{L\,D}{100\,l}\right) - q_2 \Sigma\left(\frac{L^2}{100\,l}\right)}{\Sigma\left(\frac{D^2}{100\,l}\right)\Sigma\left(\frac{L^2}{100\,l}\right) - \Sigma\left(\frac{L\,D}{100\,l}\right)^2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (18)'$$

$$\left.\begin{aligned} v_1 &= \frac{L_1}{100}x' + \frac{D_1}{100}y' \\ v_2 &= \frac{L_2}{100}x' + \frac{D_2}{100}y' \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)'$$

$$\Sigma(v) = x'\,\Sigma\frac{(L)}{100} + y'\,\Sigma\frac{(D)}{100} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)'$$

$$\left.\begin{aligned} v_{L_1} &= \frac{L_1}{l_1}v_1 = \frac{L_1^2}{100\,l_1}x' + \frac{L_1\,D_1}{100\,l_1}y' \\ v_{L_2} &= \frac{L_2}{l_2}v_2 = \frac{L_2^2}{100\,l_2}x' + \frac{L_2\,D_2}{100\,l_2}y' \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)'$$

$$\left.\begin{aligned} v_{D_1} &= \frac{D_1}{l_1}v_1 = \frac{L_1\,D_1}{100\,l_1}x' + \frac{D_1^2}{100\,l_1}y' \\ v_{D_2} &= \frac{D_2}{l_2}v_2 = \frac{L_2\,D_2}{100\,l_2}x' + \frac{D_2^2}{100\,l_2}y' \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)'$$

$$\Sigma(v_L) = -q_1, \qquad \Sigma(v_D) = -q_2. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)'$$

With the following notation:

x' and y' may be called auxiliaries;

q_1 = error in latitude;

q_2 = error in departure;

$L_1, L_2, L_3, \ldots,$ = latitudes of sides $l_1, l_2, l_3, \ldots$;

$D_1, D_2, D_3, \ldots,$ = departures of sides $l_1, l_2, l_3, \ldots$;

Σ denotes the summation of a number of similar terms;

$v_1, v_2, v_3, \ldots,$ = corrections to sides $l_1, l_2, l_3, \ldots$;

$v_{L_1}, v_{L_2}, v_{L_3}, \ldots, v_{D_1}, v_{D_2}, v_{D_3}, \ldots,$ = corrections to latitudes and departures, respectively.

The application of the method is best shown by a problem.

TABLE No. 2.—PROBLEM FROM DATA PREVIOUSLY GIVEN.

Station.	Deflection to the R.	Correction.	Azimuth.	Dist.	Latitude, L.		Departure, D		$\frac{L^2}{100\,l}$	$\frac{D^2}{100\,l}$	$\frac{L\,D}{100\,l}$
					+	−	+	−			
1.....	92° 00′	+0.6′	92° 0.6′	236	.....	8.3	235.9	.....	0.0	2.36	−0.08
2.....	46 00	0.6	138 1.2	244	.....	181.4	163.2	.....	1.35	1.09	−1.21
3.....	82 00	0.6	220 1.8	324	.....	248.1	.....	208.4	1.90	1.38	+1.60
4.....	51 57	0.6	271 59.4	183	6.3	.....	.....	182.9	0.0	1.83	−0.06
5.....	88 00	0.6	360 0.0	437	437.0	.....	.....	0.0	4.37	0.0	0.0
	359 57	3.0	Totals...	.....	443.3	437.8	399.1	391.3	7.62	6.66	+0.25
					437.8	.....	391.3				
				q_1	=+5.5		q_2	=+7.8			

The observed angles foot up 3 minutes too small. To each angle 0.6 minute is added, and the azimuths computed with 5–1 as an axis. The latitudes and departures are computed and added as usual, giving $q_1 = +5.5$, and $q_2 = +7.8$. The three columns at the right are used in finding x' and y'. To fill them out, set the slide rule to latitude divided by distance ($L \div l$) for each horizontal line and multiply by one-hundredth of the latitude (0.01 L) for the corresponding term in the $\frac{L^2}{100\,l}$ column; with the same settings multiply by one-hundredth of the departure for the corresponding terms in the $\frac{L\,D}{100\,l}$ column. Then set to departure divided by distance and multiply by one-

hundredth of the departure for the $\frac{D^2}{100\,l}$ column. Add the three columns for $\Sigma\left(\frac{L^2}{100\,l}\right)$, $\Sigma\left(\frac{D^2}{100\,l}\right)$, and $\Sigma\left(\frac{L\,D}{100\,l}\right)$, and substitute in (18)', giving

$$x' = \frac{7.8 \times 0.25 - 5.5 \times 6.66}{6.66 \times 7.62 - (0.25)^2} = -0.68$$

$$y' = \frac{5.5 \times 0.25 - 7.8 \times 7.62}{6.66 \times 7.62 - (0.25)^2} = -1.15.$$

If the corrections to the latitudes and departures only are required, as in computing areas or co-ordinates, use (21)' and (22)'. To do this, set x' on the slide rule and multiply by each number in the $\frac{L^2}{100\,l}$ column for the first term in each latitude correction, and by each $\frac{L\,D}{100\,l}$ for the first term in each departure correction. Then set to the value of y', and multiply by each $\frac{L\,D}{100\,l}$ for the second term in each latitude correction, and by each $\frac{D^2}{100\,l}$ for the second term in each departure correction, giving:

$v_{L_1} =$	$0.0 + 0.09 = +0.1$	$v_{D_1} = +0.05 - 2.71 = -2.7$	
$v_{L_2} = -0.92 + 1.39 = +0.4$		$v_{D_2} = +0.82 - 1.25 = -0.4$	
$v_{L_3} = -1.29 - 1.84 = -3.1$		$v_{D_3} = -1.09 - 1.59 = -2.7$	
$v_{L_4} =$	$0.0 + 0.07 = +0.1$	$v_{D_4} = +0.04 - 2.10 = -2.0$	
$v_{L_5} = -2.97 + 0.0 = -3.0$		$v_{D_5} =$	$0.00 + 0.0 = 0.0$
Check, (23)'			
$\Sigma(v_L) = -q_1$	$= -5.5$	$\Sigma(v_D) = -q_2$	$= -7.8$

If the corrections to the distances only are required, as for a geometrically consistent description or record map, use (19)'; the x' setting of the slide rule giving all the first terms when multiplied by one-hundredth of the latitudes, and the y' setting the second terms when multiplied by one-hundredth of the departures. Thus:

$v_1 = +0.06$	-2.72	$= -2.66$
$v_2 = +1.23$	-1.88	$= -0.65$
$v_3 = +1.68$	$+2.40$	$= +4.08$
$v_4 = -0.04$	$+2.11$	$= +2.07$
$v_5 = -2.97$	$+0.0$	$= -2.97$

$\Sigma(v) = -0.68 \times 0.055 - 1.15 \times 0.078 = -0.13$ by (20)'

If corrected co-ordinates and distances are both required, it will be less work to find the distance corrections first and the co-ordinate corrections from these by the first portions of (21)′ and (22)′.

TABLE No. 3.—Problem from Actual Survey.

Angles closed without adjustment.

Station.	Bearing.	Distance.	Latitude, L.		Departure, D.		L^2	D^2	$L\ D$
			+	−	+	−	$100\ l$	$100\ l$	$100\ l$
1.....	N. 44° 38.5′ E.	287.24	204.37	.....	201.83	.....	1.45	1.42	1.44
2.....	N. 52 42.5 E.	451.75	273.70	.....	359.40	.....	1.65	2.85	2.18
3.....	S. 2 00 W.	921.60	.....	921.04	.....	32.16	9.20	0.01	0.32
4.....	N. 88 34 W.	212.00	5.30	.....	.....	211.93	0.00	2.11	−0.05
5.....	S. 88 50.5 W.	317.30	.....	6.41	.....	317.24	0.01	3.17	0.06
6.....	North.....	443.60	443.60	.....	.....	.....	4.44	0.00	0.00
	Totals.....	.....	926.97	927.45	561.23	561.33	16.75	9.56	3.95
				926.97		561.23			
			$q_1 =$	− 0.48	$q_2 =$	− 0.10			

$$x' = \frac{-0.10 \times 3.95 + 0.48 \times 9.56}{160 - 15.55} = +0.029$$

$$y' = \frac{-0.48 \times 3.95 + 0.10 \times 16.75}{160 - 15.55} = -0.001$$

$v_1 = +0.05$;	$v_{L_1} = +0.04$;	$v_{D_1} = +0.04$
$v_2 = +0.08$;	$v_{L_2} = +0.05$;	$v_{D_2} = +0.06$
$v_3 = -0.26$;	$v_{L_3} = +0.26$;	$v_{D_3} = +0.01$
$v_4 = 0.00$;	$v_{L_4} = 0.00$;	$v_{D_4} = 0.00$
$v_5 = 0.00$;	$v_{L_5} = 0.00$;	$v_{D_5} = 0.00$
$v_6 = +0.13$;	$v_{L_6} = +0.13$;	$v_{D_6} = 0.00$
$\Sigma(v) = 0.00$;	$\Sigma(v_L) = +0.48 = -q_1$;	$\Sigma(v_D) = +0.11 = -q_2$

In some cases base or reference lines and stations are carefully located, and the other work adjusted to them; in others, a survey is extended after an adjustment has been made which it is desired not to disturb. These cases are treated by omitting the terms and corrections for the perfect lines, which forces all the corrections into the unadjusted parts.

Thus, in the foregoing problem, if 6–1 be regarded as perfect,

$$\Sigma\left(\frac{L^2}{100\,l}\right) = 12.31;\ \Sigma\left(\frac{D^2}{100\,l}\right) = 9.56;\ \Sigma\left(\frac{L\,D}{100\,l}\right) = 3.95.$$

$$x = \frac{-0.10 \times 3.95 + 0.48 \times 9.56}{9.56 \times 12.31 - (3.95)^2} = +0.041$$

$$y = \frac{-0.48 \times 3.95 + 0.10 \times 12.31}{9.56 \times 12.31 - (3.95)^2} = -0.007$$

$v_1 = +0.07;$	$v_{L_1} = +0.05;$	$v_{D_1} = +0.05$
$v_2 = +0.09;$	$v_{L_2} = +0.05;$	$v_{D_2} = +0.07$
$v_3 = -0.38;$	$v_{L_3} = +0.38;$	$v_{D_3} = +0.01$
$v_4 = +0.01;$	$v_{L_4} = 0.00;$	$v_{D_4} = -0.01$
$v_5 = +0.02;$	$v_{L_5} = 0.00;$	$v_{D_5} = -0.02$
$\Sigma(v) = -0.19;$	$\Sigma(v_L) = +0.48;$	$\Sigma(v_D) = +0.10$

By (20)′, $\Sigma(v) = -0.041 \times 4.44 + 0.007 \times 0.001 = -0.18$, for a check.

The method thus lends itself readily to the fitting of new to old work which has already been adjusted.

The application of the method of adjustment is given in considerable detail so that it can be used mechanically the same as the old compass survey method without looking up its theoretical basis. It cannot be stated so concisely as the old, and the numerical work involves more labor if corrected co-ordinates only are required. In case corrected distances are required, to give with the bearings geometrically consistent data for a record map or description, this requires less labor than the old method, for the corrected distances are used directly with the bearings, while in the old method neither corrected distances nor bearings can be obtained without a special computation after the co-ordinates have been adjusted.

The method has a good theoretical basis for the conditions assumed, viz.: small errors of closure for the angles and an inaccuracy of chaining increasing as the square root of the distance, while the old method has none for transit work.

DISCUSSION.

GEORGE W. TUTTLE, Esq. (by letter).—It is stated on page 454 that the most probable corrections will be those the sum of whose squares is a minimum after each correction has been divided by its probable error. It is the square of the probable error, however, that should be and has been used in the development of the formulas. Mr. Tuttle.

It would be inferred by a reader of the paper that Dr. Bowditch developed his method from least-square considerations, although such is not the case. In *The Analyst*, Vol. 1, No. 4, 1808, Dr. Bowditch presented his well-known method, in answer to a question proposed in a previous number of that journal, as to the best method of correcting a compass survey. Dr. Bowditch did not give the solution in mathematical form, but presented the now common graphical one, stating, in substance, that it was reasonable to assume that the corrections should be proportional to the length of the lines, and parallel to the closing side.

It remained for Dr. Adrian, the editor of *The Analyst*, in the same number, to demonstrate Dr. Bowditch's method by least squares, and this work is remarkable as the earliest application of the method of least squares to geodetic work, as well as for the general and instructive form in which it is given. This problem, and its solution by Dr. Adrian, forms an important and interesting part of the early history of mathematics in the United States. Dr. Adrian demonstrates the following formulas, which are put into the notation of the present paper:

$$\left.\begin{array}{l} x\,(\Sigma\, M\cos.^2 B + p^2\,\Sigma\, M\sin.^2 B) + y\,(1-p^2) \\ \Sigma\, M\sin.\ B\cos.\ B + q_1 = 0 \end{array}\right\} \ldots\ldots\ldots(1)$$

$$\left.\begin{array}{l} y\,(\Sigma\, M\sin.^2 B + p^2\,\Sigma\, M\cos.^2 B) + x\,(1-p^2) \\ \Sigma\, M\sin.\ B\cos.\ B + q_2 = 0 \end{array}\right\} \ldots\ldots\ldots(2)$$

$$\text{or, } x\left[\Sigma\left(\frac{L^2}{l}\right) + p^2\,\Sigma\left(\frac{D^2}{l}\right)\right] + \Sigma\left(\frac{L\,D}{l}\right)(1-p^2)\,y + q_1 = 0 \ldots(3)$$

$$x\,\Sigma\left(\frac{L\,D}{l}\right)(1-p^2) + \left[\Sigma\left(\frac{D^2}{l}\right) + p^2\,\Sigma\left(\frac{L^2}{l}\right)\right] y = 0 \ldots\ldots(4)$$

$$\text{where } p = \frac{\text{Probable error in bearing} \times \text{length of line.}}{\text{Probable error in length of line.}}$$

Equations (3) and (4) are more general forms of the author's Equations (17), which are obtained by making $p = 0$. Dr. Adrian notes, first, that when $p = 1$,

$x = \dfrac{-q_1}{l_1 + l_2 + l_3 + \ldots}$ and $y\,\dfrac{-q_2}{l_1 + l_2 + l_3 + \ldots}$, then $V_{L_1} = x\,l_1$, $V_{D_1} = y\,l_1$ and motion of angle points is in the direction of closing line. Second, when $p = 0$, motion of angle points is in the direction of l_1, l_2, l_3.....

Mr. Tuttle. Third, when $p = \alpha$, motion of angle points is in a direction perpendicular to the distances l_1, l_2, l_3, ...

It will be seen that the first case is the method of Dr. Bowditch; the second is that of the author, and supposes all the errors distributed among the sides only, the bearings remaining unchanged. The third solution takes up all the errors of the traverse in the bearings, and the measured distances remain unchanged.

Now, the foregoing general equations can be used with substantially the same amount of work as those proposed by the author, and with the use of his forms of computation. They have the advantage that the errors can be distributed among the sides and angles in any proportion, according to the relative accuracy of the measured sides and angles; these formulas have the defect however, that the correction to the bearing of any line is inversely proportional to the square root of the length of that line, while in a transit survey, where the lines are not excessively short, it should be independent of the distance.

The fundamental defect in the formula proposed by the author is, that it requires the errors to be distributed in the sides only, while the angular error, where many angles are involved, is often greater. When the errors have to be taken up, either in sides or angles alone, the corrections will, in general, be double what they would be if distributed through both.

That which follows refers largely to city work, with transits of good make, and reading to 20 seconds, and tapes having spring balance and temperature adjustment.

The probable error in chaining appears to increase less rapidly than the distance, and faster than the square root of the distance as assumed in the foregoing formulas. The law of the square root would probably be true if all errors to which distance measurements were subject were compensating, and made up of a large number of purely accidental errors, an equal number of positive and negative ones occurring. But as most of them are not compensating, and, as for any one line measured in a brief period, temperature and other errors proportional to distance may be considerable and cumulative, it would seem to be quite as accurate to take the error of chaining as being proportional to the distance, in which form the error is usually stated.

It would seem that for an accurate and economical survey, some relation should exist between the accuracy demanded of the chain, and that of the transit. No adjustment that can be devised is so perfect or satisfactory that it can take the place of accurate work. It is quite important, therefore, that the proper relation between the precision of the line and angle measurements should be determined.

Were the angle and distance measurements equally difficult, for the sake of economy, the allowable error should be divided equally between them, as Dr. Bowditch's method assumes. The angular measurements

being usually less difficult, some greater precision may be observed in their measurement, but the respective errors, usually, should not depart widely from equality, and in this particular it appears desirable that the assumption implied in the usual rule, *i. e.*, that errors due to chaining and angle measurements are equally probable, should obtain in practice. Mr. Tuttle.

Assume a closed traverse of n sides and angles, with an angular error of closure of $a\sqrt{n}$, where a = allowable error in measurement of one angle; then, to make the figure geometrically consistent, as far as the angles are concerned, a correction of $\frac{a\sqrt{n}}{n}$ should be applied to each angle. This is the same, in effect, as assuming the bearing of one line in the traverse, from which the bearings of the other sides are derived, and weighting the two determinations of the bearing of any given side found in passing from the assumed line, round to the right and left, respectively, inversely as the number of angles passed through.

Let m be the value of any bearing determined by passing through z angles to the right, and m_1 by passing through $(n - z)$ angles to the left, and m_m and r_m the value and probable error of bearing, after the error of closure is applied.

Then,
$$m_m = \frac{\frac{m}{r^2} + \frac{m_1}{r_1^2}}{\frac{1}{r^2} + \frac{1}{r_1^2}}$$

where r and r_1 represent the probable error of determination in passing round the closed figure to the right and left, respectively.

It may be observed that the probable error, as here used, is that obtained from the allowable difference of angular closure, *i. e.*, $a\sqrt{n}$, and is consequently the probable error corresponding to this difference, and the maximum value which the probable error should have.

Now,
$$\frac{1}{r^2} = \frac{1}{(0.674a)^2 z}, \text{ and } \frac{1}{r_1^2} = \frac{1}{(0.674a)^2 (n - z)},$$

so that
$$m_m = \frac{\frac{m}{z} + \frac{m_1}{n - z}}{\frac{1}{z} + \frac{1}{n - z}} = \frac{m(n - z) + m_1 z}{n} \quad \ldots\ldots\ldots\ldots (5)$$

and
$$r_m = \frac{1}{\sqrt{\frac{1}{r^2} + \frac{1}{r_1^2}}} = \frac{0.674a}{\sqrt{\frac{1}{z} + \frac{1}{n - z}}} = 0.674a\sqrt{\frac{n z - z^2}{n}} \quad \ldots\ldots (6)$$

From the foregoing, it is seen that the probable error of the bearing distant $z = \frac{n}{2}$ from the assumed side is $r_{m\left(z = \frac{n}{2}\right)} = 0.34a\sqrt{n}$,

Mr. Tuttle. while the probable error of a single determination is $r_{\left(z=\frac{n}{2}\right)} = 0.47a\sqrt{n}$. The allowable error, $a\sqrt{n}$, is, therefore, 2.1 times the probable error of a single determination as here defined and used.

It is easily seen from Equation (6) that the probable errors of the corrected bearings vary from zero at the base to $0.34\ a\sqrt{n}$ at the bearing $\frac{n}{2}$ angles distant from the base, in proportion to the ordinate of a circle of diameter n, divided into segments z and $n - z$.

As it is impracticable to assign different probable errors to bearings in the determination of corrections, or to measure the angles so as to secure bearings having a uniform probable error, the square root of the mean value of the squares of the probable errors should be taken as the constant probable error of an angle; this is equivalent to making p^2, in Equations (3) and (4) constant, and equal to its mean value. We will define the square root of the mean of the squares of the probable errors as the mean probable error. The mean probable error of bearing, in a traverse of n angles, will therefore be, approximately,

$$0.674a\sqrt{\frac{1}{n}\int_0^{\frac{n}{2}}\left(\frac{n\,z-z^2}{n}\right)dz} = 0.275\ a\sqrt{n},$$

and the mean value of the allowable error, being 2.1 times the latter, will equal $0.58\ a\sqrt{n}$. While this is the mean value of the allowable error in traverse work connected with fixed bearings, it is the actual allowable error in a closed circuit where no bearings are fixed, since each bearing is similarly circumstanced relatively to all the others.

From the record of the angles measured with several good 20-second transits, under service conditions, and by different observers, the probable error of a single angle measurement was found to be less than 12 seconds. This quantity, therefore, is taken as the maximum probable error of a single measurement, and consequently, it is found that 12 seconds $\times$ 2.1 = 25.0 seconds is the allowable error or difference between measurements.

The allowable error of chaining with the tape before described has been found to be about 1 in 10 000, and an angular error of 20.6 seconds will cause an error of an equal amount in the location of the end of any given line. Therefore, when the mean allowable error of a traverse equals 20.6 seconds, the error due to angular measurement will be approximately equal to the error due to chaining, when measured with the accuracy of 1 in 10 000. We have already found the mean allowable error in a traverse of n sides to be $0.58\ a\sqrt{n}$; therefore, to find the value of a for equal precision in angles and chaining, we have the equation $0.58\ a\sqrt{n} = 20.6$ seconds.

Table No. 4 has been prepared from this equation. The first column gives values of n, the number of angles in the traverse. The second, the values of a, the allowable error in angular measurement, *i. e.*, the difference between two measurements should not exceed this amount for equal precision in sides and angles. The third column gives the approximate number of repetitions of angle necessary for equal precision in sides and angles when $p = 10\,000 \times 0.58\, a \sqrt{n}$ arc 1 second = 1. Mr. Tuttle.

TABLE No. 4.

n.	a.	Repetitions necessary.
4	18	2
9	12	3
16	9	4
25	7	6
36	6	8

From a considerable experience in adjusting careful traverse work to triangulation, it has been found that where many angles are involved, the divergence due to angular errors may be considerably greater than errors due to chaining, and in this way the foregoing deductions as to the necessary care in measuring traverses of many angles has been verified.

Equations (1), (2), (3) and (4) may be easily modified to make the probable error in distance measurement proportional to the length of line, and the probable error in bearings independent of the same. The equations then take the form:

$$\left.\begin{array}{l} x\,[\Sigma\ M^2 \cos.^2 B + p^2 \Sigma\ M^2 \sin.^2 B] + y\,(1-p^2) \\ M^2 \sin.\ B \cos.\ B + q_1 = 0 \end{array}\right\} \quad \ldots\ldots(7)$$

$$\left.\begin{array}{l} y\,[\Sigma\ M^2 \sin.\ B + p^2 \Sigma\ M^2 \cos.\ B] + x\,(1-p^2) \\ M^2 \sin.\ B\ \cos.\ B + q_2 = 0 \end{array}\right\} \quad \ldots\ldots(8)$$

or,

$$x\,[\Sigma\ L^2 + p^2 \Sigma\ D^2] + \Sigma\ L\ D\,(1-p^2)\,y + q_1 = 0 \ldots\ldots(9)$$

$$x\,(1-p^2)\,\Sigma\ L\ D + y\,[\Sigma\ D^2 + p^2 \Sigma\ L^2] + q_2 = 0 \ldots\ldots(10)$$

Now, when the angles and sides are equally well measured, p will equal unity, and we have

$$x = \frac{-q_1}{l_1^2 + l_2^2 + l_3^2 + \ldots.} \qquad V_{L_1} = l_1^2\, x \ldots\ldots\ldots\ldots(11)$$

$$y = \frac{-q_2}{l_1^2 + l_2^2 + l_3^2 + \ldots.} \qquad V_{D_1} = l_1^2\, y \ldots\ldots\ldots\ldots(12)$$

It will be seen that in these equations the squares of the sides are used in precisely the same way that the sides are used in the common formulas. While the latter make the corrections to be applied to long lines comparatively slight and the corrections to short ones large;

Mr. Tuttle. Formulas (11) and (12) make the corrections to the location of any given point less when passing along a short side, and greater in the case of a long side, by assuming the probable error in the location of a given point as proportional to the distance to that point, instead of to the square root of the distance.

These equations are so simple, and so readily deduced, that the writer has some misgivings in presenting them as new, although he has no knowledge that they have ever been proposed. They appear to the writer to be superior to the common formulas for the adjustment of transit work, and also have the merit of simplicity.

Mr. Fuertes. JAMES H. FUERTES, M. Am. Soc. C. E.—During the past two years, in the investigations connected with the Philadelphia Water Supply Improvement, for the Mayor's expert Water Commission, and the New York City Water Supply investigations, for the New York Merchants' Association, the speaker has conducted rapid surveys over a large area of mountainous country, for the purpose of securing data relative to stream flow and storage facilities. A short account of the methods used and the results obtained may be of interest.

For the New York investigations the work consisted of reconnaissances for aqueduct lines, and surveys of reservoir sites and water-shed limits, in the Adirondack, Catskill, Kittatinny and Shawangunk regions, for which no topographical maps were available. As the time allowed was very short, quick approximate methods were necessarily used.

In the Catskills the information desired was the location of suitable dam sites, the capacities of the reservoirs formed by these dams, and the area tributary to each, in order to arrive at conclusions as to the available daily yield of this water-shed. It was, at first, a rather disheartening task; favorable locations for masonry dams appearing only at the lowest sites on each of the three main streams, the Esopus, Catskill and Schoharie Creeks, and the country being so rough and broken that progress was necessarily slow.

As a foundation for the work, the best available maps were secured. These were on so small a scale as to be of little use except for the purpose of estimating water-shed areas. The equipment of a party consisted of a pocket aneroid barometer, a pocket compass, a hand-level, and a horse and buggy. Each party consisted of three persons, two for the field and one to remain at headquarters and record at short intervals the readings of the stationary barometer, noting the exact time of each reading. Readings were sometimes taken at about 3-minute intervals, when the fluctuations in pressure were rapid. The time was noted at each reading, both in the field and at headquarters. At night the readings of the stationary barometer were plotted and the proper corrections of the field readings were made. Sometimes it was necessary to compensate for lapse of time, when an area of low pressure was moving toward or away from the

observers; this was done by shifting the time scale. At times the changes in the barometer would be as much as 300 to 400 ft. in a few hours, but by taking several readings at consecutive stations the effects of such variations could be eliminated. Generally, the barometric readings, after correction, could be depended upon within about 10 ft. Mr. Fuertes.

The field party first meandered all the roads in, and, crossing, the creek valleys, taking bearings at every change of direction, and elevations at every angle point, and at points where the slopes changed. The distances from point to point were measured by counting the revolutions of the buggy wheel. The corrections for the slip of the wheel, according as the roads were smooth or rough, frozen, icy, muddy, stiff, hilly or flat, were determined each day by plotting the day's work and checking the distances by counting the rails of the railroad track between road crossings. Where possible, surveys were made in complete circuits, so that distances and directions could be checked from two independent lines. After the main lines through the valleys were completed, cross-sections were taken at various points where the topography indicated that such sections were desirable.

Elevations were checked by reading the barometer at the different stations every time points were passed where elevations had been taken on previous days. The elevations of the railroad stations were assumed as starting points for the different lines. These were taken from the time table of the Ulster & Delaware Railroad, or from the "Dictionary of Altitudes" published by the U. S. Geological Survey. At Cold Brook Station, on the Ulster & Delaware Railroad, the barometer indicated a discrepancy of about 40 ft. in the elevations, as compared with the time table elevation. Every time this station was passed the same difference of elevation was found; it was therefore checked, first with a hand-level, and afterward with a wye-level, and the barometric elevation was found to be substantially correct.

Each day's work was plotted at night, the contours at 20-ft. intervals, roads, buildings, railroads, bridges, etc., being put on the sheets, which were then sent to the New York office to be worked into record shape. The sheets for the general map were plotted to a scale of about 1 mile to an inch, and the detailed maps of the creek valleys, on a scale of 500 ft. to an inch. The plotting generally checked out quite closely; closely enough to enable the probable storage capacity of the reservoirs to be estimated with a sufficient degree of accuracy. It was found that the error was generally on the side of safety; that is, the tendency was to underestimate distances rather than to overestimate them.

The total cost of the field work for the Esopus Valley, including all salaries, team hire, hotel bills and traveling expenses, was $457; the cost of making the record maps was $136; or, a total of $593 for the

Mr. Fuertes. survey and maps complete. The territory embraced in this survey, over the whole of which contours were taken at 20-ft. intervals, covered 9 500 acres; thus, the cost of the field work was at the rate of 4.8 cents per acre, and the cost of the record maps, on a scale of 500 ft. to an inch, about 1.5 cents per acre, making the total cost 6.3 cents per acre.

The reliability of the barometers was tested most severely in making the survey of the "Drowned Lands" in the Wallkill Valley. Just south of the crossing of the Erie Railroad over the Wallkill River, the valley widens out and forms a broad plain from one to five miles in width. This plain is nearly flat, both lengthwise and transversely, and extends well down into New Jersey. It was at one time, undoubtedly, the bottom of a lake. During the winter and spring the whole valley frequently lies several feet under water. In getting elevations in this valley care was exercised to take as many barometric readings as possible on points where elevations had been taken previously. Elevations were carried down from Goshen and Middletown. After the completion of the work, comparisons were made with the Geological Survey sheets of New Jersey, and it was found that the contours carried down from the Erie Railroad crossing fitted those brought up from tide-water through New Jersey.

In the case of the erroneous elevation found at Cold Brook Station, on the Ulster & Delaware Railroad, the attention of the Railroad Company was not called to the discrepancy, because the elevation was taken from the time-table, and was not official. The valley through which the railroad runs is quite beautiful, and is much frequented by tourists and summer visitors, and one of the features of the time-table was the publication of the elevations of the railroad stations. The profile of the railroad was not secured. At the time, it was thought that the error might have been simply an oversight in proof-reading. The elevation is given correctly, on the authority of the railroad in the "Dictionary of Altitudes."

Mr. Luster. W. H. Luster, Jr., Assoc. M. Am. Soc. C. E.—The speaker has done quite an amount of topographic work, with both aneroid and wye-level, and probably did some of the work near the New Jersey State line to which Mr. Fuertes refers.

Part of this topographical work was in a rough section of the Adirondacks, where the general method followed was to run lines of levels along the railroads and country highways. These levels were used as the base for all stadia and aneroid work, vertical angles being used on all stadia lines.

In taking the topography of a township the lines were first run around the tract, using transit and steel tape, the horizontal and vertical angles were read and stadia distances recorded; the latter was reduced to the horizontal and used to check the tape distance.

Mr. Luster.

A township contains 36 sq. miles and measures 24 miles around the border. The closure of the horizontal work was 30 ft. and of the vertical about 6 ft.

This work was all cross-checked by running lines on the subdivisions of the township, sometimes by transit and tape and at other times with compass and stadia, according to the importance of the line. These cross-lines always crossed a line of levels, so that perfect vertical control was secured at all times. The azimuth was checked by sun observations and triangulation.

All horizontal measurements were taken between vertical rods which were driven solidly in the ground and made vertical by the use of two attached levels. Down the center of these rods there was a slot which just allowed the entrance of a projection on a plate attached to the zero end of the tape. The tape was made horizontal by the use of a hand-level, and the distance between the centers of the slots was then read and recorded.

Some of the distances taped over had slopes of 45 degrees. Taking that fact into consideration with the distance measured, 24 miles, the closure of 30 ft. was remarkably good.

It is a peculiar fact that in all the township line work which the speaker has done in the Adirondacks, he has found that with few exceptions these lines cross the summits of all the high peaks or very close to them, and in almost every instance a triangulation station can be located on the line which will give perfect view to other stations.

At these points a small signal was erected and a round of angles measured, and at a later date it was observed on from the other stations. These triangulations were made with a 20-second Buff and Berger transit. The results of these triangulations were plotted on a projection and the lines then adjusted to these points graphically. The scale of this map was 1 in 10 000, which made the perceptible error on the map very slight.

The topography was filled in by aneroid, which, for ground of this nature, was a rapid and fairly accurate way of working. Each township was divided into lots, the lines of which were run by compass and stadia, the vertical angles being read for the vertical control. From these lines a pacing traverse was run to the interior of each lot, and aneroid readings taken at all points of topographic interest. The remaining contours were sketched in by eye. The aneroids were read at the start and finish of each traverse, and the points taken in the interior were adjusted in proportion to the elapsed time. Readings for comparison were taken at the camp during the day by the man in charge, if his other duties did not prevent; as this often happened, these readings were little used and dependence was placed on average readings taken on the same point. These aneroid

Mr. Luster. heights can be depended on within 10 ft., five or six check readings were taken on a point of known elevation, which led the speaker to take the error mentioned as a maximum.

The speed of such a survey, when all hands, fifteen men, were working on the line, was about 2 miles per day.

The foregoing method differs from that carried on in New Jersey. In that State the ground is not so rough and is not heavily timbered, so that the various features of the ground can be readily seen and no useless lines need be run. For country of this nature the wye-level was used for all topography. In the few cases where the country was hilly and timbered aneroids were depended on, but were used on only very short lines.

Formulas were never used to distribute the error, as the number of stations involved would have made the task so great that the results of the work would have been greatly delayed before publication, and, in the speaker's opinion, the scale of a map, where so small as that mentioned, does not warrant such refinement.

The paper has been read carefully, and, while admitting that a method of adjustment is necessary, the one given seems to the speaker to be too cumbersome for practical use. In making an adjustment of a survey the speaker uses the following method: The lines which can be measured without any difficulty are taken as correct and the difference then remaining is proportioned to the other lines in proportion to their lengths. If the ground is open and the located corners can be seen from two or more points in the field, a base line is carefully measured, from the ends of which these corners can be seen; the angles are measured and then the sides are computed to check the measured distances. It has been the speaker's experience that by this method very little error will be found in lines easily measured, and no refinements need be used to adjust this small difference.

Mr. Lee. W. B. LEE, M. Am. Soc. C. E.—The speaker has often used Dr. Bowditch's method of adjusting a survey, probably for the lack of something better, and has fallen into the error, common to those who have applied it to a transit survey, namely, the giving of insufficient credit to the accuracy of the angle measurements.

No survey should be accepted, the original notes of which are not sufficiently accurate within their limits, which may be anywhere between those of mountain land and city lots. This condition being satisfied, no surveyor can afford to make, nor will any owner consent to pay for, an elaborate correction. Surveys are made for the purpose of locating fixed points or corners. If the bearings and distances when re-run will not locate these points accurately, it matters not whether the error be one of bearing or distance.

Since the present practice tends to an accuracy of instrument work greater than that of chaining, the speaker would recommend that

these two be adjusted to a common basis before a survey is made. Mr. Lee.
Then Dr. Bowditch's simple and rapid method of correction may be applied without detriment to either.

Much depends upon the circumstances attending the making of a survey; that is, the conditions under which it is made, and the nature of the territory. Here arise abundant opportunities for the exercise of judgment.

The speaker recently made a survey of an industrial plant which was scattered along a railway for about half a mile, making a circuit then, of about a mile. The line along the railway was assumed as a meridian and its length as correct; then the meandering line behind the plant was adjusted to it. Numerous shots and measurements to corners of buildings and other fixed points proved the accuracy of the work and the rationality of the adjustment.

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No. 894.

THE CONSTRUCTION OF GRAVITY SAND FILTERS AT NYACK, N. Y.

By G. N. HOUSTON, Assoc. M. Am. Soc. C. E.

PRESENTED FEBRUARY 20TH, 1901.

WITH DISCUSSION.

The Village of Nyack is situated on the west bank of the Hudson River, about 28 miles from New York City. The Villages of South Nyack and Upper Nyack adjoin it on either side, and are supplied by it with water. The combined population is about 8 000.

These villages were originally supplied from springs by gravity, but in 1884 the company then owning the works found it necessary to increase this supply by pumping from Hackensack Creek, the present source of four-fifths of the supply.

As the proportion of Hackensack Creek water to spring water increased, dissatisfaction among the consumers likewise increased, until, in 1893, the Village of Congers constructed a shallow lake by damming a branch of the stream and flooding an area which had never been cleared of the stumps or surface soil. The water then became unbearable, and the villages applied to the State Board of Health for a remedy. Accordingly, the Board made a report in 1894

advising the water company to filter the supply. Nothing, however, was done at that time.

In July, 1897, the Village of Nyack acquired the plant of the Nyack Water-Works Company by condemnation proceedings, and began to consider the improvement of the supply.

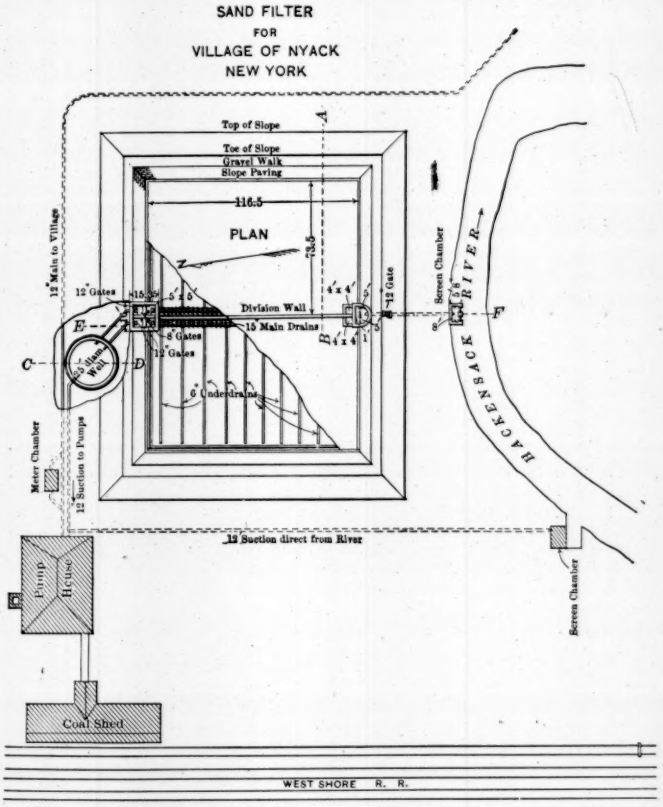

FIG. 1.

Driven wells were tried and failed. A gravity supply from a more sparsely settled water-shed was considered, but the estimated cost exceeded the available means. The filtration of the present supply was then recommended and adopted.

Description of Plant.

The filter is of the open, gravity, sand type, and was designed for slow filtration. It is located in a swamp on the Hackensack River, about 2½ miles west of the village, and just north of the West Nyack Station on the West Shore Railroad (Fig. 1).

It consists of two rectangular beds of equal size, with a combined filtering area of 16 400 sq. ft., or 0.38 acre, and a pure-water reservoir having a capacity of 44 000 galls. It is built wholly in excavation below the level of the creek, and is protected by earth embankments from high water, which floods the surrounding ground. Several cross-sections are shown in Fig. 2.

The enclosing walls are of concrete to a height of about 3 ft. 8 ins. above the bottom. Above these walls a vitrified brick paving is laid at a slope of 1½ to 1, and to a height of 4 ft. above the wall.

The bottom is of concrete, and the regulating and intake chambers are of brick. The division wall above the sand is of brick; below the sand it is of concrete.

The filtering material consists of 3 ft. of sand underdrained by 1 ft. of gravel and by 6-in. tile pipe. Each bed has a main drain of tile pipe leading to the regulating chamber.

The pure-water well is built of brick, and the bottom is of concrete. It is entirely separated from the filter, but is connected with the regulating chamber by a 12-in. cast-iron pipe.

Details of Construction.

The excavation was begun during dry weather of August, 1897. After removing 2 ft. of black muck on top, a stiff damp red clay was encountered, and the deeper the excavation was carried, the more wet and tenacious became the clay, and the more difficult it became to handle. Shovels and spades were used to cut it in blocks which were then thrown on wagons by the hands. Teaming was impossible, except on roadways of old railway ties or plank.

The depth of the cut to be made averaged 12.5 ft. After a depth of 10 ft. was reached, the sides of the excavation, which up to this time had stood nearly vertical, cracked back from the face a distance of 2 to 4 ft., and subsided, making a corresponding elevation in the adjacent bottom, which had become very soft and puddled by teaming over it.

To remedy this difficulty, excavating the easterly end at a slope of 1½ to 1 was tried, with a like result when the same depth was reached,

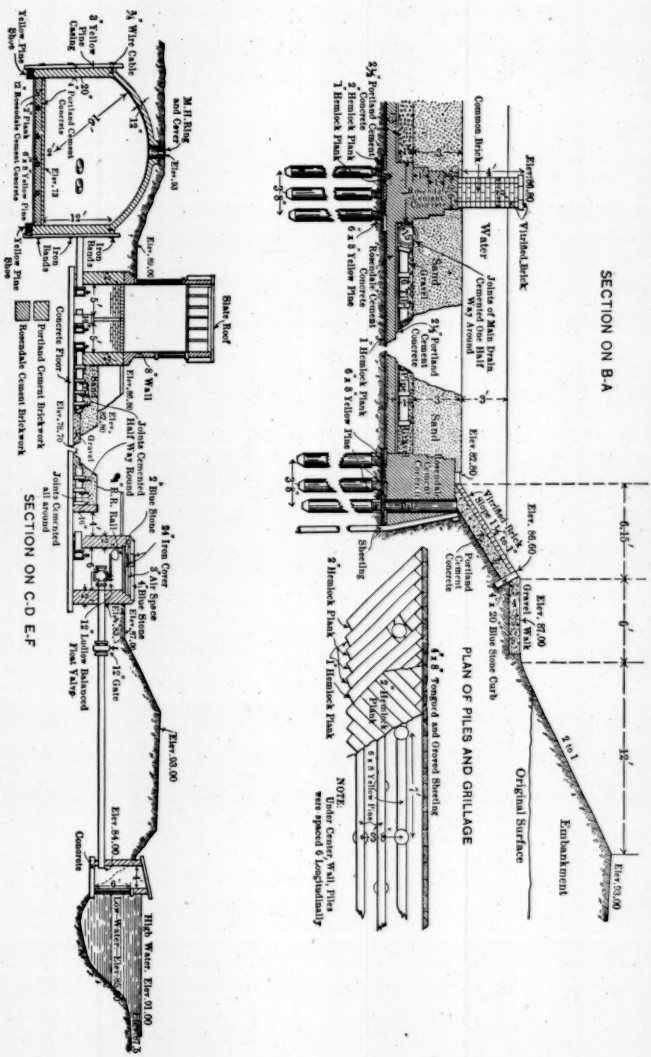
SECTION ON B-A
PLAN OF PILES AND GRILLAGE
SECTION ON C-D E-F
Original Surface
Embankment
Water
Common Brick
Vitrified Brick
Sand
Gravel
Sheeting
Portland Cement Concrete
Slate Roof
Concrete Floor
Portland Cement Brickwork
Rosendale Cement Brickwork
Joints Cemented all around
Joints Cemented Half Way Round
Joints of Main Drain Cemented One Half Way Around
Gravel Walk
Tongued and Groved Sheeting
Yellow Pine Shoe
Yellow Pine Casing
Iron Bands
Concrete
High Water
Low Water

FIG. 2.

and it became evident that, with this method of excavation, the required depth could not be reached.

The contractor had taken the excavation at the low price of 29 cents per cubic yard. It was evident that he was losing money on this part of the work, and, becoming somewhat discouraged, he finally abandoned it.

Rudolph Hering, M. Am. Soc. C. E., was then asked to report on the situation for the village. By his advice, soundings were made over the entire area. These were made with an iron rod 1 in. in diameter and 30 ft. long, on which was arranged a sliding weight. This weight was made of a short section of 6-in. cast-iron water pipe, in the center of which was a piece of 1¼-in. wrought-iron pipe, the space between being filled with lead. On the bottom was an iron plate, the whole weighing about 130 lbs.

The weight was lifted by two men and allowed to fall on an adjustable clamp. The height to which it was lifted varied from 3 to 4 ft., and a record was taken of the depth penetrated by each ten blows. The upper part of the rod was steadied by a tripod. To withdraw the rod a 3-ton block and fall was used.

These soundings showed the presence of water-bearing gravel strata varying from 2 ft. at the westerly end to 20 ft. at the easterly end below the proposed bottom of the filter. Mr. Hering then advised continuing the construction by the use of sheet-piling and sustaining piles.

Before the sheet-piling could be started, the creek broke through the bank and flooded the excavation. This made it necessary to drive a line of 4 x 8-in. tongued and grooved spruce sheeting between the creek and the excavation. This sheeting was driven to a depth of 3 to 4 ft. below the proposed bottom of the filter. The water was then pumped out, and the whole area surrounded by a single row of sheet-piling of the same kind. The latter sheeting was driven within the excavation and 1 ft. outside of the foundation line of the filter. Sustaining piles were then driven on the proposed site of the walls.

In driving the sheeting it was found that in most places it could be driven clear through the gravel strata into softer material below. For this reason the round piles were driven with blunt ends of at least 6 ins. diameter. Although some of the 332 piles thus driven evidently stopped at the gravel strata, fully two-thirds of the number depend largely on friction for sustaining power.

The maximum weight sustained by each pile under the outside wall

FIG. 1.—VIEW OF EXCAVATION, SHOWING SUBSIDENCE OF BANKS.

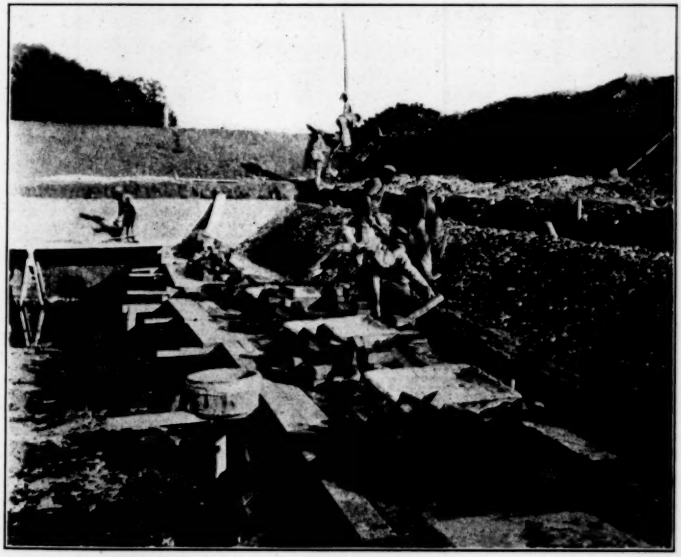

FIG. 2.—LAYING VITRIFIED BRICK SLOPE.

is about 4 tons; under the division wall, 5½ tons. The sheeting cost 5 cents per foot, board measure, driven, and the cost of the sustaining piles averaged $4.42 each, in place.

Just at this time, April, 1898, the funds available for this work gave out and the construction came to a standstill. Soon after, during high water, the excavation was flooded a second time. It remained in this condition until July, 1899, when the work was again taken up.

The original plans and specifications were modified to meet the new conditions, and a new contract was let. The changes were as follows:

First.—The level of the whole bed was raised 1 ft., and the depth of the filtering material was reduced from 5 to 4 ft. This made a total raise of 2 ft. in the bottom of the filter, and reduced the average cut to 10.5 ft.

Second.—Concrete was substituted for brickwork in the walls below the level of the top of the sand, and slope paving was substituted for the vertical walls above this level.

Third.—All brickwork exposed to the elements was faced with vitrified brick.

After pumping out the water, the excavation was carried down to the proper depth on the site of the outside walls, the piles were sawed off and capped, and the walls were started before attempting to excavate the interior.

The sheet piling was exposed by this excavation for a depth of 5 to 7 ft., and was seen to be practically water-tight. It had a tendency to crowd in at the top, on account of the pressure behind it, and, in order to brace it, the outside row of sustaining piles was left standing. Inclined braces, running to a footing of railroad ties, were also found necessary.

In one place, where the sheeting had been pushed over so far as to interfere with proper construction, it was forced back part way by screw jacks after relieving some of the pressure by excavating behind it.

At the easterly end derricks and buckets were used to handle the excavated material and to place the concrete, but were not found necessary after the wall at that end was completed. The interior was excavated with teams. Very little water was encountered until the walls and floor of the east half were nearly completed, when a large spring broke out in the center of the excavation at a point over which all the teams bringing material for concrete had passed and had con-

sequently puddled the clay. This was stopped at this point, but broke through at a point about 20 ft. distant.

Underdrains were laid leading the water to the pure-water well, which had been sunk to a depth somewhat lower than the filter, and was kept dry by a centrifugal pump.

This and a similar spring which broke through later were left until the beds were otherwise completed, and were finally stopped by filling with from 10 to 12 cu. yds. of field stone and concreting over it, leaving a 15-in. vitrified pipe through the concrete for a drain. When the concrete had set, a 6-in. pipe was placed inside the 15-in. pipe, and bagging was rammed down around it and loaded with concrete, which was covered with bagging, to prevent the water which dropped over the edge from washing out the cement. It was found that there was not head enough to raise this water above the sand line, and the spring was stopped up with cement bags and concrete.

The piles were capped with 6 x 8-in. yellow pine stringers spiked to each pile by two 14-in. drift bolts. Where the ends of these stringers met they were joined with a scarf joint and bolt and nut.

The piles in the row next to the sheeting were notched to receive the stringers, as shown in Fig. 2. On these stringers, and at an angle of 45° with them, 2-in. hemlock planks were nailed. On top of these, and at an angle of 90° with the former planks, were nailed 2-in. and 1-in. hemlock plank, laid alternately, thus making a well-braced platform sufficiently rough to give a good footing for the walls.

The cost of the piles, capping and platform averaged $3 per foot of wall sustained.

The forms for the walls were then erected. They were made from 2-in. hemlock plank. In some instances forms were used for the back of the wall, but in most of the construction the concrete was carried back against the sheeting, and the piles in the outside row were concreted into the wall.

In some instances the inclined braces supporting the sheeting were also concreted into the wall and left until just previous to putting in the sand, when they were either taken out and the hole filled up with concrete, or sawed off and cut back from the face of the wall at least 6 ins. and covered with Portland cement concrete.

The face of the outside walls below the sand line was stepped in 2 ins., and the division wall 3 ins. in each foot of rise. With the excep-

FIG. 1.—PLACING GRAVEL IN EAST FILTER BED.

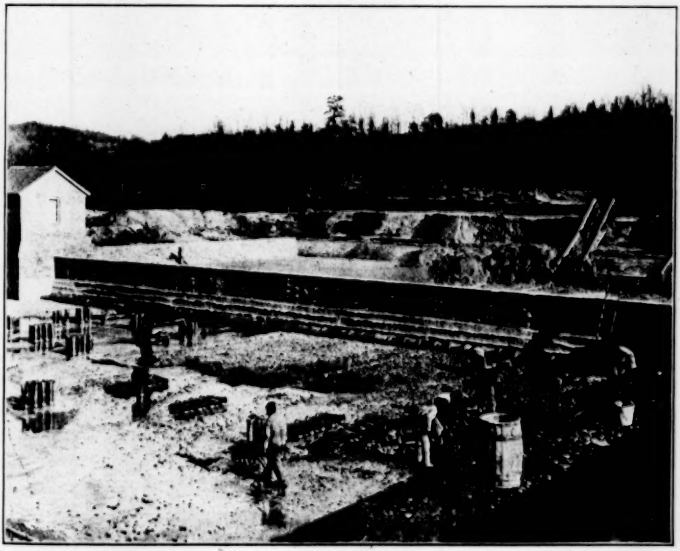

FIG. 2.—UNDERDRAINS AND MAIN COLLECTOR IN WEST FILTER BED.

tion of the division wall, which was of uniform dimensions throughout, the concrete walls varied in thickness according to their location. The concrete in the south wall averaged 0.8 cu. yd. per lineal foot; that in the east wall averaged 0.33 cu. yd. per lineal foot, and that in the north and west walls 0.7 cu. yd. per lineal foot. The division wall averaged 0.61 cu. yd. per lineal foot.

The floors were placed on 1-in. hemlock plank, laid on the clay to prevent the concrete from mixing with the soft bottom.

They were made of Rosendale cement concrete 0.66 ft. thick, topped with Portland cement concrete having an average thickness of 0.22 ft.

All concrete was mixed by hand. Rosendale cement concrete for the walls and floor was composed of 1 part cement, 2 parts sand and 6 parts broken limestone, varying from ½ in. to 2½ ins. in greatest dimensions. The Portland cement concrete in the floors was mixed in the proportions of 1 part cement, 3 parts sand and 5 parts ¾-in. broken limestone.

The faces of the concrete walls were first pointed up with 2 to 1 Rosendale cement, and then plastered with 2 to 1 Portland cement.

The floors cost $1.50 per square yard in place. The concrete walls cost $4.40 per cubic yard in place. The labor of mixing and depositing the concrete for the walls was 1 man, 0.42 day per cubic yard. For the Rosendale cement concrete in the floors, 1 man, 0.32 day per cubic yard.

The division wall, above the sand line, was built of brick in Portland cement, and faced with vitrified brick. The slope paving was laid on a foundation of 8 ins. of Portland cement concrete, composed of 1 part cement, 3 parts sand and 7 parts broken limestone, varying from ½ in. to 2½ ins. in greatest dimension. The vitrified brick slope paving and all other brickwork, with the exception of that in the house over the regulating chamber, was laid in 3 to 1 Portland cement mortar.

The average cost of the exterior concrete wall, slope paving and curb was $5 per lineal foot. The average cost of the division wall was $5.19 per lineal foot. The ordinary brickwork in the walls cost $8 per cubic yard. The vitrified brickwork in the walls cost $8.40 per cubic yard.

The intake chamber extends outside of the first row of sheeting, and is of circular form on the outside, to better withstand the pressure.

The filtering gravel is of the following sizes: The bottom 6 ins. in depth varies from 3 ins. to 1 in. in greatest diameter; the next 3 ins.,

from 1 to ½ in.; the top 3 ins., ¼ in. and less. With the exception of a small portion of the top layer, the gravel in the east half is of crushed trap rock and hard field stone; in the west half it is washed gravel from Cow Bay, Long Island.

The sand was obtained from Cow Bay, and was delivered in barges at the dock at Nyack, from which it was carted to the beds in wagons. It was placed in the beds as received, as was also the crushed stone and gravel. Both sand and gravel cost $2.15 per cubic yard in place.

During the first week the filter was operated the water was wasted, on account of its being quite turbid and tasting of the cement. After that, it was pumped into the reservoirs, although still quite turbid. After about a month it became decidedly better, and at the end of the second month the turbidity could scarcely be noticed in an ordinary glass, but still showed slightly in the regulating chamber, where there was a depth of about 8 ft.

After three months the water was free from all turbidity, but had a slight color. In a glass or small vessel it appeared to be colorless, but where there was any considerable depth, as in the regulating chamber, it could be easily noticed.

In the intake chamber there is an ordinary 12-in. Ludlow balanced float valve, which regulates the depth of water over the sand to 3 ft., when the pumps are running; otherwise, 3½ ft.

The rate of filtration is regulated by the pumps, which draw from the pure-water well and supply the reservoirs in the village. The pumps are in use almost every day for at least 9 hours, and run at a uniform rate while in operation.

The maximum consumption of water is at the rate of about 630 000 galls. per day, which, if one bed only were used, would give a maximum draft of about 3 326 000 galls. per acre per day. This condition, however, is not realized in practice, as both beds are used at the same time, except when one is being cleaned.

The development of the springs before mentioned is reducing to some extent the amount of filtered water required, and the total consumption is being decreased by the gradual introduction of meters.

The pure-water well was sunk on a shoe made of three rings of 4-in. yellow pine bolted together. Each ring is composed of six sections. This makes a solid shoe of 25 ft. internal and 28.5 ft. external diameter.

An excavation, about 6 ft. deep, was made on the site of the well,

FIG. 1.—PLACING SAND IN EAST FILTER BED.

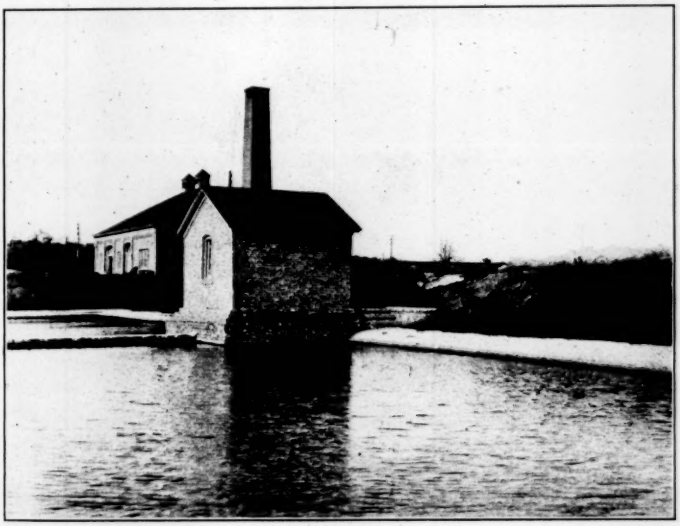

FIG. 2.—COMPLETED FILTER BASINS AND REGULATING CHAMBER.

and the shoe was put together in the bottom. Around this shoe was erected a tub or casing of 3 x 6-in. x 15-ft. yellow pine plank, spiked to the shoe and held in place by three iron hoops drawn tight by bolts and nuts. The 20-in. wall was then built on this shoe to a height of 10 feet.

Excavation with derricks and buckets was then begun, but, later, the second contractor erected a centrifugal pump, and, using a jet of water from the mains under about 150 lbs. pressure, softened the clay and pumped it out.

The floor was made by laying 2-in. planks on the bottom, and over them 6 x 8-in. yellow pine timbers, inserted in holes cut in the brick-work just above the shoe. Rosendale cement concrete 1 ft. thick and of the same character as that on the bottom of the filter, was then laid on the bottom, thus covering the 6 x 8-in. stringers 4 ins. deep. On this a 4-in. layer of Portland cement concrete was placed.

The well was covered with a spherical dome of 6 ft. rise, the radius of the sphere being 16 ft. To relieve the thrust of this dome, a $\frac{3}{4}$-in. cable was built in at the spring line. Of this cable, 125 ft. were used, it being double on the side toward the filter.

The embankments were formed with the material excavated from the original surface, without cleaning or grubbing. Where it was found necessary to make embankments upon which the slope paving was to rest great care was taken to have it thoroughly compacted by tamping and then settling with water. Nothing was done to compact the embankments above the slope paving, except the ordinary teaming over them. They were designed and constructed originally on a slope of $1\frac{1}{2}$ to 1, with the walk 1 ft. above the curb. They were completed about December, 1899, and, therefore, could not be seeded. The clay, on thawing in the spring, became semi-liquid and ran until it stood at a slope of from 2 to 1 to 3 to 1. A great deal of it ran over the curb and upon the filter beds which were then being used. This was cleaned off, and then the banks were cut back to a 2 to 1 slope, topped with 1 ft. of loam, and seeded, and the grade of the walk was lowered to the top of the curb.

In order to keep the foot of the embankment in place it was found necessary to excavate the walk to the 4-in. sub-grade and fill in with crushed stone; which furnished sufficient weight to prevent the pressure of the embankment from raising the soft material under the walk.

When the beds were cleaned for the second time it was found that the clay had never been thoroughly removed from them. In order to remove it, the sand to a depth of from 1½ to 3 ins. was taken off and replaced with clean sand. The beds were finally completed about May 1st, 1900.

The following engineers were connected with the work: James S. Haring, M. Am. Soc. C. E., was the Engineer of the Board of Water Commissioners during the design and construction under the first contract. Allen Hazen, M. Am. Soc. C. E., was consulted on the advisability of filtration, and made the original design. Rudolph Hering, M. Am. Soc. C. E., was consulted on the early part of the construction. Louis L. Tribus, M. Am. Soc. C. E., was called in consultation under the second contract. He approved the plans and specifications as modified by the writer, and acted as Consulting Engineer during the construction. The writer was Assistant Engineer under the first contract and Engineer of the Board of Water Commissioners under the second contract.

The cost of the filters is shown in the following table:

Sheet piling in place	$3 300
Sustaining piles in place	1 558
Excavation	5 270
Grading and soiling	1 500
Engineering	3 644
Inspection and tests	713
Rosendale cement concrete in floors	1 958
Portland " " "	780
Rosendale " " walls	1 892
Portland " " slope paving	484
Vitrified brick, slope paving	113
Curb in place	250
Underdrains	347
Filtering material	5 524
Hemlock lumber, capping and floors	838
Yellow pine lumber " " "	465
House over regulating chamber	150
Pipe laying, etc.	58
Miscellaneous	250
Total	$29 094

DISCUSSION.

JAMES S. HARING, M. Am. Soc. C. E. (by letter).—As outlined by the author, the origin of the change of ownership of the water-works of the village of Nyack, from a private to a municipal plant, began with the continued dissatisfaction of the consumers, in the three villages named, with the quality of the water supplied by the private company. Mr. Haring.

In 1872 Commodore William Voorhis, a large property owner in Nyack and South Nyack, constructed, principally for the supply of his own houses in the latter village, a gravity plant, taking the supply from small springs found in the hillside of these villages. Gradually the plant became enlarged by the addition of other consumers, and additional supplies were obtained from springs. In 1884, however, after much expensive and fruitless effort to obtain similar additions, it was found necessary to seek a supply by other means than that of gravity. Examinations of various other sources, including Rockland Lake and the Hackensack River, were then made, and resulted in the selection of the latter as the source of supply.

The Hackensack River originates principally from the outflow of Rockland Lake, and from a stream having its source west of New City, the latter being considered as the purer supply.

In 1895, the Village of Nyack held an election to determine upon the construction of a municipal plant. The vote was favorable, and the Board of Water Commissioners resulted. The writer was at once appointed their engineer, and designed a distribution system which was built in connection with the sewerage system, which caused its rather hasty consideration, as the pipes were laid on a shelf at the side of the sewer trench. A search was then instituted for a supply of water. Various nearby supplies were investigated, and finally a test was made by driven wells in the red sandstone, along the shore of the Hudson River in Upper Nyack, out of which many springs were flowing. These wells were in quarry sites worked many years ago, but long since abandoned. The springs from the various strata had been flowing, without any appreciable loss, for a long time, and seemed to promise results satisfactory, both as to quantity and quality. Three wells, each 8 ins. in diameter, were contracted for at a price per foot which left to the Water Commissioners much latitude as to the depth and the final number of wells to be driven. The depth of the deepest well was 285 ft. and that of the remaining two very much less, when a test was instituted, at first with the barrel pump owned and operated by the contractor, but this having a capacity of only 100 galls. per minute, the writer advised the Commissioners to apply a more thorough test, and succeeding in getting the necessary plant to pump by the use of

Mr. Haring. compressed air. This test was continued without intermission for one week, the quantity pumped averaging more than 100 galls. per minute during the whole of that period. Measurements were made from time to time by which the depression of the water table, both in the well pumped, and in each of the other wells was recorded, and, previous to the constant pumping, the length of time taken by the water-table to recover its normal height was also noted. For six days the test, both as to quantity and quality, seemed to be satisfactory, but upon the seventh day, when it was proposed to end the test, a sample of water taken for analysis was found to be salt. It would be difficult to determine at what time during the test the water became salt. It is a fact that people were using the water pumped during the whole time the test was being made, and that during that time a local chemist was analyzing the water for his own information, but, until the time mentioned, there was no discovery of the fact that the water was salt. The probability is that the salt water came in very gradually as the elevation of the water-table was lowered, the latter finally reaching a point below that of the bottom of the river, the ordinary high water line of which was about 35 ft. distant from the well tested. The shore was a continuation of the red sandstone for a considerable distance out, and was covered not by silt, as had been supposed, but by gravel and coarse sand only.

After this attempt failed, a gravity supply from Spring Valley, about 8 miles to the west, was investigated and practically decided upon. The supply came from a sparsely settled territory and from various small streams, one of which was gauged by the 10-in. meter now in use at the pumping station.

During the early operations of the Water Commissioners it was an open question as to whether or not they would take proceedings to condemn the private water company's plant. They construed the law as being adverse to any attempted negotiations for its purchase at private sale, even though an amicable adjustment could have been arranged, which was doubtful, popular sentiment in the village being adverse to such acquisition; but it was finally determined to take such action because of the immediate revenue to be derived therefrom, and also because of the franchises and distribution in the adjoining villages of Upper Nyack and South Nyack. Proceedings were begun, and a lengthy litigation before Commissioners of Appraisement ended in the acquisition of the private plant, but at a valuation so high that the available appropriation was insufficient to pay the cost of carrying out the gravity supply scheme.

During the time proceedings of condemnation were in progress, the Commissioners had paralleled the mains of the company in the village with a materially larger and more complete system.

One of the elements of the condemnation proceedings was the

attempt of the Water Commissioners to prove the Hackensack River supply to be of objectionable quality. This was, in part, admitted by the company, and the attempt was met by showing that the supply could be filtered at small cost, thereby overcoming that objection. The experts for the company were John Bogart, M. Am. Soc. C. E.; W. F. Whittemore, Assoc. M. Am. Soc. C. E., and Mr. Joseph B. Rider. These experts (with the exception of Mr. Whittemore, who gave testimony on the cost of a filter located on the hill and constructed near one of the reservoirs, but made no recommendation as to site), selected as the site of the filters, not the high-service reservoir, which was especially brought to their attention as a suggested location, but a place near the old pumping station, which was a few hundred feet southeast of the present filters. Later, when Allen Hazen, M. Am. Soc. C. E., was consulted with regard to the construction of a filter and its location, he also advised the present site, or one within a short distance of the site selected, the reasons given for such location being principally its proximity to the pumping station, the facilities for obtaining sand, and that the engineer running the pumps could regulate the filters, thereby saving the cost of additional labor in operation, the approach to the high-service reservoir being somewhat difficult and the cost of additional land at that point being also considerable. The writer calls special attention to these facts because of the criticism following the discovery of the bad bottom alluded to in the paper, and of an attempt made to hold him and the Water Commissioners responsible for an improper location of the filters. Both acted on the advice and best judgment of the expert engineers connected with the condemnation proceedings and their own consulting engineer. Mr. Haring.

When the contract for the construction was let, the price fixed for the excavation was 29 cents per cubic yard, on the supposition by the contractors that the earth, or clay, could be excavated by the use of a centrifugal pump, as was done later by the second contractor in excavating in the curb of the pure-water well. The pump was put on, but the attempt to take out the clay in this manner was a failure; the material, to use the contractors' expression, being "too much like white lead to make it soluble."

Tests were made over both sites before either was selected, one being at the old pumping station, the other at the place chosen. A steel-pointed, 1-in. round iron rod, was used, and was both churned down and driven with a sledge to resistance. The results of these tests were reported to the Water Commissioners, and their attention called to the fact that they were not satisfactory; they were, however, all that were attempted, except the test excavations mentioned later, until the conditions described by the author occurred, after which, as stated by him, tests were made on short cross-sections to the north

Mr. Haring. and west of the excavation. The results of the latter tests showed a stratification of small boulders and gravel sloping from west to east, and from south to north, the strata seeming to rise to quite near the surface of the meadow at a point about 25 ft. west of the west line of the excavation. This fact led Mr Hering to recommend that the location of the filters be changed by placing them that distance to the west. On the lines of that suggestion the location of the pure-water well was changed, which accounts for its apparently angular position with reference to the filters. The iron rod used in testing the site came to absolute resistance repeatedly in the last test, and upon being withdrawn was frequently found to be much bent, yet, when the clay and gravel was excavated, and on account of the puddling caused by teams passing over it, the bottom of the excavation became semi-liquid, thus proving that nothing short of deep test pits could have shown exactly the character of the bottom at the depth required. Before the site was purchased, test pits were dug to such depth as was possible without pumping appliances, but they were too shallow to afford material information. The geological formations, afterward considered, were generally adverse to the selection of the site.

Complications with the contractors led the Water Commissioners to retain the original location of the filters as shown on the plans, and they were constructed on that site.

From the time bad bottom was discovered, the writer has been under the belief that it would be difficult to prevent separation taking place in the concrete along the edges of the floor adjoining the stringers and board covering. The placing of unequal weight by careless handling of the sand in first filling, or afterward in cleaning the filters, will be found to be a dangerous element in affecting this result; or, with the semi-liquid earth forming the bottom, on which the 1-in. flooring used for the area of the filters alone rests, may accomplish this also by the wave action due to unequal loading, which was constantly in evidence during the operations of the first contractor. Its dangerous character is shown by the spring described in the paper. The first appearance of this spring was the result of a test hole. When the rod was withdrawn a stream of water spouted up some distance above the surrounding bottom, and it in time became an enlarged boiling spring. During the writer's connection with the work, which ended in May, 1898, and while the excavation was free from water, this spring continued to be active, and was only one of several evidences that an underground flow of water was sealed up by the clay bottom only a very short distance below the point reached by the excavation. It would not be surprising to find, when the filters are emptied, that cracks, admitting considerable unfiltered water, exist along these lines, and that in time they may become very troublesome.

Mr. Haring.

The writer questions the advisability of the change in Mr. Hazen's plans, from a vertical wall of brick to a slope covering of concrete faced with brick. The embankment material is especially subject to upheaval by frost, the angle of slope and the reduced thickness of the concrete, while it economizes cost somewhat, seems to the writer to do so with the loss of stability.

It may be interesting to note the writer's experience in endeavoring to have brick of proper quality selected for use under the original contract. Mr. Hazen's specifications required the use "of the best quality of hard-burned brick, burned entirely through and of uniform shape and size." The contractor purchased and brought upon the ground brick of a quality which the writer refused to accept for the pumping station structure, and would not consider at all for the filters. He condemned the brick, and the matter was appealed to the Water Commissioners, and by the writer to the Consulting Engineer, Mr. Hazen, who declined to pass upon the matter as not connected with his duties. In this condition matters remained until Mr. Hering was called in. He at once sustained the writer, and the brick were rejected. The writer, believing at all times that vitrified brick with a small percentage of absorption should be used, at least for the facing of the walls exposed to the action of frost by the fluctuation of the water in the filters, insisted upon their purchase and use. This was opposed by some of the Water Commissioners, and consent to their use could not at first be obtained. The writer then personally visited the yards manufacturing the best hard brick at Haverstraw, obtained samples of their brick and submitted them to absorption tests; but finding such proof unavailing, he sent the samples, with one of the inspectors employed on the work, to an ice plant where they were submitted to alternate freezing and thawing. These samples were then submitted to the Commissioners, and, the proof being then no longer to be disputed, the writer was ordered to purchase vitrified brick for the facing of the exposed walls, and the best obtainable hard brick for the interior of the walls. This was done, and all the brick required for the completion of the work was delivered upon the ground under the first contract.

During all the writer's operations at Nyack, the author was his assistant, and the writer desires to bear testimony to his painstaking efforts to bring a difficult problem (for he has had many obstacles to contend with, both in and out of his profession) to a successful conclusion.

Mr. Fuertes.

JAMES H. FUERTES, M. Am. Soc. C. E. (by letter).—The manner in which the difficulties encountered in making the excavations for these filters were overcome is interesting and instructive. The method of construction, however, was quite costly, the amount chargeable to the excavation, sheet piling, foundation piles and other earth work, being

Mr. Fuertes. at the rate of about $30 500 per acre of filter surface, out of the total cost of $46 700 per acre for the filters complete.

In the fall of 1895 the writer visited the sewerage works at Newport, Isle of Wight, which were then in construction under the direction of Baldwin Latham, M. Inst. C. E. One of the works was a covered precipitation reservoir, about 70 x 83 ft. inside, divided by longitudinal walls into three basins each 22 ft. wide. The conditions were somewhat similar to those at Nyack, the soil being a slippery clay with a strong tendency to slide and cave, the ground-water level being several feet above the bottom of the reservoir.

The method there successfully adopted was to first build the side-walls, on wide footings, in sheeted trenches. After their completion the interior was excavated, the concrete bottom laid and the partition walls and posts for supporting the roof built upon it.

A description of the method used in building the dome over the filtered water reservoir at Nyack would be of much interest. A little reservoir, somewhat similar to that at Nyack, was built last summer for the Tome Institute, at Port Deposit, Md., under the writer's supervision. This reservoir was circular in plan, 16 ft. in internal diameter, and had a capacity of 5 000 galls. The side-walls and bottom of the reservoir were of Portland cement concrete 6 ins. thick, and the average thickness of the dome, which had a rise of 2 ft., was about 5 ins. Sheets of No. 10, 3-in., diamond, expanded metal were built in the walls, bottom and dome as the concrete was put in place. The dome was turned on a mound of earth, built upon a platform erected inside of the reservoir, and tamped and trimmed to the proper shape. The earth mound was covered with heavy building paper before laying the concrete. When the centering was removed, the dome was found to be perfect in form and smooth on the interior surface, thus justifying the methods used. The total cost was about $250.

Mr. Kaufman. GUSTAVE KAUFMAN, M. Am. Soc. C. E.—The speaker desires to be enlightened as to why it was deemed necessary to locate this filter below high water, and thus require a deep excavation in swampy ground, to do which difficulties might have been expected to be encountered? Why was not the filter placed a great deal higher, and a small centrifugal pump used to deliver the river water to the filter?

In the absence of information or reason to the contrary, it would appear that placing the filter above high water would have been by far the better arrangement. A large amount of money would have been saved in the construction, and the cost of pumping would not have been affected, because the head between the filter and the village would have been reduced by the height of the filter above the ordinary stage of the river.

MORRIS KNOWLES, Assoc. M. Am. Soc. C. E.—Mr. Kaufman's remarks about the advantage of using a centrifugal pump bring to mind an occasion of the use of such a pump for a somewhat opposite purpose, namely, that of pumping water off the surface of the filter. Mr. Knowles.

The author does not state the reason for placing the filter with the water level below that in the river, but in several instances it has been done for the purpose of saving the continuous maintenance cost of pumping.

As is probably well known, the filter at Lawrence, Mass., is placed in such a manner that the river water flows to it by gravity, and, thus far in its history, there has been only one period when this has not been possible. This was during the cold weather in January, 1900, when, for some unusual cause, accompanying long continuous dry weather, like holding the water back in some of the storage reservoirs or mill ponds above, the river was so low that it was not possible to send enough water to the filter to supply the city's use.

Two centrifugal pumps (8-in. and 6-in.), were secured, hurriedly placed in position, and used for a period of about ten days, after which the water level arose, and ever since has been sufficiently high. The 6-in. pump was then returned.

At about this time it was thought advisable to test the usefulness of such pumps for removing the water from the surface of the filter, when desiring to draw down prior to scraping. When the filter is clogged at the surface and needs cleaning, much time has been wasted, after shutting off the inlet water, and before the water has drained down into the body of the sand and the surface is dry enough to work upon. The 8-in. pump, therefore, was equipped with a double suction and discharge, together with suitable valves, so that water could be drawn from the river or from the surface of the filter and discharged to either the filter or the river, as might be desired.

When used as an aid in draining the filter the advantage is so manifest and the saving in time so great, that the wonder is that it was not put into practice before. Thus the pump has two uses, one in case of emergency, and the other, originally secondary, but now the regular one, whenever scraping is necessary; and the investment has proven a profitable one.

ALLEN HAZEN, M. Am. Soc. C. E. (by letter).—This paper describes the application of sand filtration to a small water-works plant. The original plans, which were drawn by the writer, showed the walls of brick, of the full height of the filter, and without the paved slopes. The arrangement, in this respect, was much like that of small filters of the Rumsen Improvement Company, at Red Bank, N. J., which had just been completed, and that of the filters of the Queens County Water Company, at Far Rockaway, Long Island. Mr. Hazen.

Mr. Hazen. The general location of the plant was unquestionably the best. Numerous and serious difficulties would have been encountered in putting the filters on the hills above the distributing reservoirs, and the cost of pumping to filters on higher ground near the pumping station would have represented more than the interest charges on the capital required to put them below the river level. The exact site was selected by Mr. Haring, after making soundings, which showed "very firm" material at depths not exceeding 6 ft. This site was selected because of the convenience of getting a sidetrack from the railroad to it.

The method of construction required by the specifications was to lower the ground-water level at the site to a point below the foundations by underdrains placed low enough to accomplish that purpose, to surround the site, if necessary, with sheet-piling, and to hold the water at that level until after the completion of the structures. This method, apparently, was not carried out at first.

The writer's connection with the work ceased with the completion of the plans and specifications, and at the time which Mr. Haring mentions, in speaking of the brick, he was not in any way connected with the work; and, while he expressed a willingness to pass upon the brick, he was not willing to do so under the conditions suggested by Mr. Haring.

Mr. Tribus. LOUIS L. TRIBUS, M. Am. Soc. C. E. (by letter).—The author has presented a very clear and full summary of the main events and chief facts attending the construction of the Nyack sand filters, one of the closing chapters in a series of incidents interesting to the onlooker, but rather trying at times to the parties directly concerned.

Taken with Mr. James S. Haring's discussion, the whole Nyack water question has been outlined. It is a public improvement, which, with the sewerage system, has been followed from the start by difficulties, political, structural and legal.

In the condemnation proceedings, for acquiring the private water plant, the writer was called in consultation for the village, and his valuation closely approximated the award finally made by the arbitrators. Mr. Haring's criticism that the award was excessive is true, perhaps, as to the physical value of the plant acquired, but there was a moral obligation or value which it was only proper to recognize; a value that the experts for the company, whom Mr. Haring mentions, put at many thousands of dollars more than the arbitrators thought wise to grant.

The questions of new supply and filtration were not brought before the writer at the time of the proceedings, only existing conditions being discussed.

About two years later, when the resumption of work was considered on the previously begun and practically abandoned filter plant, the

writer was again called upon, this time to take consulting supervision of construction, the author continuing as the direct Executive Engineer of the Board and Superintendent of the water plant. Mr. Tribus.

Various amendments to the original plans seemed advisable, though the essential points, as designed by Mr. Allen Hazen, remained unchanged.

The springs that broke out in several places in the floor area, naturally gave much concern, and received much attention during construction, as to their flow, head and permanence.

Examination of the floor and walls after the springs were shut off, failed to find any breaks or cracks, even when one of the filters was loaded with its quota of broken stone, gravel and sand, thus bringing nearly the maximum unequal strain on the foundation stratum, the other bed remaining practically unloaded at the time.

To avoid as much of the upward pressure as possible, was one reason for sloping the banks instead of retaining the vertical walls, with consequently heavier embankments; the other, economy, in saving the borrowing of earth beyond that from the filter bed area, and the lesser cost of even the substantial concrete and brick-paved slope over vertical walls of sufficient weight to hold the banks securely.

It is unlikely that as the work stood intact in its construction stage, when the whole was fairly saturated with successive rains, that in its completed, settled, graded and well-drained condition, any harm will come to it, as has been intimated; unless some near-by plot should be excavated to considerable depth for foundations or other purpose; not a likely contingency.

The clear-water well had been partly built at the first construction time, some 10 ft. or so of the brick curbing being in place; in the months intervening before work was again commenced, this curb careened in the soft soil, until very seriously out of plumb. By careful loading on the high side, however, and dredging out beneath the curb on the same side, the whole structure was gradually brought back to line without a crack developing; rather a neat piece of work.

It is to be regretted that regular analyses of the raw and filtered waters are not being made, for, while the general public may rest content with a water freed from its sedimentary matters, and probably, it is supposed from analogous cases, from undesirable bacteria as well, it is of real value to the public to have results known and recorded, whenever conditions have warranted the actual construction of purification works.

Monthly analyses would, in regular conditions of operation, be sufficient, with others made at the time of any special disturbance. This would not entail any great annual expenditure, but might be productive of much good; they certainly would add to the as yet small stock of similar knowledge. Perhaps the authorities will yet see their way clear for such investigations.

Mr. Houston. G. N. HOUSTON, Assoc. M. Am. Soc. C. E. (by letter).—In his discussion, Mr. J. S. Haring calls attention to the unequal loading of the bottom while filling the filters with sand. That no serious trouble should be anticipated from this cause will be evident from the following: The maximum inequality of loading took place when placing the sand. This amounted to 330 lbs. per square foot, or less than the increase of pressure caused by a man walking on the concrete bottom.

Again, as the operation of filling occupied more than a month, any uneven settlement would have occurred while it was in progress, for, when completed, there was no unequal loading. As there was no visible rising of the unloaded portion, it is fair to assume that there was no serious settlement of the loaded portion. The actual load on the bottom is about 900 lbs. per square foot. Assuming the angle of repose of the material as 10°, Rankin's formula gives 2 000 lbs. per square foot as the safe load.

The conditions which led to the changing of the vertical walls to slope paving were as follow: When the plans were being prepared for resuming the work of construction, it was found that the banks of the excavation above the sheeting were eroded to a slope varying from 3 to 1 to 2 to 1, and that the quantity of material to be excavated would not be sufficient for the proposed embankment. As material to make up this deficiency would cost from 60 to 80 cents per cubic yard, it was proposed to change the vertical wall to slope paring, thus saving $\frac{1}{3}$ cu. yd. per linear foot of wall at 70 cents per yard, or 23 cents. This, with 71 cents per linear foot in favor of slope paring, made a total saving of 94 cents per running foot of wall. This slope paving was practically completed in November, 1899, and has been through two severe winters without showing any signs of lack of stability, and is in excellent condition.

The method of building the enclosing walls in sheeted trenches, as described by Mr. James H. Fuertes, was suggested to the first contractor and his engineer during the early part of the work, but was not adopted by them.

The dome of the pure-water well was built on centering composed of 40 ribs radiating from a 2-ft. cylinder at the crown. These ribs were sawed to the proper arc from 2-in. plank, and notched so as to rest on the inner edge of well. The 12-in. dome was built on this, after covering it with small boards. No supports were used below. The cost of this centering, in place, was about $60.

As Mr. Hazen, in his report to the Commissioners, does not discuss the relative merits of supplying the beds by gravity and by low-lift pumps, but simply recommends the first method, the writer cannot state the original reason for so locating the beds. He supposes, however, that it was to avoid the maintenance of the duplicate centrifugal pumping plant which would be necessary. The filters have been

running continuously since they were completed, and are giving the village excellent water. Mr. Houston.

Inasmuch as mention has been made of legal difficulties in connection with the early development of the municipal sewer and water systems of Nyack, it may be of interest to state that a decision has just been given by the Referee, in the case of the Sewer Contractors *vs.* the Village, for recovery for alleged extra work. The contractors recover $800 of $41 000 for which they sued, none of the amount allowed being disputed by the village.

The writer was not employed in connection with the filter construction until after the site had been selected and ground broken. In the light of the discussions, however, it seems evident that the difficulties met in the early part of its construction may be traced to two causes:

1.—The time allowed the engineer in which to make an examination of the proposed site was not sufficient to enable him to determine the exact character of the materials to be excavated and the conditions to be met.

2.—The contract was let to a man with an abundance of good intentions, very little capital and less experience. Had the first contract been let to a thoroughly experienced contractor, the filters would have been completed at a much lower figure.

The importance of periodic bacteriological examinations of the raw and filtered waters is realized by the writer, but he has not yet been able to arrange for them, and, as he has already resigned as Superintendent and Engineer of the Board, he hopes that his successor may soon be provided with the means for carrying them out.

The writer wishes to acknowledge the hearty support and co-operation given him by the Board of Water Commissioners, and also the courteous, careful and workmanlike manner in which the second contractors, Messrs. James Duell & Co., of Tarrytown, N. Y., carried on the work, and which contributed largely to its successful completion.

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TRANSACTIONS.

NOTE.—This Society is not responsible, as a body, for the facts and opinions advanced in any of its publications.

No. 895.

THE PRESERVATION OF RAILWAY TIES IN EUROPE.

By O. CHANUTE, Past-President, Am. Soc. C. E.

PRESENTED OCTOBER 17TH, 1900.

WITH DISCUSSION.

The writer chanced to become engaged practically in the preservation of railway ties by chemical injections, in consequence of his having acted as a member of the Committee which reported to this Society on "The preservation of timber," June 25th, 1885. Having continued in the business fourteen years, he felt the need of information concerning many anomalies which had puzzled him, and thought it desirable that he should investigate the experience and methods of doing the best work in Europe, where the industry has been carried on for the last sixty or seventy years. He accordingly went abroad in October, 1899, and proposes to give to this Society some of the results of his enquiries in three different countries. The same methods are practised in the other countries of Europe.

GREAT BRITAIN.

England took the lead in preserving timber by chemicals more than one hundred years ago. Her wooden ships forced this subject

upon the attention of her builders and chemists, and thousands of experiments were tried with scores of substances. M. Paulet, in his book, "Conservation des Bois," enumerates 173 processes or methods. Most of them have proved failures, but some four or five proved to have value, as stated in the report of the Committee to this Society in 1885.

These four or five processes were brought into practical use, especially after the "Railway Era" rendered the preservation of ties an important economy, but all antiseptics, save tar-oil, have now been abandoned in England, where "creosoting,"* so-called, has grown to be recognized as the best process to use. Practically all the ties (they call them sleepers) now laid in Great Britain are treated. A few, cut in Scotland, are laid in their natural state, but at least 90% are imported and creosoted.

Mr. S. B. Boulton, author of a paper awarded the Telford medal by the Institution of Civil Engineers in 1884, and universally recognized as the best British authority on the subject; Mr. A. Ross, Chief Engineer of the Great Northern Ry.; Mr. C. L. Morgan, Engineer of the London, Brighton & South Coast Ry.; Mr. Harry Footner, Engineer of the London & North Western Ry., and others, have kindly furnished the writer with data concerning modern British practice, from which the following account has been compiled.

Almost all the sleepers are of "Baltic redwood" from Russia, Sweden and Norway. They are cut in the late autumn or winter, and summer cutting is generally barred in the specifications. The tie makers are generally engaged in farm work during the summer. When the spring and summer floods occur, the sleepers are floated and rafted down the streams to the shipping ports, much of the sap being washed out by this procedure. The logs or ties are then shipped to English ports, arriving some four to six months after felling, and they are frequently left to soak in water until sawed. They are generally imported as "Blocks" or "Balks," cut to a length of 9 ft., and squared to 10 ins., requiring sawing in two, which many railways specify must be done only at the port of delivery. After being made into sleepers, generally 8 ft. 11 ins. long, 10 ins. face and 5 ins. thick, containing 3.11 cu. ft., they are stacked in cribs, with 4-in. spaces between the sleepers and with sloping top layers,

* Creosote, properly so-called, is a medicinal product of vegetable distillation.

and allowed to season from 4 to 6 months more. Thus they are from 10 to 12 months, and sometimes 18 months, old when "creosoted."

Many of the railways have their sleepers adzed and bored for the fastenings by machinery before creosoting, while others do this by hand in laying. It is conceded that boring before treatment is good practice, but sometimes inconvenient when several patterns of chairs are used.

The specification for creosoted sleepers of the London, Brighton & South Coast Ry. is given herewith, in Appendix A, as fairly representing British practice. The mode of injection is practically the same on all roads; the "Bethell" process is used, in which there is no previous steaming, as the sleepers are already well cleared of sap, and seasoned, but the quantities of tar-oil injected and the results differ a little on various roads.

From an elaborate report of M. V. Herzenstein, presented to the International Railway Congress in London, 1895, giving data obtained from about half of the British railways, Table No. 1 has been compiled by the writer. The kind of timber is omitted, as it is almost all "red-wood"; and the reported cost, reduced to our money, is only applicable where long contracts previously existed, as tar-oil has much advanced in price since 1894, when the data were gathered. This advance has been due to its increasing recognition as the best timber preservative. Some years ago it was so cheap that it was used as fuel, and while the normal price in England (so far as a fluctuating commodity can be said to have a normal price) was about 4 cents per gallon, it is now 7 cents per gallon, hard to find, and with indications of going higher. The present price increases the cost of creosoting very materially from the prices given in the table.

Most of the railways are getting their creosoting done by contract, but the London and Northwestern is doing its own work, and reports the cost at 20 to 25 cents per sleeper. It also reports the longest life (16 to 20 years), save the London, Tilbury and Southend, which is a small road, of light traffic. The British are not injecting nearly as much tar-oil as the French roads, and they obtain a lesser life, as will be shown later.

Many of the results obtained with sleepers in England are due to the system of track. The rails are of the "Bull-head" type, laid on chairs with about 108 sq. ins. of bearing surface, and hence there is no

TABLE No. 1.—Abstract of Answers of British Railways to Mr. Herzenstein.

Number.	Railway.	Number of sleepers annually renewed.	Process of preservation.	Amount injected.	Reported cost, 1894. Cents.	Life, in years.	Cause of failure.
I.....	Belfast and N. Counties Ry	.....	Creosoting.	1 gall. per cubic foot.	12	15	Splits.
II....	Furness Ry.....	40 000	"	8 lbs. per cubic foot.	15	15	
III...	Hall Barnoley Ry...	4 000 to 24 000	"	8 lbs. per cubic foot.	.....	.....	
IV....	Great Eastern Ry...	90 000 to 100 000	"	2½ galls. per tie.	16	12–15	Wear.
V.....	Great Northern Ry..	.....	"	0.7 gall. per cubic foot.	13	12	
VI....	Great Southern and Western Ry.	.....	"	3½ galls. per tie.	.....	.....	
VII...	London and North Western Ry.....	300 000	"	30 lbs. per tie.	.....	16–20	Wear.
VIII..	London and South Western Ry.....	170 000		2½ galls. per tie.	18	12	Wear & splits.
IX....	London, Tilbury and Southend Ry.	Variable.	"	7 to 10 lbs. per cubic foot.	.....	25–30	Decay, etc.
X....	Manchester S. and L. Ry.....	20 per mile.	"	10 lbs. per cubic foot.	.....	16	40% Decay.
XI....	Midland Ry.....	.....	"	.....	.....	.....	
XII...	North British Ry....	.....	"	1 gall. per cubic foot.	14	.....	
XIII..	North London Ry...	.....	"	28 lbs. per tie.	.....	15	
XIV..	South Eastern Ry...	97 000	"	28 lbs. per tie.	.....	8–9	Wear.
XV ..	Taff Vale Ry.....	17 000	"	1½ galls. per cubic foot.	30	15	Natural causes.

Particular attention is called to the quantities (28 to 30 lbs.) of creosote injected per tie. The creosote used practically conforms to the specifications of Dr. Tidy, published in Mr. Boulton's paper, of 1884, and is to be entirely free from water, bone oil, shale oil, or other inferior oils.

cutting by the foot of the rail. The fastenings consist of trenails, of round drift bolts called spikes, and of lag screws, and are thoroughly effective.

France.

France, which was once so abundantly wooded, has been partly denuded, the present area of the forests being estimated at about one-fifth of what it was in the days of Julius Cæsar. The forests have, moreover, been culled, yet they furnish about 3 000 000 ties annually to the French railways, leaving about 1 000 000 to be imported. Of the aggregate about 2 600 000 ties per annum are now consumed in renewals, and it is a notable feature that the number renewed has been diminishing for the past 15 years,* in spite of the fact that the mileage to be maintained has been constantly increasing.

* The renewals averaged 3 072 605 ties per annum for the eight years 1878 to 1885 inclusive.—H. Mathieu, in *Revue Générale des Chemins de Fer*, August, 1887.

This result, which has been estimated as producing an annual economy of about $3 000 000 to the French railways, has been wholly due to the impregnation of the ties and to constant improvements in the processes. The French began treating wood about 1837 by the "Boucherie" process of expelling the sap by hydraulic pressure, using sulphate of copper as an antiseptic. Many of the railways took this up, but most of them abandoned it about 1878, and the only road which had continued the use of this substance, the "Midi," has now finally given it up in favor of creosote. The latter substance is now practically used exclusively by all the roads, save those of the State, which use a mixture of chloride of zinc and creosote, which will be more fully noticed when the German practice is described. "Burnettizing," *i. e.*, the injection of chloride of zinc alone, has been extensively used, but is now abandoned, as the zinc is found to wash out in time, especially in moist situations.

The kinds of wood used in renewals and construction have approximately been in the following proportions: Oak, 60%; beech, 22%; pine and sundries, 18 per cent. Of these, the supply of oak is diminishing, and the use of beech and pine is increasing. Formerly, the creosoting of oak was confined to sappy ties, but now all oak is treated, although but small amounts of tar-oil can be forced therein. The quantities of this substance injected have steadily been increased. While the British railways were content with 22 to 30 lbs. per tie, the French railways injected from 31 to 40 lbs. in beech and pine, with correspondingly better results in duration, and they have lately still further increased the amounts. Thus the "Ouest" Railway now requires 44 lbs. per tie; the "Est," which does its own work, injects 60 lbs. of tar-oil per tie, and the "Paris, Lyon et Mediterranée" follows the practice of the "Est."

The results heretofore attained upon the French railways are given in Table No. 2, which is compiled from Mr. Herzenstein's report, already quoted. It will be noticed that the life obtained is considerably in excess of that in England, and is nearly in direct ratio to the quantities injected and to the cost incurred. As this treatment is applied to beech ties, which cost some 70 cents each unprepared; to pine, costing 60 cents each, and to oak, costing from 95 cents to $1.10 each, it is seen that thorough injection produces important economies.

TABLE No. 2.—ABSTRACT OF ANSWERS OF FRENCH RAILWAYS TO MR. HERZENSTEIN.

No.	Railway.	Number of ties annually renewed.	Kind of wood.	Process of preservation.	Amount injected.	Cost, 1893. Cents.	Life, in years	Causes of failure.
XXVII...	State	161 213	Pine.	Zinc. Creosote.	66 lbs. per tie.	....	10–15	Decay.
"	"	.....	Oak.	Zinc. Creosote.	9 lbs. per tie.	....	.....	
XXVIII..	Eastern.....	356 650	Beech.	Creosote.	60 lbs. per tie.	42	25–30	Wear.
XXIX....	Meridional.....	10%	Oak.	None.	.....	....	.....	
XXX.....	Southern.....	284 511	Pine.	Sulphate of copper	0.4 lb. dry per cubic foot.	....	8–10	Decay and Wear.
"	"	.....	Oak.	Creosote.	9.5 lbs. per tie.	....	10–15	Since increased.
XXXI....	Northern.....	285 000	Oak.	Creosote.	11 lbs. per tie.	11	.....	
"	"	.....	Beech.	Blythe Process.	24 lbs. per tie.	21	.....	Going over to straight creosoting.
XXXII...	Western.....	242 050	Oak.	Creosote.	11–13 lbs. per tie.	....	15–20	Decay and Splits.
"	"	.....	Beech.	Creosote.	31–33 lbs. per tie.	....	18–25 (?)	Now inject 44 lbs.
XXXIII..	Orleans.....	460 000	Oak.	Creosote.	12 lbs. per tie.	....	15	Decay.
"	"	.....	Pine.	Creosote.	35–44 lbs. per tie.	30	13–16	"
XXXIV..	Paris, Lyon & Mediterranean	700 000 (?)	Oak.	Creosote.	10–11 lbs. per tie.	12	12	Decay and cutting.
"	"	.....	Beech.	Creosote.	26–35 lbs. per tie.	18	Prior to 1890.	Now copies Eastern.

All the French roads but one have been creosoting by the "Bethell" process. The exception, the "Nord," has been using the "Blythe" process, which consists in first heating the wood with a mixture of steam and creosote vapor, and then injecting by pressure about 24 lbs. of liquid tar-oil per tie. This method was adopted to economize the amount of this costly substance used. The specifications of this road are given in Appendix B, but the Chief Engineer of Maintenance of Way, Mr. Agnellet, when handing these to the writer, stated that the quantity of creosote injected had lately been increased to 35 lbs. per tie, and that the present cost was 35.6 cents, as against 21 cents per tie in 1893. The road will probably go over to straight creosoting, and adopt the methods of the "Est" Railway, which is recognized as doing the best work of that kind.

The "Est" Railway does its own creosoting, and has two plants with an aggregate capacity of over 500 000 ties a year. It purchases chiefly oak ties (95 cents), and beech ties (65 cents), upon rigid specifications, carefully enforced, and piles them up at its works to season. The piles are isolated and contain 100 ties each, with 10-in. air spaces between the sticks, and the top layer is made of sawed ties laid close and sloping. These piles are inspected from time to time, and if cracks appear a hoop-iron **S** is driven in at the end, or a hole is bored and a bolt with washers is inserted. Oak ties are thus seasoned 15 to 20 months, and beech ties 6 months or more, and are further desiccated before treatment in special drying ovens from 60 to 80 hours. Repeated weighings show that the oak loses 16% of its weight in 12 months, and 2.7% more in the drying oven, while the beech loses 25% in 10½ months, and 3.5% more in the ovens. The use of the latter is deemed important, not only to expel remaining moisture and to ensure regularity of treatment, but to obviate the chilling of the hot tar-oil upon its admission. For this purpose, the ties, loaded upon buggies, are run direct from the ovens, where the final temperature is 176° Fahr., into the treating retort, whereupon, without previous steaming, a vacuum of 26 ins. is produced, and the tar-oil then admitted. Before going into the drying ovens the ties are adzed for rail seat and bored for lag-screw fastenings. This is done by steam machinery, at a cost of 2 cents per tie.

The time occupied by a treatment is 2½ hours for oak and 3 hours for beech, of which time ½ hour is under a pressure of 75 lbs. per square inch for oak, and 1 hour of the same pressure for beech, the tar-oil being admitted at a temperature of 176° Fahr. Under these conditions, the oak ties absorb from 14.22 to 16.65 lbs. of creosote each, and the beech ties take from 59.40 to 66.60 lbs. each. More could be forced into the latter, but this has been found sufficient, being, it will be noticed, double as much as is injected in England.

Every invoice of creosote is chemically tested at the works. The requirements are: (1) That it shall be greenish in hue. (2) Naphthalene not over 30%. (3) Distillation not to begin below 392° Fahr. Moreover, there should be 10% of carbolic acid (and attending water) and 10% of green oil, the density of the whole being 1.05 to 1.10 specific gravity. This tar-oil comes generally from the Paris Gas Co., but the specifications cannot be rigidly enforced just now, for

tar-oil is so scarce that the engineers have to accept what they can get. The total cost of treatment is 33.8 cents per tie for oak, and 64.2 cents per tie for beech, as stated by M. V. Dufaux, Engineer of Maintenance of Way, of the "Est" System, who kindly gave the writer his elaborate paper, published in "*Revue Générale des Chemins de Fer*" in January and March, 1898. This cost is about double that on other roads, but the results are correspondingly good, the creosoted oak lasting 25 years, and the creosoted beech being estimated to last 30 years in the track, as evidenced by data for 27 years, these data also showing that untreated oak lasts about 15 years.

The road uses "*Vignoles*" or foot rails, and, as already intimated, fastens them to the ties by lag-screws. Tarred tie-plates, of animal felt, have heretofore been in use, but are now replaced by creosoted poplar tie-plates, which cost less than 1 cent each, and are said to be giving perfect satisfaction. Two dating nails are driven in each tie, one at the works, and the other when laid in the track.

GERMANY.

By far the largest proportion of wooden ties laid in Germany are home grown, and almost all are chemically treated before laying. The raw ties of the first class are generally 8 ft. 10¼ ins. long, 10¼ ins. face and $6\frac{5}{16}$ ins. thick, and cost $1.32 to $1.49 each for oak, and 82 to 90 cents for pine. White beech costs about $1.00 per tie, and red beech is in controversy. About 500 000 oak ties are creosoted annually, and about 1 600 000 pine ties are treated annually with chloride of zinc and creosote for the Prussian State Railway, the whole of Germany probably taking about double those quantities. The German specifications are very strict, and the raw ties are minutely inspected. If there are incipient cracks, a sharpened band of hoop iron, bent into the shape of an S is driven in, and the ties are piled up in isolated piles, well ventilated, to season in the receiving yards at the treating works. There they remain, with casual inspection and more driving in of S-irons, or insertion of bolts if required, from 8 to 12 months before they are treated, having in the meantime become perfectly seasoned.

At the beginning of the "Railway Era" each railroad company in Germany erected its own works and treated its own ties, either with corrosive sublimate, with sulphate of copper, with chloride of zinc or

with creosote, but gradually there was an evolution which curiously recalls that of the Pullman Car Company in the United States. Mr. Julius Rütgers' father, who had learned the business in some French tie-treating works, erected a plant or two, took his son in with him, and did so much better work than the railroads did for themselves that the business gradually came into his son's hands. At the present time, Mr. Rütgers controls some twenty plants, while the Prussian State Railroad has four, and there are five more in other hands.

The general officer in charge of maintenance of way on the Prussian railways told the writer that although the State treated ties at its own works somewhat cheaper than Mr. Rütgers, yet the latter had so much better facilities and controlled the proper quality of chemicals so thoroughly, that it was preferred to give him most of the work by contract, and to operate the State plants as a check. The original reason seems to have been that the railways which did their own work attempted undue economies by skimping the chemicals and employing cheap men, while Mr. Rütgers made a specialty of the business.

Corrosive sublimate and sulphate of copper are now practically given up in Germany, and the State Railway also abandoned Burnettizing or the injection of chloride of zinc by itself in 1897. There are now three processes in use:

(1) Impregnation with chloride of zinc and tar-oil.

(2) Creosoting after seasoning and drying in ovens.

(3) Creosoting after desiccation in hot tar-oil.

The latest German specification (1898) for the first and third of these processes is given in Appendix C. The method of desiccating in hot creosote, although introduced by Mr. Rütgers, was originally proposed by Mr. S. B. Boulton, in England, in 1885.

As the German railways did not answer the questions of Mr. Herzenstein, no such tabular statement can be given as those given herein for England and France, but through the kind assistance of Mr. Rütgers, of Privy Councillor Wetz, of M. F. Holbein, and many others, the important facts were gathered, and access was obtained to many tie-treating plants.

The most notable thing in Germany is the painstaking care with which every operation is performed. The ties are not treated until they are thoroughly seasoned, this generally taking six months to one year after cutting and piling in open piles in the yards, most of which

yards will hold one year's supply. The chemicals are tested constantly, a laboratory being attached to each plant, each buggy load of 32 ties is weighed before and after treatment, to make sure that the ties have absorbed enough, and every little while each individual tie of a buggy load is weighed in and out. Thermometers are consulted constantly, and automatic recording gauges preserve a diagram of the vacuum and pressures attained, as well as of their duration; the ties are allowed to dry after treatment before they are placed in the track, and, after they get there, well bedded in deep ballast, further care is exercised to see that they are well drained. The result is that ties prepared by the zinc-creosote process, mostly pine, now last from 12 to 18 years, and creosoted ties, mostly oak, are expected to last from 24 to 28 years. In past time it was not always thus, some beech ties creosoted having perished about as soon as some ties injected with chloride of zinc alone, but the results developed upon the roads in Alsace-Lorraine, where beech ties, creosoted by the French, were found to be sound after 21 years of exposure, have again brought the Germans to favor the use of beech creosoted, there being a surplus of that timber, heretofore disesteemed, in the forests of that country.

An able paper by Mr. A. Schneidt, formerly Superintendent of the Alsace-Lorraine Lines, published in the German Railway Organ in 1896 and 1897, shows that the chloride of zinc undoubtedly leaches out of the ties in time, and he advocates the creosoting of beech wood, either by the process of boiling in hot oil, or by the zinc-creosote process. He shows that the former is the more economical of the two, notwithstanding its greater cost, taking in view the high first cost of the untreated ties in Germany.

The prices paid in Germany for treatment, when reduced to American currency, are shown in Tables Nos. 3 and 4.

TABLE No. 3.

Timber.	With Zinc Chloride.		With Zinc Creosote.	
	1st class. Cents per tie.	2d class. Cents per tie.	1st class. Cents per tie.	2d class. Cents per tie.
Pine	15.60	12.00	19.20	14.40
Oak	12.00	9.12	15.60	12.00
Beech	18.80	12.48	20.40	15.36

TABLE No. 4.

Timber.	With Creosote and Drying Oven.		Boiling in Creosote.	
	1st class. Cents per tie.	2d class. Cents per tie.	1st class. Cents per tie.	2d class. Cents per tie.
Pine	53.76	40.32	56.64	42.00
Oak	26.85	20.16	28.80	21.60
Beech	56.64	42.00	59.28	44.40

These prices are based upon the various amounts of the antiseptics which the different woods absorb, with careful work. As already stated, treatment with chloride of zinc alone has been given up, and boiling in creosote is growing in favor, as computations of annual charges for the renewals exhibit the fact that, notwithstanding the higher cost, impregnation with tar-oil is the most economical, in the long run.

In days gone by, some parties asked for guarantees before having ties treated by the zinc-creosote process, and in such cases Mr. Rütgers guaranteed the following lives:

Guarantee A.	Guarantee B.
95% to last.....10 years.	95% to last.....10 years.
80% " .....11 "	75% " .....11 "
70% " .....12 "	65% " .....12 "
	50% " .....13 "
	25% " .....14 "

In computing the results, 5% of all ties treated is first deducted, to cover internal diseases which even the strict German inspection cannot detect, and the guarantee applies on the remainder alone, Mr. Rütgers agreeing to make the deficit good to the extent of either refunding the treatment price paid per tie, or treating another tie free of charge, at his option. He charges extra a bonus of 10 pf., or 2.40 cents, a tie for this, and in the case of Guarantee B an extra bonus of 10 pf., or 2.40 cents, a tie a year, for every year's life over the guarantee. In point of fact the character of his work is now so well established that his customers prefer to save the bonus and take their own chances as to life of ties.

Particular attention is called to the German specifications for creo-

sote, and to the amount of chloride of zinc injected in connection therewith. The latter amounts to ½ lb. dry, per cubic foot.

The leading features of tie treatment in Europe are, therefore, the following:

1. Close inspection of raw ties, rejecting all defects.
2. Thorough seasoning for 6 to 12 months before treatment.
3. Constant testing of chemicals; strength, purity, etc.
4. Injection of liberal quantities of the chemicals.
5. Minute care in all the stages of impregnation.
6. Drying the ties after injection.
7. Methods of fastenings superior to our own.
8. Deep ballast, and thorough drainage.
9. Marking with dating nails, and careful records.

United States.

Thus it appears that the Europeans are now getting a longer service out of their ties than is obtained in the United States, Mr. Curtis having shown, in his paper* read before this Society May 17th, 1899, that an average life of 10 to 12 years is being obtained by the use of zinc chloride in this country. It would be possible to obtain a life of 15 to 30 years by the use of creosote, but it will be seen, from the figures given, that this would cost three to four times as much as zinc chloride. Thus, at present prices, it would cost 45 cents each to creosote according to English practice, and 15 to 16 years' life would be obtained; it would cost about 85 cents each to creosote after the best French or German practice, and 27 to 30 years' life would be obtained in thoroughly drained ballast; but it would not be economical to spend such sums upon ties costing 20 to 40 cents each untreated, while it is economical to spend them upon ties costing from 90 cents to $1.50 each abroad.

We must be content, therefore, either to allow our cheap ties to decay in the good old way, or to adopt for the present some of the cheaper and inferior methods which will produce shorter lives than obtained in Europe. By the light of past experience, those cheaper methods may be said to be three in number: 1st, straight Burnettizing; 2d, the zinc-tannin process, and 3d, the zinc-creosote process.

* *Transactions*, Am. Soc. C. E., Vol. xlii, p. 288.

The writer is satisfied that the zinc-tannin process, as modified by himself in 1896, is superior to straight Burnettizing, and that the record of the next few years will demonstrate this, yet he is desirous of doing still better work, and he went abroad chiefly to investigate the zinc-creosote process. He now thinks that it is probably superior to the zinc-tannin process, although part of the greater life shown by records is attributable to other causes, such as the better ballast and drainage, and the better modes of fastening, as well as the climatic conditions. There are, however, some serious difficulties to be overcome before the process can be introduced here. Suitable tar-oil, as described in the specifications of Appendix C, is just now very scarce and high in price, so high that the freight, the leakage and the cost of the barrels render the cost almost prohibitory. The writer took over with him two samples of American creosote from different makers, and had them analyzed in Berlin, where they were pronounced by Mr. Rütgers' chemist quite unfit for tie preserving by the zinc-creosote process. The writer brought back samples of the German tar-oil, and is now endeavoring to procure a similar product in this country. He is also investigating, so far as he can, the merits of various new processes which are being advanced from time to time, with the hope of finding some method which he can recommend.

The principal dilemma, with reference to new processes, is the fact that it takes half a business lifetime (15 to 18 years) to ascertain beyond peradventure whether an antiseptic or a method is thoroughly efficient to preserve ties in the track; yet it may be possible by isolating the bacteria and fungi which are most destructive to wood, and inoculating chips and shavings with the cultures, to draw some approximate conclusions in the course of a few months. Meanwhile, it is recommended that the railroads shall give trial orders to the various parties who offer plausible processes, and then expose the treated ties in such locations as admit of careful record and watching of the results.

The principal new processes are as follows:

1. The Creo-Resinate Process.—This process was described in a paper presented to this Society by F. A. Kummer, Jun. Am. Soc. C. E., on June 6th, 1900.* He proposes the use of creosote and resin, both preservative substances, to which a small percentage of formal-

Transactions, Am. Soc. C. E., Vol. xliv, p. 181.

dehyde is to be added. It may be questioned whether the latter volatile substance will stay in the wood, but the process is well worth trying, provided sufficient quantities of the chemicals are injected.

2. *The Water-Creosote Process.*—This consists in injecting an emulsion of creosote and water, and is being experimented with in Berlin. The writer expects to have samples sent to him this year.

3. *The Hasselman Process.*—This consists in boiling the wood in a solution of the sulphates of copper and iron, with alumina and "Kainit." It possesses the merit of being cheap, and some ties prepared in that way have now been three years in the yard tracks in Berlin. Works have been started at Perth Amboy, N. J., to work this process.

4. *The Allardyce Process.*—This consists in the injection of chloride of zinc, followed by a second injection of tar-oil. A similar process was patented by the late J. P. Card, Assoc. Am. Soc. C. E., in 1882, but neither he nor the writer, who subsequently became his partner, ever succeeded in doing good and regular work therewith, notwithstanding many experiments.

5. *The Naphthenic Acid Process.*—This consists in injecting the wood with a solution of copper which has been dissolved in an acid obtained by a peculiar process in the distillation of Russian petroleum. It is stated to be theoretically effective and cheap, but American petroleum differs so much from the Russian in chemical constituents, that it is yet a question whether the naphthenic acid can be produced in this country.

Laboratory tests are sometimes misleading. They show, for instance, that bichloride of mercury and sulphate of copper, as antiseptics, are superior to chloride of zinc, and yet the latter preserves ties better. So with carbolic acid, which is stated by Mr. Boulton to be less efficient than the heavy oils of creosote. Such criticism, however, does not apply to the laboratory tests of the strength and purity of the materials used, and there are three features of the European practice which it would be well to imitate in this country. They are the following:

1. The careful testing, chemically, of the antiseptics to be injected.

2. The uniform injection of the wood with stated and liberal quantities of the antiseptics.

3. The adequate seasoning of the wood before treatment. This is

now generally neglected in the United States, and yet it is the most important requirement in obtaining good results, for, otherwise, the antiseptic will not be uniformly distributed, and some portions of the ties will decay before others.

Originally, at the works of the writer, ties were treated in the order of their arrival, and without regard to their condition. The result was that in 6 or 7 years some were found to decay much in advance of others. Experiments, made by weighing considerable numbers of individual ties before and after treatment, disclosed the fact that there were great differences in the absorption, and in later years arriving ties have been tested, sorted out, and seasoned in case of need, so as to obtain uniformity of treatment, with the result that the average absorption of chloride of zinc is now two and one-half times as much as it was 14 years ago.

From his experience, the writer is satisfied that if the ties are injected with reasonable uniformity and with the equivalent of ½ lb. of dry zinc-chloride to the cubic foot, as is done in Germany, straight Burnettizing makes them last 10 to 12 years in the track, with ordinary exposure; while perhaps half of that quantity will produce the same result in the more arid regions of the United States; that the new zinc-tannin process will impart to them a life of 12 to 14 years, and the zinc-creosote process may extend this to 14 or 16 years.

It cannot, however, be too strongly insisted upon that the work must be well and skillfully done, for, otherwise, the results are sure to be disappointing.

APPENDIX A.

LONDON, BRIGHTON AND SOUTH COAST RAILWAY COMPANY.

Specification and Conditions of Contract for Creosoted Redwood Sleepers.

1. This proposed contract is for the supply and delivery of
.....sleepers at Deptford Wharf,
.....sleepers at Newhaven Wharf,
or any less number that the Directors may decide to accept.

2. The dimensions are to be the customary 9-ft. length (say $8\frac{11}{12}$ ft.) 10 by 5 ins. rectangular, cut from square blocks out of which neither more nor less than two sleepers can be sawn (no centers will be accepted).

3. The quality shall be the best Baltic redwood fir in good condition, free from shakes, dead knots, and other defects.

4. Sixty per cent. of the sleepers to have on one side a flat surface not less than 9 ins. wide throughout the length, and the remainder to have on that side a flat surface not less than 8 ins. wide throughout the length. All sleepers to have a flat surface not less than 10 ins. wide on the other side with sharp edges throughout the length.

5. Ninety per cent. of the sleepers to have not less than $8\frac{1}{2}$ ins. and the remainder not less than 7 ins. diameter of heart at both ends.

6. The blocks from which the sleepers are cut must be of last autumn's deflotation at the port of shipment; any delivered of an earlier deflotation will be rejected.

7. The sleepers are to be cut and stacked from four to six months (or until they are considered sufficiently dry by the Company's Engineer or his Inspector) before they are creosoted. They are to be adzed to a true plane for a width of 17 ins. at each end for the chair seating, and 40%, or such other percentage as may be required, are to be bored with eight holes, namely, two $1\frac{1}{8}$ ins. diameter at each end (for trenails) and two $\frac{5}{8}$ in. diameter at each end (for spikes). A template, showing the position of these holes, will be provided by the Company, and the Contractors must bore the holes exact to it, and perfectly true through the sleepers.

8. The sleepers will be inspected at the Contractor's wharf before being creosoted, and the Engineer shall have power, personally or by deputy, to reject any sleepers he may consider inferior, either in quality of timber or from any deviation from the specification, and his decision in the matter shall be final.

9. The sleepers, when sufficiently dry, are to be placed in a wrought-iron cylinder, and when closed a vacuum is to be created by air-pumps. The creosote, at a temperature of 120° Fahr., is to be allowed to enter the exhausted cylinder, and afterward maintained there by pumping at a pressure of not less than 120 lbs. to the square inch. The sleepers are to be kept under this pressure until each sleeper has absorbed at least 3 galls. of creosote on the average, the quantity to be ascertained by weighing.* Any charge of sleepers not giving the average impregnation of at least 3 galls. to be returned to the cylinder for further treatment.

10. The creosote to be a pure coal-tar distillate of the very best quality, free from water and all impurities, and on analysis to give the following results:—

To be entirely liquid at a temperature of 120° Fahr., and remain so on cooling to 93 degrees.

To contain not less than 25% of constituents that do not distil over at a temperature of 600° Fahrenheit.

To yield, to a solution of caustic soda, not less than 6% by volume of tar acids.

The specific gravity at 90° Fahr. to range between 1.040 and 1.065, water being taken as 1.000 at the same temperature.

11. The Contractor is to supply a copy of the analysis of each delivery of the creosote oil used, in the terms of the Specification, and the Engineer shall be at liberty to take samples of the oil from time to time and have the same tested, the Contractor paying the cost of the analysis to the extent of one analysis for each 10 000 sleepers. Any additional analysis to be made at the Company's expense.

12. Delivery shall be made alongside the Company's New Haven and Deptford wharves, at either of the rates mentioned below, at the option of the Company's Storekeeper, until the Contract is completed. Delivery at Deptford will be taken by open barges containing not more than 1 800 sleepers each. The craft to take regular turns for discharging, and conform to the regulations of the Company's wharves:—

At Deptford—To commence..... and
continue at the rate of ..... sleepers per week.
At Newhaven—To commence ..... and
continue at the rate of .....sleepers per week.

13. Should the Contractor fail to deliver the sleepers, or any portion of them, as stipulated in Condition No. 12, the Directors may cancel the Contract or the residue thereof, and obtain other supplies in such manner as they think fit, and the Contractor shall pay to the Company any extra cost and expenses incurred by such failure, or the

* In practice one trolley is weighed out of each charge —O. C.

Directors may deduct the amount from any sum then due or becoming due to the Contractor.

14. The shipping port or ports must be named in the Tender; and if more than one port, the number of sleepers proposed to be shipped at each port must also be given. Bills of lading to be produced by the Contractor when required.

15. The price per sleeper is to include every charge except wharfage and landing at the Company's wharves.

16. For sleepers delivered and approved during one month, payment will be made at the Company's next monthly pay day by cash, less 2½% discount, provided the Company have no claim on the Contractor as specified in Condition No. 13. In case of any dispute arising between the Contractor and the Company or their agents as to the meaning of any of the terms and conditions of this Contract, the decision of the Company's Engineer shall be final and binding upon all parties.

17. The Contractor shall, if required, enter into and sign a formal Contract with the Directors, and find good and sufficient surety to guarantee its proper fulfilment; the expense of such Contract and Bond to be paid by the Company.

18. The Directors do not bind themselves to accept the lowest or any Tender.

19. The Tenders are to be returned by post, addressed to "The Secretary, L., B. & S. C. R., London Bridge, S. E.," and must reach him not later than first post on .....
endorsed on the outside cover "Tender for Sleepers."

Signed,

.....

Storekeeper.

General Stores Office,
New Cross, S. E.

APPENDIX B.

NORTHERN RAILWAY COMPANY, FRANCE.

SPECIFICATIONS.—1893.

For furnishing beech ties of usual shape, creosoted by the new "Blythe" process, called Thermo-Carbolization.

Article 1.—The present specifications refer to the furnishing of ordinary ties for the extension and maintenance of tracks upon the Company's various lines.

Article 2.—The ties shall be of beech, creosoted by the new "Blythe" process (called Thermo-Carbolization).

Article 3.—The ties shall be rectangular, or present one of the sections shown in Figs. 1 and 2.

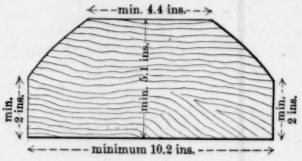

FIG. 1.

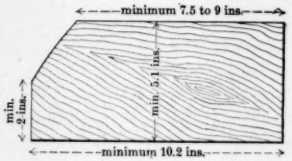

FIG. 2.

The top and bottom faces shall be sawed, the sides may be hewed.

The bottom face, which rests on the ballast, shall be square edged; the two lateral sides shall be without wane, for a minimum height of 2 ins. The top face shall be at least 4.4 ins. wide in the middle, and for the whole length of the tie.

The minimum dimensions of the ties shall be as follows:

Length (2.60 m.)	8.53 ft.
Width (0.26 m.)	10.2 ins.
Thickness (0.13 m.)	5.1 "

Article 4.—The ties shall be practically straight. If bowed sideways the incurvation shall not be more than $\frac{1}{20}$ of the whole length. All ties will be rejected whose bottom face is longitudinally bowed, being either convex or concave.

The ties shall be sawed off square at the ends.

Article 5.—The beech wood must be perfectly sound and of the best quality. It shall be neither heart-shaken, nor frost-split, nor

brashy, nor worm-eaten. It shall be exempt from doziness, rotten knots, cracks, splits, bad knots, red-heart or any other defect. The trees shall be felled between the 1st of November and the end of March. They shall be worked up into ties continuously, which work shall be completed by the end of April. Timber will be refused which has been felled before the 1st of November, or not worked up before May 1st.

All the ties shall be completely barked. As fast as the ties are made they must be piled up carefully, cob-fashion. The Chief Engineer of the Company reserves the right of putting on inspectors to follow up the felling and manufacture of the ties at various points.

Article 6.—The untreated ties will all be most carefully inspected, both as to quality and dimensions. Those accepted will be stamped at the ends with the Company's marking hammer.

All the sticks with any dimensions less than the limits stated in Article 3 will be rejected. Those with greater dimensions than required may be accepted, but no allowance will be made for oversize, the ties being purchased by the piece, and not by the cubic meter.

If a tie of good quality, and otherwise acceptable, shows a crack likely to spread open, the Contractor shall bolt the two parts together, or insert an **S** in the end, at his own expense. Every tie split open at the end for its whole width or thickness shall be rejected.

The rejected ties will be left on the hands of the Contractor at the point of inspection; they will be so marked as to preclude them from being again offered for inspection.

Article 7.—The untreated ties must be well seasoned before preparation, and, so far as possible, adzed and bored by the Company, to fit the rails and the lag screws. The seasoning will be tested by weighing a number of ties haphazard. The weight of the timber, thus ascertained, must not be more than 46.7 lbs. per cubic foot.

Article 8.—The ties will be injected with creosote by the new "Blythe" process, called "Thermo-Carbolization." In this process the ties are subjected to the two following operations:

1. They are enclosed in a cylinder of boiler plate and subjected to a current of steam mixed with creosote oil vapor for a length of time sufficient to insure, during the second operation, the absorption of the prescribed quantity of creosote.

2. The cylinder containing the ties is then filled with a sufficient quantity of crude creosote. This liquid, maintained at a temperature of at least 140° Fahr., is compressed in the cylinder by steam to five atmospheres from the generator, during a sufficient length of time so that the total quantity of creosote injected into the wood, both as a vapor and as a liquid, shall be at least 24.4 lbs.* per tie.

* Since increased to 35.2 lbs. per tie.

If the Engineer of the Company deems it advisable to increase this quantity of creosote, the Contractor will comply with the indications which he may receive relating thereto.

Article 9.—The creosote shall consist of the mixture of volatile products heavier than water distilled from coal-tar produced by gas-works. It shall contain at least 6% of carbolic acid, or of analogous products soluble in caustic soda. It shall be entirely soluble in benzine, and completely liquid at a temperature of 122° Fahrenheit. It must, moreover, conform to the samples which the Contractor shall submit to the Company's Engineer.

The Contractor shall specify from what gas-works each shipment of creosote is received.

Article 10.—The Company shall take cognizance of all the operations relative to the inspection of the wood, through an inspector appointed by the Chief Engineer. Such Inspector shall satisfy himself as to the thorough application of the "Blythe" process, shall keep accounts of the wood injected, and shall verify the results of the injection. He will report, upon a special blank, the following points for each operation:

1. The length of time of application of the mixed steam and oils coming from the vaporizer, as well as the final pressure in this operation, which must be at least five atmospheres.

2. The time occupied in filling the cylinder with crude creosote, the final pressure and the quantity of liquid absorbed during this operation.

3. The quantity of crude creosote introduced in the vaporizer and in the lower reservoir. This quantity shall not be less, for ten consecutive operations, than an average of 24.2 lbs. per tie injected.

4. Finally, the inspector shall keep an exact account of the total quantity of creosote which the Contractor shall receive at the works for injection, so as to check the amount of liquid absorbed during each season's work.

The Chief Engineer of the Company reserves, moreover, the right of using any other checks which he may deem desirable to control the quantity of creosote injected into the wood, either as a mixture of vapors or in the liquid state.

The royalties to be paid to Mr. Blythe or others for the use of the patents, etc., will be at the sole charge of the Contractor, who guarantees the Company against any claim of this nature.

All experiments made by the Company to ascertain whether the creosote is of good quality, and if the injection of the wood is complete, shall be at the charge of the Contractor.

Article 11.—Upon being withdrawn from the cylinder the ties shall be inspected one by one in order to ascertain whether the injection is homogeneous and whether the quantity of 24.2 lbs. has been duly

absorbed by each. This last verification shall be effected as described in Section 3 of Article 10.

Such ties as may be incompletely injected, a fact which shall be established by cutting into them with a gouging adze, shall be subjected to a second operation, or even to a third, after which they may be declared, in case of need, as unfit for injection, and finally rejected, as well as those ties which may be deformed by the action of heat in the cylinder.

The Contractor shall make no extra charge for these repeatings of treatment, and shall, moreover, either insert a bolt or an **S** into any tie which shall split during the treatment.

The ties accepted shall be counted and either immediately loaded on cars or piled up at some point designated by the Company's agent. The ties rejected shall receive a special mark at the rail seat and will be piled at special points to be indicated to the Contractor. These ties shall only be taken away upon authority given by the Engineer of the Company, who may hold them until the season's contract is filled, so as to avoid their being again presented for inspection by agents.

*Article 12.**—The final acceptance of the ties shall only take place six months after the full delivery of the season's contract.

Until this final acceptance the Company reserves the right of rejecting any ties which may possess defects not detected upon a first inspection, or which may split by reason of a bad quality of wood.

The ties so rejected will be surrendered to the Contractor at the point of delivery, who shall either deduct them from his bill or furnish other ties if required by the Company.

Article 13.—The Contractor shall be governed, save in such modifications as result from the present specifications, by the clauses and general conditions imposed upon contractors doing work for the Northern Railway Company, through the Rules drawn up September 26th, 1892, by the Chief Engineer of the Ponts et Chaussées, Chief Engineer of Maintenance of Way, approved October 21st, 1892, by the Executive Committee of said Company, and registered in Paris the 28th of the same month.

* The same contractor furnishes the ties and treats them.

APPENDIX C.

IMPERIAL RAILWAYS OF ALSACE-LORRAINE, GERMANY.

SPECIFICATIONS FOR IMPREGNATING WOODEN RAILROAD TIES.—1898.

Section 1.—For pine ties, the impregnating fluid is a solution of chloride of zinc with an addition of coal-tar oil containing carbolic acid; for beech or oak ties hot coal-tar oil containing carbolic acid must be used.

Section 2.—The process of impregnating by chloride of zinc solution, with addition of coal-tar oil containing carbolic acid, is divided into three parts.

1. Steaming of the ties.
2. Production of a partial vacuum and admission of the impregnating fluid.
3. Compression (forcing in) of the impregnating fluid.

The ties are loaded on iron cars, which are pushed into the impregnating cylinder, this is closed air-tight, and they are exposed to the action of steam; steaming is continued for a longer or shorter period, according to the time of year and the condition of the ties. The admission of steam into the impregnating cylinder must be regulated in such manner, that an inside pressure of 1.5 atmospheres (22 lbs. per square inch) above air pressure is reached within 30 minutes. For dry ties it will suffice to maintain this pressure in the impregnating cylinder for 30 minutes longer, but for green ties it should be kept up for another hour. For dry ties, therefore, the steaming takes at least 1 hour, while for green ties at least $1\frac{1}{2}$ hours are necessary. A gauge attached to the cylinder indicates existence of the specified pressure. The valve at the bottom of the cylinder must be opened on admitting the steam, in order that the air contained in it may be driven out, but should be closed when steam begins to blow out. This valve should be opened repeatedly, as fast as steam condenses; open it at least every half-hour to draw off the water, and for the last time just before exhausting the air. When steaming is finished, the steam remaining in the impregnating cylinder is allowed to escape.

After steam is discharged a partial vacuum is produced in the cylinder containing the ties, until the vacuum gauge shows at the least a column of mercury of 60 cm. (23.6 ins.); this partial vacuum

must be maintained for ten minutes. On expiration of this time, while continually preserving the partial vacuum, allow the impregnating fluid, which meanwhile has been prepared in a separate vessel and heated to at least 65° Cent. (149° Fahr.), to enter the impregnating cylinder, filling it entirely. To prepare the impregnating fluid, add while heating, 1 kgr. of coal-tar oil to every 15 kgr. ($6\frac{2}{3}\%$) of the solution of zinc chloride.

To insure as perfect a mixture of the solution of zinc chloride with the coal-tar oil as possible, an effective stirring apparatus, combined with injection of steam and air, must be applied.

Next, a pressure pump is used to exert an excess of seven atmospheres above air pressure. This pressure to be maintained for not less than 30 minutes; if necessary, continue it for a longer time, until the ties have absorbed a certain amount of impregnating fluid as specified hereafter. The impregnating fluid is then run off.

The chloride of zinc solution intended for impregnating must be as nearly as possible free from foreign substances, and there must be no free acid. An admixture of other metals, notably iron, can only be allowed in a very slight percentage and only if it cannot be avoided in the manufacture. The solution must have a strength of 3.5° Beaumé = 1.0244 specific gravity at a temperature of 15° Cent. (59° Fahr.). The solution contains 1.26% of metallic zinc.

The coal-tar oil used must not contain over 1% of oils that boil below 125° Cent. (257° Fahr.). It must be so little volatile that its boiling point lies mainly between 150° and 400° Cent. (302° and 752° Fahr.). In no case is it permissible to have more than 25% of its weight volatilized below 235° Cent. (455° Fahr.). It must contain at least 20 to 25% of acid substances (creosote or oils resembling carbolic acid) that are soluble in caustic lye of soda of 1.15 specific gravity. The coal-tar oil must be entirely liquid at + 15° Cent. (59° Fahr.), and as much as possible free from naphthaline, so that on evaporation (fractional distillation) produced in a glass vessel in groups of 50° each, it shall leave a residue of not more than 5% of naphthaline. Its specific gravity should not be less than 1.020 at a temperature of + 15° Cent. (59° Fahr.) and should not exceed 1.055. To remove such impurities from the impregnating fluid as are due to the process, suitable settling (clarifying) apparatus should be provided.

The Contractor is required to report where he obtains his supplies of zinc-chloride solution and of coal-tar oil, intended for use, and to furnish samples of the same to the Supply Office of the Imperial Railways at Strassburg in Alsace before commencing to impregnate. He will be permitted to purchase the solution of zinc chloride and the carbolized oil of coal-tar only from such factories whose samples have been approved by the Management of the Railways. The Railway Management reserves the right to test the fluids used at any time.

It is specified that the average absorption of impregnating fluid contained in every charge of the cylinder shall be the following:

A. Absorption of 35 kgr. (77 lbs.) for each tie of the first class, length of 2.70 m. (8.85 ft.).

B. Absorption of 26 kgr. (57 lbs.) for each tie of the second class, length of 2.50 m. (8.2 ft.).

C. Absorption of 310 kgr. per cubic meter (19 lbs. per cubic foot*) for ties of other dimensions.

To determine the amount of impregnating fluid absorbed by the ties, the following method must be adopted:

Weigh all ties on a platform scale placed under roof immediately before steaming them, and again after impregnating when dripping has ceased. The difference in weights equals amount of impregnating fluid absorbed. A deduction of 15 pfennigs per 10 kgr. (16 cents per 100 lbs.) will be made for shortage shown by this weighing test. In case the shortage amounts to more than one-sixth of the absorption specified, the impregnation must be repeated. If, on the other hand, the weighing shows that the ties have absorbed more than the amount specified, a bonus of 15 pfennigs for every 10 kgr. (16 cents per 100 lbs.) will be paid for such increase, up to a maximum of 15 per cent.

Section 3.—The work of impregnating with hot carbolized oil of coal-tar (*i. e.*, oil of coal-tar containing carbolic acid) must be divided into two parts.

1. Drying of the ties, *i. e.*, withdrawing water from them.
2. Introduction of oil of coal-tar under pressure.

The ties are run into the impregnating cylinder and this is closed air-tight. Next, a partial vacuum, equal to at least 60 cm. (23.6 ins.) column of mercury, is produced in the impregnating cylinder and maintained for 10 minutes, and thereupon, while keeping up the vacuum, the hot oil of coal-tar is made to flow in until it rises to a level that will prevent sucking over by the air pumps. The flowing in of the coal-tar oil may be accomplished all at once or at intervals, according to the dryness of the ties. While thus filling up, and afterward, the coal-tar is heated up inside the cylinder to at least 105° Cent. (221° Fahr.), but not higher than 115° Cent. (239° Fahr.), by means of steam coils. This heating should be accomplished during a space of time of not less than 3 hours. When this temperature is reached in the impregnating cylinder, it must be kept up for another hour, either with or without the partial vacuum, as may be judged necessary, in order that the ties may absorb the specified amount of oil of coal-tar.

The impregnating cylinder is connected with a pipe condenser from the instant that filling with hot coal-tar oil commences, and all the

*3.5° *B.* corresponds to 2.62% dry zinc chloride. Hence, 19 × 2.62% amounts to 0.498 lb. of dry zinc chloride per cubic foot.—O. C.

aqueous vapors driven out of the ties are condensed in this, the water being carried to a tank. This receiver must have a water gauge from which one can read off the amount of water evaporated from the ties.

After the drying of the ties or the extraction of water from them is finished, the impregnating cylinder is filled completely and the pressure pump started, which must produce a pressure of at least 7 atmospheres. This pressure is to be maintained for at least 30 minutes in treating beech ties and 60 minutes for oak ties, unless it proves necessary to prolong the time to obtain the amount of absorption specified. The oil of coal-tar is then drawn off.

The coal-tar oil used must be heavy oil, derived from the distillation of coal-tar, of greenish black color, specific gravity of 1.045 to 1.100 at 15° Cent. (59° Fahr.), boiling point between 150° and 400° Cent. (302° and 752° Fahr.).

While making fractional distillation no oils must pass over below 150° Cent. (302° Fahr.) and not more than 25% of the volume at temperature up to 235° Cent. (455° Fahr.).

The coal-tar oil must contain by volume at least 10% of carbolic acid and, at a temperature of 15° Cent. (59° Fahr.), must be free from naphthalene and show no sediment.

To determine percentage of carbolic acid apply agitation to the oils heated to 400° Cent. (752° Fahr.) with a caustic solution of soda having specific gravity of 1.15. The difference in volume of oil before and after agitation gives percentage of carbolic acid.

The Contractor is required to state source of supply for his coal-tar oil and to furnish samples to the Supply Office of the Imperial Railways at Strassburg before he commences work of impregnation. The coal-tar oil can only be purchased from factories whose samples have been approved by the Railway Management. The Railway Management reserves the privilege of at any time testing the coal-tar oil used.

It is specified that the average absorption of coal-tar oil for every charge of the cylinder shall be:

a. For one railroad tie, 1st class, 2.70 m. (8.85 ft.) long, of oak wood, 11 kgr. (24 lbs.); of beech wood, 36 kgr. (79 lbs.).

b. For one railroad tie, 2d class, 2.50 m. (8.20 ft.) long, of oak wood, 8 kgr. (18.6 lbs.); of beech wood, 28 kgr. (61.6 lbs.).

c. For ties of other dimensions per cubic meter (35.3 cu. ft.), of oak wood, 100 kgr. (220 lbs.); of beech wood, 325 kgr. (715 lbs.).

To determine the amount of coal-tar oil absorbed by the ties, these are weighed before the impregnation and again after it, when dripping of oil has ceased, using a platform scale placed under a roof. The difference in weight is amount of coal-tar oil absorbed. Correct the weight of the ties before impregnation by deducting from it weight of water delivered by condenser to the tank and obtained from the vapors distilled while drying in hot coal-tar oil, as weight of ties is reduced

to this extent by drying process. If on examination it is proved that absorption amounts to less than five-sixths of that specified, the impregnation must be repeated.

For every shortage in coal-tar oil shown by above test, a deduction of 50 pfennigs for 10 kgr. (54.5 cents per 100 lbs.) will be made, but, on the other hand, an increase in absorption will be paid for at the same rate, a maximum of 15% increase being the limit of such payment.

Section 4.—The Contractor is required to give eight days' notice to the Supply Office of the time of intended commencing to impregnate ties, in order that the office may send an official to supervise same. This official must be freely admitted at all times to the plant of the Contractor, and all desired information must be readily furnished him. The Contractor must furnish all necessary appliances, apparatus and labor to make tests without charge.

Section 5.—In case the Contractor does not supply his own ties, the parties furnishing them will be required to deliver f. o. b. cars at the station nearest to the impregnating works, provided they are shipped by rail; ties delivered by wagon or other conveyance will be delivered loaded at storage yards of the factory without charge.

The hauling of ties from the station to factory will be at the expense of the Contractor for impregnation. He has also to provide for unloading, piling and handling of ties as per regulations. The Contractor will be paid for this labor the amount of 8 pfennigs (1.92 cents) for each track tie and 4 pfennigs (0.96 cent) for each switch tie of 1 m. These prices cover the expense of labor and tools required in receiving green ties, as well as that of reloading rejected ties; payment for a tie to be made only once.

Section 6.—The Contractor for impregnating is held liable for all damages and loss of ties that may occur from the time they are delivered to him at the railroad station, or at his works, as long as ties remain at his works. This liability includes losses by fire occurring at the impregnation works and by theft committed while ties remain there. The Contractor must pay the value of all missing ties or of such as become unserviceable previous to their return after impregnation, but is not liable for splitting. He is, however, required to furnish without charge all necessary **S**-hooks and bolts for drawing together the cracks occurring during storage, and has to drive or put these in according to directions of the supervising official.

When ties are turned over to the Contractor for impregnation, they are already supplied with **S**-hooks needed to draw together all existing cracks. Each beech track tie is also fitted with two iron bolts running through it, about 10 cm. (4 ins.), from each end in the direction of its breadth. It is his duty, therefore, to supply, without charge, only such **S**-hooks and bolts as may be needed thereafter, and of the same kind, and to fasten them.

Section 7.—On receiving the green ties they must be piled at the factory in such a way that air will circulate freely around each one. Each pile only to have length and breadth equal to length of one tie, and must contain 100 ties. The lowest layer of ties must rest on solid supports, so that they will never touch the ground. Storage yards must be thoroughly drained and have ditches if needed. Open spaces are to be left between the piles, which spaces must measure 80 cm. (32 ins.) in one direction and have a width of 40 cm. (16 ins.). This piling of ties must be finished at the latest in 14 days from receipt of same. Date of piling to be plainly marked on each pile.

For delay in completing impregnation of ties beyond time fixed by contract, unless previous express and written permission of the Imperial General Management has been obtained, the latter will collect a penalty from Contractor for such delay, amounting to 1% of the Contract value of the unfinished impregnation per week of such delay.

Section 9.—The Railroad Management reserves the right to employ the Contractor for impregnation to adze surface of ties in places for bed-plates of rails, as well as to bore holes for fastenings, if such work becomes necessary. This work to be done by direction of supervising official and before impregnating.

Strassburg, February....1898.

Imperial General Management of
Railways in Alsace-Lorraine.

Acknowledged: The preceding contract of.....
this day the..... 1.....

D.....

Contractor.

APPENDIX D.

TESTING OF IMPREGNATING SOLUTIONS FOR IMPERIAL PRUSSIAN RAILWAYS.

For the impregnation of timber there are at present two products in use: 1st, chloride of zinc; 2d, tar-oil.

Testing of Chloride of Zinc.

The chloride of zinc for impregnating purposes will be manufactured as a concentrated solution, containing about 50% of anhydrous chloride of zinc. It is best to use such a strong solution for testing, and for that purpose, samples are to be taken directly from the shipping tank or carboy.

The zinc chloride solution used must be as free from impurities as possible, particularly from iron and free acid. Therefore, it is to be determined whether or not iron and acid are present.

Test for Free Acid.

Twenty grammes (by weight) of the above strong zinc chloride solution are to be mixed with distilled water; the whole to amount to 100 cu. cm. (by measure), the mixture to be well shaken.

a.—There is no free acid present if the mixture, by shaking, becomes cloudy, and, particularly if after a short period of rest, flakes settle down, which will again dissolve to a clear fluid, upon the addition of a few drops of muriatic acid (H Cl). No further test is then required.

b.—If, after shaking, the mixture remains clear, then an excess of acid is present, the amount of which can be determined by the following manipulation:

Take several reagent bottles and put in each 10 cu. cm. of the above-described mixture, then add to each bottle a measured successively increasing quantity of a solution of one-tenth normal soda. For example: Add to the first reagent bottle 0.1 cu. cm., to the second 0.2 cu. cm., to the third 0.4 cu. cm., and so on. Shake well and observe in which bottle a remaining white flaky precipitation will settle. The proportion of soda which lies between the mixture where a precipitation is produced, and that where no precipitation is produced, exactly represents the quantity of free acid present in the solution. For example, the mixture in the bottle to which 0.2 cu. cm. of the soda solution was added, remains clear, while in the following reagent bottle, where 0.4 cu. cm. soda solution was added, a precipitation is produced; then, 0.3 cu. cm. soda solution is exactly the quantity corresponding to the free acid present in the chloride of zinc solution.

Should there be required for this test more than 0.4 cu. cm. of the one-tenth normal soda solution, then the percentage of free acid is too high in the chloride of zinc solution, and such solution must not be used for impregnation.

Testing for Iron.

Take 10 cu. cm. from the above-described mixture of zinc chloride solution and distilled water, and add a few drops of concentrated nitric acid ($H\,N\,O_3$) and shake well. Divide this mixture into two equal parts. To one part, without diluting, add a quantity of ammonia ($N\,H_4\,O\,H$) and shake well. If this mixture remains clear, no iron is present. Through the presence of iron in the mixture, more or less brown flakes will precipitate, corresponding to the amount of iron present. Should there precipitate in the mixture a quantity of gray-white (not brown) flakes, then not only iron, but also another impurity (nearly always magnesia) is present. In this latter case a more complete test has to be made, and, therefore, the zinc chloride solution must be sent to a chemist. But this case will happen very seldom.

The second part of the mixture of 10 cu. cm. to which nitric acid was added, should be diluted with distilled water, and 5 cu. cm. of a solution of 10% yellow prussiate of potash added, the whole to be well shaken. A very ample precipitation will be produced, which will look snow-white, or very light yellowish, if the zinc chloride solution is free from iron; but in the presence of iron it will look more or less blue, according to the amount of iron. If the precipitation shows a corn flower blue color, then the zinc chloride solution surely contains a high percentage of iron and must therefore be rejected.

To avoid, in testing, the weighing of the 20 grammes of the strong solution, the use is recommended of the easier method of measuring. First find the specific gravity, at 15° Celsius, of the strong concentrated zinc chloride solution. The quotient of this specific gravity into 20 grammes shows the number of cubic centimeters which must be measured off and which represent exactly 20 grammes by weight. For instance, the specific gravity of the strong zinc chloride solution is 1.6, then 1.6 divided into 20 grammes gives the number of cubic centimeters $\left(\frac{20}{1.6} = 12.5 \text{ cu. cm.}\right)$ which have to be measured off to be used for testing as described before.

Testing of Tar-Oil.

At a temperature of 20° Celsius the tar-oil must be limpid, and to test it, shake the tar-oil well, pour a few drops on a folded filter paper, and observe whether after absorption there remain undissolved particles on top of the paper. If the amount of these is large, the tar oil must not be used for impregnation. To find the specific gravity, the

tar-oil must be heated, or cooled off, to a temperature of 15° Celsius; then drop slowly an hydrometer into the same, and read the number at the surface of the oil. This number indicates the specific gravity of the tar-oil at 15° Celsius; small variations in temperature are of minor importance, and can be corrected closely enough by adding or subtracting 3 to the figure in the third place of the specific gravity for every 2° variation from 15° Celsius.

Laboratory Distillation of the Tar-Oil.

By means of a funnel, 102 cu. cm. of tar-oil at about 15° Celsius are to be filled into a retort, a thermometer is to be inserted, but in such a manner that the quicksilver ball shall be in or below the neck of the retort but shall not touch the oil, or will not be covered by the same. The retort must be heated slowly, until all the water, which is contained in nearly every tar-oil, is evaporated. Stronger heat can then be applied to the retort, but it must be so regulated that in one second two drops will distill over. The distilled product will be caught in a graduated glass cylinder, and the different quantities are to be read and noted which distill over from the oil (become volatile), within the various intervals of temperature, say to 125° Celsius (150°) from 150° to 235°, and again from 150° to 355° Celsius, and which are specified in the "Description of the Process, and Specifications" as to the composition and proportions of the impregnating fluid.

Finding the Percentage of Carbolic Acid.
(Acid Constituents of the Oil.)

The entire amount of the distilled tar-oil is to be mixed in a separating funnel with 50 cu. cm. of caustic soda of 1.15 specific gravity at 15° Celsius, shaken well for about five minutes, after which let it stand and settle. The caustic soda absorbs the carbolic acid and precipitates; the stopcock of the funnel is to be opened and the precipated caustic soda is caught in a 200-cu. cm. graduated glass cylinder. The same operation must also be repeated with 50 cu. cm. of fresh caustic soda, to make sure that all carbolic acid is extracted from the oil. The caustic soda of both manipulations is then to be combined, about two tablespoonfuls of salt (Na Cl) added and this dissolved by means of stirring; the required quantity of concentrated muriatic acid (H Cl) added, and the combination again stirred up until well mixed. After cooling off the hot mixture, read the quantity of the separated carbolic acid in percentage of cubic centimeters, and add to this number ½% for the small amount of carbolic acid still remaining in the acid solution.

All the figures obtained are to be compared with those specified in the description of the composition and proportions of the impregnat-

ing fluids. Small differences should not be cause for rejection, as small variations in testing, resulting from barometric changes, cannot be avoided, and the result of the test is influenced by them. However, the figures obtained by the above-described tests are sufficiently close to judge of the quality of the impregnating fluids. It is not advisable that the tar-oil for testing be taken directly from the shipping tank, but it is better to take the samples from the receptacle of the apparatus in the impregnating plant from which the mixing vessels, or the impregnating cylinder (in the impregnation with pure oil) will be supplied.

The Chief of the Operating Inspection 3.

(Signed.) SETTGAST.

BERLIN, June 14th, 1899.

DISCUSSION.

Mr. Croes. J. JAMES R. CROES, M. Am. Soc. C. E.—This interesting paper contains many valuable facts and figures, but the variety of notation used in presenting them renders it somewhat difficult to compare the results attained in different countries and by different processes of preservation. Thus, in Table No. 1, which gives data from English railroads, the amount of creosote used is given in pounds per cubic foot, gallons per cubic foot, pounds per tie and gallons per tie, indiscriminately, without any statement as to the relation of the tie, the cubic foot, the pound and the gallon. Then, too, the number of ties renewed annually is given without any reference to the total number of ties in use or the length of the track, except in one case where the number of ties renewed annually per mile is given. So, also, in Table No. 2, giving data of French railroads, the length of track on which the observations were made is not given at all; simply the total number of ties renewed annually. However, the cost of treatment per tie is given uniformly in American cents per tie and that is, after all, what it is necessary to know, in connection with the life of the tie, to determine the relative advantage of using different methods of treatment or no treatment at all. As the author well says, it is not economical to spend as much money on the preservation of a cheap tie as might be spent in extending the life of one of great first cost, and, therefore, cheaper and more inferior methods of preservation than are found advantageous in Europe, where the timber costs more, may well be applied in this country.

The author gives as the first two of the leading features of tie treatment in Europe, (1) close inspection of raw ties, rejecting all defects, and (2) thorough seasoning for six to twelve months before treatment. It appears that he has found it desirable to introduce these features in his own works in this country. A number of railroad managers and engineers have found it to their advantage to take the same precaution with reference to the ties to be used in their roads, without treatment. Indeed, some engineers go so far as to say that this preliminary treatment appears to prolong the life of the average tie more than the subsequent injection of a small quantity of cheap chemicals. It would seem as if this question was worth a careful investigation and a comparison of two distinct sections of road, one laid entirely with untreated ties selected and seasoned under cover for a proper length of time, and the other laid with treated ties from some one of the commercial tie plants. One difficulty connected with the determination of the whole question is, as the author says, the fact that it requires nearly half a business lifetime to arrive at any results, and the proportion of an official lifetime is much greater. In carrying

out any series of experiments or inspection, uniformity of method is desirable, and that is not easily maintained under changes of administration. From the Society lists for the years 1890 and 1900 it appears that of one hundred members holding the position of chief engineer of a railroad in the former year only 30% held the same position in 1900. Promotion, transfer, resignation or death, had caused the changes of administration, and, in the absence of any uniform and standard system of recording facts and statistics, had doubtless produced changes of method which caused previously collected data to be of little or no use. The valuable papers of Mr. Chanute and Mr. Curtis* emphasize the need of a concert of action among the engineers of railroad maintenance, in the collection and recording of important statistics. Mr. Croes.

W. B. REED, Assoc. M. Am. Soc. C. E.—There are a few vulcanized ties on the Metropolitan Railway, and, as nearly as the speaker can ascertain, these were laid about eight years ago. In taking out some of them recently, they were found to be so soft that they would not hold a spike. They did not seem to be decayed, but had evidently absorbed more or less moisture from the ground, and were not fit to be used again. On the other hand, some long-leaf Georgia pine or southern pine ties, without any treatment, which had been laid by the speaker six years or seven years ago, are perfectly fit to be used again, and are thought to be good for five or six years' more service. Of course, the conditions are different from what they would be on a steam railroad, or on an elevated railroad. The ties are protected, more or less, by the paving of the streets, and are surrounded by the gases of various kinds under the street pavements of New York. Whether or not that is a preservative, the speaker is unable to say. Mr. Reed.

Eight or nine years ago, the speaker used 5 000 vulcanized ties in a surface railroad in the northern part of New York State. Five or six years later, owing to a change in the paving, some of these ties were dug up and were found to be so soft that they would not hold a spike, although they did not show much decay.

These ties were of short-leaf yellow pine, which may be one cause for the trouble. The wood was certainly softer in the beginning than the hard pine now being used in the streets of New York where tie construction is used.

The speaker is not able to say whether or not these soft ties would have hardened when dried out by exposure to the air. He did not try them, as he did not think they were worth saving. The whole tie was very soft, and there was not a great deal of moisture. The ties referred to were taken out at the intersection of Twenty-third Street and Ninth Avenue, where the ground is not low, but is fairly well drained, although the paving at that point had no concrete foundation, and more or less moisture could get through the pavement into

* *Transactions*, Am. Soc. C. E., Vol. xlii, p. 288.

Mr. Reed. the ground. The other ties taken out (long-leaf southern pine, not treated) were on South Street. They were protected more or less by a concrete foundation under a granite block pavement, with tar and gravel joints, but South Street is very wet a great deal of the time; in fact, it is flooded with water for a long time after a storm, so that the ground underneath is a good deal more moist than at Twenty-third Street and Ninth Avenue.

Mr. Tillson. G. W. TILLSON, M. Am. Soc. C. E.—The speaker was interested some time ago in looking up the methods and history of the preservation of timber, to see how long ago it was first taken up, and the many and curious experiments which were made. It is known that as far back as 1657 a German recommended the use of tar as a preservative of timber, and Baumeister, in 1798, was very much in favor of the use of salt. It was found that the timber used in ships engaged in the salt-carrying trade, as well as that used in the shoring up of salt mines, lasted a much longer time than timber under ordinary conditions, and the speaker was surprised this last summer, when visiting the wooden shipbuilding section of Maine, to find that they made use of that practice there now. The salt is distributed in great quantity among the timbers in the framework of the vessel, between the outer and the inner planking, and adds materially to the life of the vessel. This value of the salt is considered more on account of its hygroscopic than its antiseptic properties.

Boucherie, of whom the author speaks, had a system of treatment in the early part of the nineteenth century, which was certainly novel and unique. The timber to be treated was chosen while standing in the forest. Large cuts were made at the bottom of the trunk and a receptacle built around it; then the liquid proposed to be used was put in this receptacle and the respirative action of the tree—the suction—drew the liquid up into the trunk and branches and thus preserved it. Another method, practically the same, although somewhat different in detail, was to cut the tree down first and then insert the butt of the tree in a vat containing the preserving liquid. It is stated in Parnell's Applied Chemistry that a poplar tree 90 ft. high absorbed 10 cu. ft. of acetate of iron in six days, the liquid having a specific gravity of 1.056, and that it would go up as far as the leaves. If engineering is admitted to be the adaptation of the powers of Nature to the uses of man, this would seem to be a case of the pure science.

In Mr. Kummer's paper* some reference is made to the treatment of wood for pavements. The speaker thinks that Indianapolis, Ind., is the only city in this country in which wood as a paving material for an improved pavement is being considered seriously. Experiments

* "A Proposed Method for the Preservation of Timber," *Transactions*, Am. Soc. C. E., Vol. xliv, p. 181.

have been conducted there for three or four years, and this year a large quantity of it has been laid. The timber is treated by the ordinary creosoting process, the specifications requiring 10 lbs. of oil per cubic foot of wood. The pavements treated by this method cost about $2.50 per yard. In London and other English cities which use wood as a pavement, soft wood, Baltic deal, is treated, while hard or Australian woods are untreated. Of course, the principle by which they determine whether they shall or shall not creosote the wood is practically the same as with railroad ties. If the traffic is such that the wood will wear out before it would naturally decay, it is not treated; but if the traffic is light, so that the wear will not be very much, but the life of the pavement be determined by its ability to resist the action of the weather, then the question of creosoting comes in, and when the relative lives of the creosoted and the uncreosoted blocks are known, and also the cost of the treatment, whether it shall or shall not be used is simply a question of mathematics, as in the case of ties. Mr. Tillson.

MENDES COHEN, Past-President, Am. Soc. C. E.—When this subject was under discussion before the Society some years ago, the speaker stated as the result of his experience with the Burnettizing process on the railways of the Lehigh Coal and Navigation Company, while under his charge, that the process was effective, not only in increasing the durability of the ordinary tie timber, but that, in a measure, it tended to lessen the original average cost of the ties by making available for such use some of the softer timber, greatly toughened by the treatment, which could not, otherwise, have been used in the track. Mr. Cohen.

He also then stated that the use of the Burnettizing process was suspended on that work only because the location and arrangement of the yard and plant was not consistent with economical handling. Before its re-location was effected, the railway property passed into the control of another corporation, and the speaker had no further information in regard to it.

SAMUEL WHINERY, M. Am. Soc. C. E.—From the speaker's observation in a number of cases, he thinks that a great deal more importance attaches to the proper selection of the wood to be treated than has often been devoted to the matter. In some of the work which he has observed, the timber was cut without much reference to whether it was long-leaf yellow pine or scrub pine. It was brought to the works in all conditions of seasoning, and he has no doubt, from the observations made since, as well as from what Mr. Chanute states in the paper, that the careful selection of the timber and its proper preparation by seasoning before it is treated has much to do with the success of the process. In the case of paving blocks, there can be no question that properly selected wooden blocks, properly creosoted, will give much better results in all cases where untreated blocks would fail Mr. Whinery.

Mr. Whinery. by natural decay before they are worn out by the travel over them; but it is very doubtful whether creosoting adds anything to the ability of the blocks to withstand the abrasion of travel.

From what one often sees in practice, it would seem that anything is considered good enough to creosote. That is, that almost any kind of wood, even if partly decayed, is all right if creosoted. The idea that creosoting is a cure for all the evils that wood is heir to is undoubtedly wrong. If great care were taken to properly select the timber, and to properly prepare it for the process, there would be fewer records of failure.

Those who have had experience in using yellow pine ties on southern railroads know very well that, in the first place, there are not many inspectors who can distinguish, when a tie is dressed and delivered on the road, whether it is of long-leaf yellow pine or bastard pine, that is, in appearance one merges so gradually into the other that it is sometimes difficult to decide. We know very well, though, that of a lot of ties put into a track at a given time, some will begin to fail in 4 years, more in 5 and many more in 6 years, and that some will last 10 years, and a few will last even 11 and 12 years. The conditions of use being practically the same, the difference in durability must be due to a variation in the quality of the timber.

Mr. Rowe. SAMUEL M. ROWE, M. Am. Soc. C. E. (by letter).—Mr. Chanute's paper, embodying, as it does, a fair epitome of European experience and knowledge up to the present time, is of much interest to those concerned in timber preservation in this country, and seems to deserve more than a passing notice.

The following is a synopsis of the salient points to which the writer will revert, and, for the sake of brevity, and to avoid the necessity of referring directly to the subject in each individual case, he will take them up in the order given:

(*a*) Greater cost of creosoting than of Burnettizing.
(*b*) Life of treated ties in Europe as compared with this country.
(*c*) Selection of timber.
(*d*) Drainage of road bed.
(*e*) Thorough permeation of the timber by the preservative.
(*f*) Increasing the amount of chemicals injected.
(*g*) Minute care during all stages of treatment. (Impregnation.)
(*h*) Treatment of oak. (White oak.)
(*i*) Test of strength and purity of chemicals.
(*j*) Timber supply.

(*a*) In regard to the creosote process, Mr. Chanute's observations in Europe seem to confirm the correctness of the position taken by the writer and many others in this country, that, owing to the difficulty of procuring suitable oil, and from its large and continually increasing

cost, the process is virtually ruled out in this country, except, perhaps, in special work required to protect timber against the teredo. Mr. Rowe.

This is also true, in some degree, with the zinc-creosote process by Mr. Rutgers, of which process Mr. Chanute seems to think very highly.

(*b*) In regard to the life of treated ties in this country, the experience on the Atchison, Topeka and Santa Fé R. R. and the Southern Pacific R. R., after 12 to 15 years, seems to fix the mean at 10 to 12 years, where the zinc-tannin process was used.

In this connection, the writer's experience may be of value. The original mountain pine ties were laid on a portion of the Southern Division of the A. T. & S. F. R. R. in 1880, and by 1886 the right of way was strewn with removed ties, or rather, with masses of rotten wood, having but little semblance to the ties originally laid. There was then a standing order that removed ties should be piled and shipped to the division points for locomotive fuel. Finally, the accumulation became so great and its unfitness for fuel so obvious, that many huge bonfires were the result. From this experience with untreated pine ties, a life of 4½ years was all that could be deduced. Some of these ties failed in the third year, and the company was face to face with a serious problem. In July, 1885, the Las Vegas Timber Treating Works were installed, and have been in almost constant operation since, but no systematic count of treated ties removed was made until early in 1897, since which time the company has worked quite carefully to secure a record. Although this record is too limited as yet for definite estimate, it furnishes a basis for a statement (by the laws of averages) by which the probable mean life can be estimated. On the assumption that the mean life is 12 years, it is found that of the earlier treated ties, not anything like the numbers that should have perished have been removed, so that it is proper to assume that 12 years is too low. The writer is informed that very many ties laid in 1885 are still in the track, and he would not be surprised to find some still in at the end of the twentieth year. To settle this definitely, however, will require the record of the next 3 or 4 years. We have, nevertheless, sufficient to show the marked contrast between the untreated and the treated ties, and there are frequent acknowledgments from the management that the substitution of the treated ties has resulted in marked economy. It should be further remarked, that the condition of the track is now very much better than was possible when untreated ties were being used, and that the treated ties, when removed, are in much better condition than the untreated, having still considerable sound timber remaining, whereas the untreated ties when removed are entirely gone.

There is no parallel by which to compare the results with those in Europe, except one case of pine in France, and in the case of Mr.

Mr. Rowe. Rutger's zinc-creosote, where the result as to life is somewhat better than is given here with the zinc-tannin process.

(*c*) The advisability of drawing high requirements as to the quality of the timber in this country is not clear, except merely for comparison with work in Europe where the specifications are severe, which is here of little consequence practically. The fact is, that the tie supply in this country has been and will continue to be drawn from inferior woods; and even with the oaks, only the inferior portion is being made into ties, while the select portions go to other purposes. This is true as to other and less durable timbers, not heretofore considered fit for ties; hence the railroads will continue to buy the cheaper grades. It is in consequence of these conditions that the soft and inferior woods, by the aid of treatment by which their lasting qualities are made greater than those of the best oak, will be sought. Thus is opened a much enlarged field of supply, together with a reduction in cost and a very marked reduction in the cost of maintenance.

Another point which may properly be noticed is the sawing of ties from large logs, which should make good ties on account of the superiority of the timber and its lasting qualities compared with the sap-pole tie made from the same kind of timber. The timber should last one-half longer at least, other conditions being equal. The practice now common is to cut the ties 6 x 8 ins., and 8 ft. long, containing $2\frac{2}{3}$ cu. ft., while the mean volume of the pole tie, upon which the number and distribution of ties under the rail has been founded in railroad practice, is more than 4 cu. ft. These sawed ties are so small that they do not form a sufficient support for the rail, unless the number is materially increased; and they are apt to fail, from rail wear or from insufficient body to hold the spikes, long before the timber decays. It would seem that a tie 7 x 9 ins., with a volume of $3\frac{1}{2}$ cu. ft., would be as small as should be made. Such a tie, properly treated, would last 15 years or more.

In connection with English timber preservation, Mr. Chanute mentions "Baltic redwood." In discussing these questions it would be well to state, for each kind of wood, the characteristics by which it might be judged. These characteristics have much to do with the lasting qualities, and, if some standard could be devised, for instance, the oak, heavy, close-grained, strong and lasting, the beech, walnut, etc., close, even grain, but not so strong or lasting, and lastly, the pines, etc., so as to enable those interested to judge somewhat as to their character, it would be a great aid in comparing results. Mountain pine, to which western railroads are obliged to resort, runs largely to open-grained timber, easily penetrated and taking the treatment readily if dry, yet it is an inferior timber, with many imperfections. Of course, some portions of the large, well-grown trees give sound, compact, well-matured fiber, and therefore,

should be quite lasting, but taking all together, the timber treated in Europe, high priced, cut and shipped under severe specifications, forms hardly a fair parallel. The very fact that these A. T. & S. F. R. R. ties have been made to last 12 years, or anything like it, speaks well for the efficiency of the treatment and methods followed at Las Vegas. Mr. Rowe.

(*d*) Mr. Chanute mentions, as one of the conditions favorable to the life of the tie, the thorough drainage of the road-bed.

In this country, in almost all cases, the treated ties are shipped directly from the works to the locality where needed, and are there racked, ready to be put in place. In the interval, they receive a certain, or rather, an uncertain, amount of drying, and are then put in track. In a dry climate this is not so important, but in a moist climate it is conceded that the ties should be quite dry before being placed. Taking this condition as requisite, the effect on the tie of exposure to the elements will be considered. The generally accepted theory of the capillary process of absorption of water when the wood comes in contact with it, is probably correct. The chloride in the treated tie, being readily soluble, is dissolved by the water, and carried with it as it penetrates the piece. By the contrary operation of surface evaporation in drying out, the chloride will be gradually dropped, and finally, such of it as reaches the surface of the wood, will be deposited on its surface, or in the soil in which the tie is bedded and there be lost. It is easy to see that this may result in loss and final wasting away of the antiseptic to a greater or less degree, depending upon the frequency of the exposure. Then, when drainage is imperfect and the tie lies in the water all the time, the absorption being continually from the bottom and the evaporation from the top, it is easy to see that the waste must be more rapid than when it is alternated, as by showers of rain.

In confirmation of this general theory, the Department of Agriculture has made some careful investigations on alkali soils, by which it is found that alkali sinks into the soil on free application of irrigating water, allowing cultivation of the soil, but when the irrigation ceases and the ground is exposed to extended drought, the alkali again rises and the land becomes sterile as before.

The alkali (sodium sulphate) existing in the soils and ground-waters of the western plains has the same characteristics as chloride of zinc, being only a little more readily soluble, and increases the specific gravity of the water. At Edgemont, S. Dak., the water used has nearly 125 grains of sodium sulphate per gallon, giving, at 60° Fahr., a hydrometric reading of 0.25° Beaumé, so that this correction has to be made in reading the strength of the chloride solution.

It would seem, too, that the effect of water on creosoted timber, while not so deleterious as on the chloride (as it only affects the lighter

Mr. Rowe. and less valuable parts of the oil), is still sufficient to demand attention. Creosote not being at all soluble, a different cause for its expulsion must be sought. It is well known that wood expands when exposed to dampness, the wood fiber absorbing the water and opening up, while the wood swells. The result is that the oils are, to some extent, squeezed out.

In this connection the writer has under examination samples of paving blocks treated by the "Creo-resin process," furnished by the kindness of City Engineer Erickson. These blocks seem to be well impregnated with creosote oil, reinforced by a quantity of resinous pitch, and they have every appearance of being an excellent paving material. These, with similar untreated blocks, were immersed in water on September 4th, 1900, and have now been in 60 days. At 30 days the absorption by the treated blocks was only about 33% of that of the untreated, the resinous pitch exuding but slightly; but at the end of 60 days the absorption of the treated blocks had gained considerably, as compared with those untreated, and a large amount of pitch had come to the surface, appearing in large globules adhering to the blocks. So marked was this action that it was deemed desirable to extend the experiment another month, and note further its extent.

Reverting to the original question of thorough drainage of the roadbed, it would seem unnecessary to insist on this in this advanced age of railroading, as such neglect is so destructive to the track as a whole and to the reputation of the roadmaster. It is easy to see that if the action just described takes place, it would be likely to soon exhaust any antiseptic that could be put into the timber.

(*e*) The most thorough permeation of the timber by the antiseptic is a fundamental part of the work, so far as the zinc-tannin or the Burnett process is concerned. The application of a stronger solution superficially will not make up for the lack of permeation, as it is not known that the chloride will diffuse in time. Steaming under pressure is the best and most effective means to secure through permeation, and it must be gauged or prolonged to meet the character and condition of the timber. Oversteaming sometimes will reduce the amount of absorption of the solution, but, on the other hand, the permeation will be more perfect. Thorough permeation by the steam can be best observed by noting the character and condition of the condensations blown from the retort from time to time. First will be observed the condensations caused by the steam coming into contact with the more or less cold charge; next, will follow a discharge of water tainted with the juices of the timber, increasing in its turbid and frothy condition as the steaming progresses, and then losing this character gradually and flowing off in the form of pure condensed steam or water. This last stage indicates that the steam has reached the heart of the timber, which is then in proper condition to receive

the solution, whether creosote oil, chloride solution or whatever is used. From the quantity of this highly charged water thus thrown off, the impression is that the timber has been cleared largely of its natural juices and of fermentable matter inherent therein. The vacuum drawn at the termination of steaming, of course, will still further free it from any remnants of saps, but its chief function is to draw the air and vapors from the retort and prepare the timber for the introduction and absorption of the solution, which is let in while the vacuum is still on. Mr. Rowe.

How absorption takes place when this point is reached, the following observations made at Las Vegas in 1885 will best show:

	Hewn ties.	Sawed ties.
First half-hour	0.42 per cent.	0.54 per cent.
Second half-hour	0.24 " "	0.31 " "
Third half-hour	0.16 " "	0.07 " "
Fourth half-hour	0.11 " "	0.04 " "
Fifth half-hour	0.07 " "	0.03 " "

(*f*) The question of the injection of liberal quantities of chemicals is of great interest just now, when managers are looking to reduction of cost. The chemicals form the most costly part of the treatment, and, while it must be insisted that enough be used to get the best results, there is an extreme which cannot be passed without wastefulness. At the time the writer was in the a-b-c of this matter, he was instructed that $\frac{25}{100}$ of 1% of the weight of the timber was the minimum amount of chloride allowable. On this basis, a careful determination of the mean weight per cubic foot of the dry ties for treatment at Las Vegas was 31 lbs. or thereabout. One-quarter of 1% would be 0.075 lb. of pure chloride, or a little over 1 oz. per cubic foot of timber. In view of the statements in the preceding paragraph, it might seem to be enough. In any case, the mean absorption of the ties treated in 1885 was about $\frac{1}{4}$ lb. per cubic foot. The absorption of some of these ties probably did not exceed the minimum mentioned, and yet, these 1885 ties are being pointed to as eminently good. The strength of the chloride solution then used was 0.015, or 1½%, and the average absorption, compared to volume of timber, was about 30 per cent.

In approaching this question of quantity, the operation must be studied closely. A dry tie having a volume of 4 cu. ft. would weigh 125 lbs. A green, freshly cut tie of this volume, with all its sap, would weigh 187 lbs., having about 16 lbs. of sap per cubic foot, the sap presumably containing almost all the fermentable agents. As before noticed, a large proportion of the juices are expelled by the steaming and the vacuum, say 75%, leaving 4 lbs. of sap to be neutralized by 1 oz. of pure chloride, or a 1½% solution.

Mr. Rowe. Mr. Chanute explains that by the rule referred to the amount was later found to be too low.

Attention is called to this, with the hope that some one who is favorably situated will take up this investigation, as any proposed increase in the cost of chemicals, in the face of the calls for cheaper work now being pressed, makes it very desirable that something definite and authoritative should be determined.

(*g*) In closing his valuable paper, Mr. Chanute says: "It cannot, however, be too strongly insisted upon that the work must be well and skillfully done, for, otherwise, the results are sure to be disappointing."

This is most heartily emphasized, and it follows that skillful operators must be employed. In Germany, as Mr. Chanute states, about the only successful and satisfactory work has been done by a private firm who makes it a business and conducts it on business principles, and an important part of whose assets consists of honest work. There is no doubt that any party operating in this country on the same principles can succeed. In either case, the skilled operator is the first requisite, and it is the same with any great railroad company that decides to do its own work. In Germany, it is said that the skilled operators can be counted on the fingers of one hand, and probably the same is true in this country. In the installation of a plant of this kind, a matter of the first moment is that an inexperienced and untrained operator should no more be put in control and direction than that a college student should be put in charge of one of the big steel plants. The proposed "operator" may have read every word written on the subject of timber preservation, and, if he has escaped the lunatic asylum, he has reached but the a-b-c of the matter, and must still have intelligent training through practical contact with the work. He must have the rules fixed by an experienced teacher and be made to apply them; and, above all, he should not allow himself to go out on any theorizing excursions, as all such are likely to prove disastrous, both to himself and his employers.

(*h*) The writer is pleased to note that oak (presumably something like our white oak) is not barred out from the woods treated in Europe Regarding the selection of tie timber, it seems to be axiomatic that "the better the timber, the better the tie it will make," regardless of treatment. Experience and reason show this, and if the life of an oak tie of the best quality can be prolonged but 2 years, it is economy to treat it. Taking the white oak tie as it runs, there is a strong probability of doubling its life. The objection that the oak will not absorb sufficient to do any good, is, in the light of experience, a mistake, and is illogical, when the nature of the process and its effect is understood.

(*i*) Mr. Chanute mentions the "constant testing of chemicals, as

to strength, purity, etc.," in connection with the requirements in European practice. It is presumed that what is meant is tests which may be performed on the work during operation, where it is not always practicable, to call in the aid of the chemist. It is necessary that the operator be able to determine quickly, approximately at least, the strength of the solution, whether chloride, tannin or glue, without the elaborate processes of the chemist, as, in the course of the operations, these solutions are constantly varying. We have been indebted to Mr. Chanute in this respect, and would be glad to have him give, at length, such facts as he has gathered, couching them in such terms as would be within the comprehension of any intelligent operator. Such information would be valuable indeed. Mr. Rowe.

(*j*) Mr. Chanute's remark in relation to the exhaustion of the timber supply introduces a theme, the importance of which, in this country especially, should be impressed at all times. In the last 30 years we have seen such destruction of our great forests as seems appalling. With the exception of a small territory in northern Maine, some small areas in the South and the region of the extreme Northwest, the forests have been invaded and the most valuable timbers have been more or less cut away. In many cases, shameful waste and destructive fires aid in the devastation. The tie renewals amount to nearly 70 000 000 annually, to-day, compared with 4 000 000 required in France, and the ratio will increase as the mileage increases. Although our forests seem boundless, they are not so, relatively; and, at the present rate of consumption, the scarcity of cross-ties, now beginning to be felt, will soon be serious. A parallel to this can be cited, quicker in its consummation, but by no means so far reaching and disastrous—that of the bison, thirty years ago teeming in millions over the western plains, but now hardly a representative remaining. This is a lesson to ponder.

WARD BALDWIN, M. Am. Soc. C. E.—Mr. Chanute's exhaustive *résumé* of the present practice in Europe in the preservation of wooden railway ties suggests to the speaker what is generally regarded as the next step in the evolution of railway track, namely, the use of metal ties. It is true, in general, that, as the cost of timber increases, it becomes more economical to use, first, preserved ties, and then, metal ties. Now, the statistics presented in the paper show that in Europe preserved ties can be procured at a cost which would apparently make them more economical than metal ties. It is nevertheless true that metal ties are used extensively there. The observation of the speaker, in passing over many miles of European roads laid with metal ties, is that the riding qualities of a track with metal ties are not superior to those of a track with wooden ties; and, therefore, the reason for their use would not appear to be that they give a smoother track. A comparison of the practice in Europe in the use of creosoted Mr. Baldwin.

Mr. Baldwin. ties and in the use of metal ties, now that the latter have been in extensive and growing use there for a number of years, would be interesting, and it is hoped that Mr. Chanute may add to his paper a discussion of this feature.

Mr. Howe. HORACE J. HOWE, M. Am. Soc. C. E.—The author shows, among other things, how ties are seasoned before treatment in Europe, and emphasizes the necessity for so doing in any and all of the processes which he mentions.

Possibly, seasoning for treatment is one thing and seasoning for direct track use is another. That is, the more the wood is made susceptible to the antiseptic, the better are the results, within certain limits; while the less alteration the surface undergoes, in the other case, the better will the tie stand the weather afterward.

It seems reasonable, therefore, to insist that even more care should be taken, in the latter case, to protect ties held over a season, from one cause or another, by the railroad company.

It is not uncommon to see bricks roofed over after receipt from the yard, and if that is considered economy by private concerns, why should not covering ties be economy for corporations? Bulk for bulk, the cost is about the same.

As to the saving by preserving ties, it is evident that everyone must calculate for himself. At present prices, the wolf does not seem to be at the door.

Assuming that a raw tie will last 9 years in first-class ballast (and no railroad hereabouts would consider seriously the treatment of ties unless its ballast was approximately first class), and that a treated tie will last 18 years, the following figures may serve as a basis for comparison:

One raw tie in place, 60 cents + 20 cents =	$0.80	
Interest at 4%, compounded annually for 18 years	0.82	
One raw tie in place, 60 cents + 20 cents =	0.80	
Interest for 9 years	0.34	
Total cost to maintain one raw tie for 18 years		$2.76
One treated tie in place, 60 cents + 50 cents + 20 cents =	$1.30	
Interest at 4% for 18 years	1.34	
Total cost to maintain one treated tie for 18 years		2.64

Roughly speaking, the costs are the same. No allowance is made for tie-plates. The item for treatment (50 cents) includes extra freight on that account.

If the foregoing does not focus with the facts, it is hoped that the author will indicate the reason.

O. CHANUTE, M. Am. Soc. C. E. (by letter).—The writer feels thankful to the gentlemen who have discussed his paper for bringing up a number of points regarding current European practice which seem to require further elucidation. Mr. Chanute.

President Croes is quite right in saying that the variety of notation used in Table No. 1 is puzzling, but the answer is that the table was compiled and quoted from the actual figures given to Mr. Herzenstein by the British railways, and illustrates in how many different ways it is possible to twist answers to one and the same question. In point of

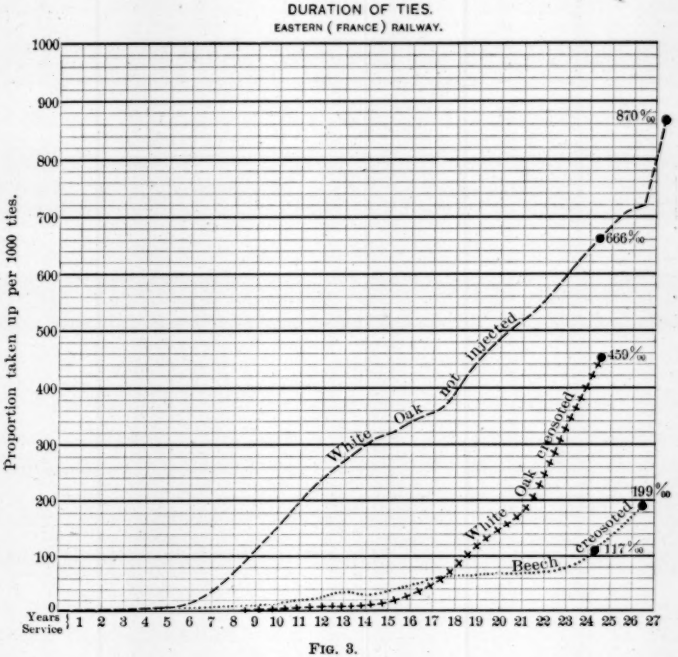

FIG. 3.

fact, the amount of creosote injected in Great Britain varies from 20 lbs. per tie on the Great Northern Railway, giving a life of 12 years, to 37 lbs. per tie on the Taff Vale Railway, with a reported life of 15 years. As all the ties are treated, there are no data to apportion the benefits due to the preliminary seasoning and to the subsequent injection in England, but the record diagram, Fig. 3, reproduced from the paper of Mr. M. V. Dufaux, showing the percentage of ties annually renewed upon the Eastern Railway of France, exhibits the fact that well-

Mr. Chanute.

seasoned but untreated white oak ties average some 15 years, while the same ties, when treated, will average about 25 years in the track.

The resulting inference as to the comparative insufficiency of seasoning alone would seem to be confirmed by the facts stated by Mr. W. B. Reed, that vulcanized ties, 8 years old, taken out of the Metropolitan Street Railway, in New York City, were not fit to be used again. Vulcanizing is an excellent seasoning process, and the circumstance that it has been practically abandoned for ties would seem to indicate that the seasoning alone is relatively less efficient than the injection of chemicals.

Mr. G. W. Tillson's allusion to the Boucherie process makes it desirable to state the main reason why it was finally abandoned in France. It was found necessary to do the work in the woods, a short time after the felling, and before the bark was taken off, because, otherwise, the sap was found to have solidified so much as to make the wood much more refractory, and because there was waste of the chemicals when hewn or sawed timber was treated. The late Henry Flad, M. Am. Soc. C. E., of St. Louis, also tested the process, with some improvements, in 1882, but he too gave it up as impractical.

Mr. Mendes Cohen's mention of the Burnettizing on the Lehigh Coal and Navigation Company's road, while under his charge, recalls the fact that the writer, while on the Committee on the Preservation of Timber, in 1883, asked L. L. Buck, M. Am. Soc. C. E., to examine the ties remaining which had been so treated in 1867. They were found to have been well preserved, and greatly hardened by the process, so as to cut but little under the rail.

The differences which Mr. Samuel Whinery notes in the individual lives of untreated ties of the same species of wood, obtain with treated ties in Europe, and also in this country, but the writer has confined himself to the statement of averages. These individual differences are doubtless "due to a variation in the quality of the timber." Those ties which absorb the most antiseptics are longest preserved. This is well illustrated by the French beech, which is nearly all sap wood, which readily takes 60 to 67 lbs. of creosote per tie, and then lasts on an average 30 years, while it would decay in 4 or 5 years if laid in the track in its natural state.

Mr. S. M. Rowe is, therefore, probably in error in assuming that "the better the timber, the better the tie it will make," if he means thereby, as he seems to, that it is preferable to treat white oak rather than the inferior woods, and that heart wood is preferable to sap wood for treatment. On the Eastern Railway of France the creosoted white oak averages 5 years less life than the creosoted beech. The fact seems to be that the woods which last best untreated are close-grained and those whose sap ducts contains the most tannin, oil or resin, while the woods best fitted for treatment are rather coarse-grained and their ducts

contain more of the fermentable elements of the sap. These will absorb more of the antiseptics if seasoned. Mr. Chanute.

Mr. Rowe's statements as to the life of treated ties on the Atchison, Topeka and Santa Fé Railway are confirmed by the report of the Committee on Ties, read March 12th, 1901, in the convention of the American Railway Engineering and Maintenance-of-Way Association, which assigns to these ties an average life of 10½ to 11 years, in the track, and thereby indicates comparatively as good economical results as obtained abroad. If we estimate that these mountain pine ties, untreated, would last 4½ to 5 years, it is seen that an additional life of 6 years is given to them by the treatment, at a cost of about 15 cents each, and the additional life has therefore cost $\frac{15}{6} = 2.50$ cents a year per tie, or about one-half its value to the road. In France, on the other hand, the untreated beech would last about 5 years; creosoting, at a cost of 64.2 cents each, increases their life by 25 years, and the cost is therefore $\frac{64.2}{25} = 2.57$ cents a year per tie, leaving out all questions of compound interest, or cost of replacing in the track. This would seem to confirm the opinion of the writer that the cheaper processes are best adapted to the present conditions in the United States.

Mr. Rowe is doubtless right in advocating a larger size of tie for the future. The sizes here and abroad compare as follows:

Ordinary Standard tie in United States	8	ft. x	6	x	8	ins.	= 2.67 cu. ft.
Pennsylvania Railway in United States	8.5	" x	7	x	7	"	= 2.89 " "
First-class tie in Great Britain	8.92	" x	5	x	10	"	= 3.11 " "
First-class tie in France	8.53	" x	5.1	x	10.2	"	= 3.08 " "
First-class tie in Germany	8.85	" x	6.31	x	10.25	"	= 3.97 " "

The "Baltic redwood" which is so largely used abroad, and which Mr. Rowe inquires about, the writer believes to be a species of pine. The samples which he saw looked to him very much like the short-leaf pine of the South, or the Norway pine of the North.

The writer does not believe, with Mr. Rowe, that the washing out of mineral salts will be more rapid when the tie "lies in the water all the time" than when it is alternately wet and dry. His experience is that it is the repeated soakings from rains which leach out the salts; each soaking taking up some of the chloride of zinc and each evaporation dragging some of it out. He has satisfied himself of this by many alternate soakings and drying of treated blocks, and has also succeeded in floating out by that process some of the lighter portions of creosote; the latter is, however, much more refractory. Somewhat

Mr. Chanute. similar experiments have also elicited the fact that zinc chloride does diffuse itself in timber. The analysis of a chip bored out from the very center of a tie, immediately after treatment, shows no trace of zinc; after a week a trace of the salt is shown by taking the adjoining chip, and a month or two later much more zinc is found by further boring in the same hole. The salt has been diffused by endosmosis; and it is found that after exposure for some years in the track it has been leached out by exosmosis.

Mr. A. Schneidt, while Superintendent of the Imperial Railways in Alsace-Lorraine, took up some ties which had been Burnettized in 1893, with known quantities of zinc chloride, and exposed in the track three years. He had many chips bored from them, mixed and analyzed these, and concluded that from the oak 90 to 97% had leached out, 80 to 85% from the pine, and 88% from the beech. There was yet perhaps enough of the chloride of zinc in the pine and beech to prevent the development of living organisms, but elimination slowly progressed. Partly in consequence of this investigation, the German specifications for injection have gradually been made more rigid. That of 1895 required that the zinc-creosote solution should be 3° Beaumé strong, while the 1899 specification requires it to be 3.5° Beaumé strong. The amount of solution to be injected was 6.2 lbs. per cubic foot for oak and 18.7 lbs. per cubic foot for pine and beech, while in 1899 the amount required is 6.2 lbs. per cubic foot (the same) for oak and 19.3 to 20.25 lbs. per cubic foot for pine and beech. The amount of seasoning required has also been increased. In 1895 oak was not to be injected until it had dried so as to weigh but 53 lbs. per cubic foot, beech 51.4 lbs. and pine 39.2 lbs. per cubic foot; while in 1899 it is specified that it shall not weigh more than 49.8 lbs. per cubic foot for oak, 45.1 lbs. for beech and 39.2 lbs. per cubic foot for pine.

In answer to Mr. Rowe's request, the writer herewith furnishes another appendix (D)* containing the new rules for testing chemicals and solutions adopted by the Imperial Prussian Railways. These are as yet unpublished in Germany, having been devised and adopted in 1899.

The writer does not believe, with Mr. Ward Baldwin, that the next step in this country will be to resort to metal ties. Not only would the interest on their cost at 4% be more than the annual cost of the decay of wooden ties, especially if treated, but the use of metal ties is not increasing abroad. In England they have been laid experimentally, but not adopted. In France they have been found less economical than wooden ties, mainly in consequence of wear in the fastenings and lack of stability after tamping. In Germany the writer gathered the impression that they were used, in great measure, for the sake

* For convenience, this appendix has been added to the original paper after Appendix C.

of furnishing work to the steel mills. The writer is of the opinion that the next step for American railways will be to resort to chemical preparation of wooden ties, and that this will be done on a large scale. Mr. Chanute.

Mr. H. J. Howe's estimate of the comparative economy of untreated and treated ties is not correct. Granting his mode of comparison, he has overestimated the probable cost of the raw ties to be treated. In sections where the durable woods, which last 9 years, cost 60 cents, the inferior woods suitable for injection will cost 30 to 35 cents per tie. The cost of treatment with zinc-creosote will be about 25 cents, probably including extra freight, and when the treating works are at a gathering point, such as New York or Chicago, there will be no extra freight. His comparison, therefore, should be amended as follows:

One durable tie in place, 60 + 20 cents	$0.80
Interest at 4% compounded for 18 years	0.82
One renewed tie in place, 60 + 20 cents	0.80
Interest at 4% compounded for 9 years	0.34
Cost of maintaining one raw tie 18 years	$2.76
One treated tie in place, 35 + 25 + 20 cents	$0.80
Interest at 4% compounded for 18 years	0 82
Cost of maintaining one treated tie 18 years	$1.62

In point of fact, the assumed average life of 9 years for durable ties and of 18 years for treated ties are both probably overestimates for American practice.

Mr. F. A. Kummer, in closing the discussion of his paper,* raises the interesting question whether steam or compressed hot air is most efficient in preparing wood to receive an antiseptic. In England neither is used; a vacuum is produced in the cylinder and the hot creosote is then admitted; the ties being thoroughly seasoned, moisture from steaming would resist the entrance of the oil. In France the best practice is to desiccate the ties with hot air in drying ovens, and then to run them quickly into the treating cylinder. In Germany no prior steaming is used in creosoting, but this method is used in the zinc-creosote process, as the latter involves a watery solution. Last year the German railway officials questioned whether such steaming was beneficial for well air-dried ties, and made many experiments. These showed that only 2 to 11.6% of the sap contained in the seasoned wood was extracted by the steaming, and there is an inclination

* *Transactions*, Am. Soc. C. E., Vol. xliv, p. 213.

Mr. Chanute. to give it up. The results might have been quite different if the experiments had been made upon freshly-cut or half-seasoned ties, such as are now furnished for injection in the United States. The writer has made a great many experiments with various methods for increasing the receptivity of such fresh ties, but has not reached such absolute conclusions as to warrant publishing them.

Mr. Kummer, however, is mistaken in assuming that "the tannin treatment, it is claimed, completely seals the pores against the entrance of moisture from the outside." The claim really made is that the tannin treatment simply retards the entrance and the escape of moisture, so as to retain the zinc chloride somewhat longer. In point of fact the data presented at the convention of the Maintenance-of-Way Association in March, 1901, showed that on the Southern Pacific Railway the average life of removed ties, which had been treated by chloride of zinc alone, was from 6.83 to 9.16 years, while on the Atchison, Topeka and Santa Fé Railway, which used the zinc-tannin process, the average life was 10½ to 11 years, and on the Chicago, Rock Island and Pacific Railway, with zinc-tannin, the average life was 10½ to 11½ years. The chief objection which has been made to the zinc-tannin process is that microscopic examinations do not show a well-defined plug to be formed by the leatheroids. The exact value of such objection will have to be determined by actual experience on the large scale.

It is well established that both Burnettizing and the zinc-tannin process materially increase the spike-holding power of ties and retard their cutting under the rail. No exact comparison of these results can be made with results of the German zinc-creosote process, because the European rail fastenings are greatly superior to our own. Screws are very generally used; these hold the rail in close contact to the tie and quite prevent that slapping up and down of the rail which causes much of the cutting. As the zinc is the hardening element, it is not doubted that zinc-creosote does materially harden the wood. The further theory of its action is not merely as stated by Mr. H. Goldmark in the discussion of Mr. Kummer's paper "that the dead oil of tar will prevent the zinc salt, which is soluble in water, from being washed out." That theory is stated by Mr. A. Schneidt, in the paper already alluded to, as follows:

"In the watery solution of the chloride of zinc, the carbolic (phenols) acid contained in the tar oil added, is partly dissolved; and in this diluted form it penetrates the cellular tissue of the ligneous body far more readily and surely than the less readily fluid tar-oil.

"The presence of carbolic acid also produces a certain potential solubility in the resinous constituents of the wood, whereby the chloride of zinc solution is better enabled to penetrate the fatty resinous woody strata, and into the heart.

"In the judgment of chemists it may also be assumed that owing to the greatly attenuated degree of the chloride of zinc solution, and

in presence of the carbolic acid, a basic zinc 'phenylate' is formed from the chloride of zinc oxide, through the agency of the bases of the salts which occur in wood ashes; and that this 'phenylate' of zinc, being insoluble in water, favors duration by opposing the leaching out of the impregnated ties. Mr. Chanute.

"The preservative action of the dissolved carbolic acid is decidedly more potent than that of the chloride of zinc; and as carbolic acid is also much less soluble in water than chloride of zinc, the addition of a small quantity of tar-oil containing carbolic acid will also arrest the decay of the wood which absorbs it. The heavy oils of the added tar-oil do not as readily penetrate the wood as the more fluid solution of chloride of zinc when reinforced with the dissolved carbolic acid; the former oils remain in the outer woody layers of the tie, and form a very thin stratum, more or less obstructive to the entrance of water."

During the time intervening between the preparation of the writer's paper, its discussion and the writing of this rejoinder, a material change has occurred in the commercial situation; tar-oil has gone down in price in consequence of greater abundance. If this condition continues there will be an opportunity to introduce the zinc-creosote process in this country at reasonable cost, but there is still much difficulty in obtaining the quality of tar-oil which the Germans say is requisite for success with that process. Mr. Rütgers has none to spare, and it is not produced in the United States. Perhaps by analysis of various oils in the market it will be possible to obtain a supply.

We may also hope to anticipate the results to be expected from various processes of timber treatment with the aid of the United States Department of Agriculture. Professor Hermann von Schrenk, special agent for the Forestry Bureau, has recently delivered an able address, upon "Factors which Cause the Decay of Wood," before the Western Society of Engineers, and later, also another address on "Decay of Ties and Bridge Timber," before the Maintenance-of-Way Association. He is now engaged in testing the rate of decay of various samples of wood and the action of various fungi.

A very interesting paper on the "Preservation of Sleepers in England and Continental Countries" has also been published* by Mr. P. H. Dudley, who treats the subject from a botanical point of view, so that we may expect to have our knowledge increased upon the little-understood *rationale* of wood decay.

* *Railroad Gazette*, March 8th, 1901.

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No. 896.

A DIRECT METHOD OF SPACING RIVETS AND FINDING THE POSITION, ETC., OF STIFFENERS IN PLATE GIRDERS.

By E. SCHMITT, Assoc. M. Am. Soc. C. E.

PRESENTED DECEMBER 5TH, 1900.

WITH DISCUSSION.

Although the scope and importance of this paper are not of a nature requiring phrases or mottoes to introduce it, the writer cannot refrain from stating in a few words the reasons actuating him in writing it and in constructing the accompanying diagrams and tables.

Even if one knows by heart some of the more important formulas of applied structural mechanics, the following axiom will govern their general usefulness:

"Good formulas, however short, are of almost no value in hurried daily practice, where quick and accurate results are expected, unless presented in the shape of tables."

These, again, are serviceable only when in such profusion, and so extended into final results, that, after having been consulted once, no further computations need be performed. They must, also, meet current demands regarding the information they are expected to furnish for all the ordinary problems coming under their headings.

The writer, and no doubt many others with him, when making structural computations, not having at his disposal labor-saving devices, in the shape of suitable diagrams and tables, and when not wishing to depend on results of rules of thumb, has found the foregoing axiom to be vexatiously true.

Having had to compute plate girders quite frequently, and having found the figuring of rivet spacing, buckling values of webs, location of stiffeners, etc., by formulas, a slow, time-consuming operation, the writer determined to make a short cut for solving such problems, and hopes that the burdens of those who have to compute plate girders will be lessened by the use of the method presented herewith.

It is hoped that the reader will not object to being reminded of the virtues possessed by the "method of graphical statics," which excels greatly the analytic method of solving problems in structural mechanics. It furnishes at once not only particular, but all possible, values and conditions of forces and stresses acting upon structures. Formulas of the analytic method, on the contrary, furnish only one value at a time. Graphical statics reveals to the eye the play of forces and stresses; nothing is hidden; it gives object lessons instead of abstract demonstrations. It possesses, in itself, a complete check of its operations and its results, or it can be made to do so in an easy manner. The analytic method, it is true, gives absolutely correct results, but only in theory, and for the assumed conditions. Such exact fulfilment in assumptions and results cannot be realized in any case, in the working of the forces in the universe or in any structure.

All who have used both methods agree that the graphical is shorter, more comprehensive, and in full accord with assumptions and practical conditions. The results obtained thereby are just as accurate and as true as the assumed conditions on which the computation is based.

Versatility with this method (acquired by constantly using it) is necessary to appreciate fully its perfect qualities in solving the most simple as well as the most intricate problems, statical or elastic, of applied mechanics. It is to be mentioned, however, that all graphical solutions start from analytic formulas, and can never be applied to advantage without first having had a thorough training in the analytic handling of the problems of applied mechanics. When a problem has

to be solved, both methods should be used, each at the proper time, the analytic formulas to be consulted in advice.

No explanation need be given for the manner in which the shear diagram for the vertical shearing stress has been made use of, as will be seen in what follows, for determining the rivet spacing by means of the vertical shear ordinates, directly, although the rivets are acted upon by a horizontal force. The intensity of both these stresses is the same at any point in the web, and since, for the purpose mentioned, the magnitude only of the horizontal shearing stress is of importance, no notice need be taken of its real sense and direction. The position in which the magnitude of this force is found in the shear diagram, is besides a very convenient one for the solution of the problem.

The method to be set forth to determine the rivet pitch and also that point from which on, toward the end of the girders, stiffeners will be required, is graphical, and requires the plotting of the shear curve for the particular loading of the girder, which in any case is a simple performance. For convenience and clearness in carrying out operations, it will be found best to arrange the shear curve immediately above or below the girder to be designed. The depth of the girder may be drawn to any reasonable scale which will allow accurate measurement of the rivet spacing.

Assuming the reader to be familiar with the theory and the designing of plate girders in general, and that he has, for a particular case, decided upon the thickness of the web and the size of the rivet; further, that he has determined the rivet value by means of a rivet table, to be found in any of the mill handbooks (see also the diagram and tables at the end of this paper), we can proceed to refer to the well-known rivet formula:

Let S = shear at any point X in the length of the girder;

h = depth of the girder between the upper and lower pitch lines;

p = the pitch, or spacing, of the rivets to be found;

v = the rivet value;

$p = \frac{v\,h}{S}$; transforming, we have then:

$\frac{h}{p} = \frac{S}{v}$; that is, the depth of the girder is to the rivet pitch, as is the total shear to the rivet value.

Referring now to Fig. 1, let $A\ B = h\ h$ be the span of the girder, and $C\ D$ the shear curve, drawn in reference to the base line $E\ F$. Set off vertically downward at points A and B the depth h of the girder, as Ah, Bh.

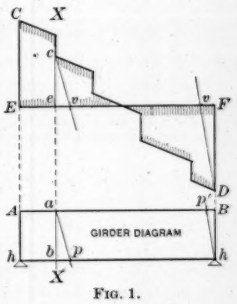


FIG. 1.

Choose any section, as X, and draw a vertical line $c\ e\ a\ b$. The ordinate $c\ e$ will then represent the total shear at X for the panel to the right of e. From the point e in the shear diagram set off, toward the right hand, the rivet value $e\ v$, in the scale of shears. Produce the line $c\ v$. Through the point a, in the girder diagram, draw a parallel line intersecting $h\ h$ in p, then the distance $b\ p$ is equal to the required rivet spacing for the shear at the point X. In the similar triangles $c\ e\ v$ and $a\ b\ p$ we have the proportion:

$$a\,b : p\,b :: c\,e : e\,v,$$

or,

$$h : p :: S : v,$$

and as before,

$$\frac{h}{p} = \frac{S}{v}.$$

To express $b\ p$ in inches, measure the length of the line $b\ p$ in the scale in which the depth h of the girder $A\ B$ has been set off.

We will now proceed to find the position of the stiffeners, or that point s, from which on, toward the abutments, stiffeners will be required.

Referring to Fig. 2, we make use of the shear diagram as before. Set off vertically, in the shear scale, above and below the base line $E\ F$, the buckling, or total shear value w of the web, whatever value the designer has determined to use.

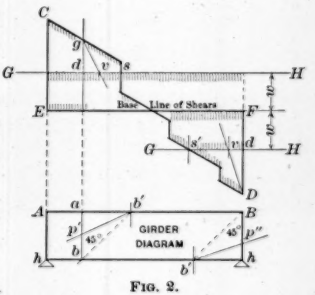


FIG. 2.

The shear stress between the lines $E\ F$ and $G\ H$ (in the upper part of the diagram) is taken care of by the web. The shear stress beyond the line $G\ H$ will have to be taken care of by stiffeners. Stiffeners will be required therefore, toward the ends beyond the point s, where the line $G\ H$ intersects the shear curve $C\ D$. (See also the lower half of the shear diagram.)

From the theory of the plate girder, it is known that stiffeners, if required at all, must be spaced at distances equal to the depth of the girder. It is also known that at any point in the web, as mentioned before, horizontal and vertical shearing stresses are of the same intensity, and that therefore the spacing of the rivets in the stiffeners can be found in the same manner as for the flange angles; considering, however, that only that part of the shearing stress above the line $G\,H$ (in reference to the base line $E\,F$ and the upper part, Fig. 2), and from the point s on, toward the abutments, has to be taken care of by the stiffeners. The same applies to the lower half of the diagram.

Proceeding now to space off from the point s, distances $d, \ldots$. etc., equal to the depth of the girder, the ordinate $d\,g$, above the points d, d, represents the surplus shears coming upon the respective stiffeners.

From the stiffener panel-points d set off again the rivet value v, as $d\,v$, produce the line $g\,v$ for any of the stiffeners, and through the point b' in the girder diagram, corresponding to the particular stiffener, draw a normal line to the direction of the line $g\,v$ intersecting the stiffener line $a\,b$ in the point p'. The point b' having been located at a distance $a\,b'$ equal to the depth of the girder, from the point a, by means of the 45° line $b\,b'$.

In the similar triangles $b'\,a\,p'$ and $g\,d\,v$, the same ratio $\frac{h}{p'} = \frac{S}{v}$ previously found for the flange angles, will obtain, as it should from the statements previously made.

The spacing of the rivets in the stiffeners, thus found, will be, usually, greater than the maximum spacing permissible in rivet work, and should be limited to this accordingly. The size of the stiffener sections may be determined by treating them as columns (with flat ends). Since, however, they are riveted at short intervals along their length, they may be computed for simple compression in ordinary cases, but they should always be applied in the form of angle irons.

The method shown for rivet spacing for particular panels, and for particular shearing stresses, can be reversed in such a manner as to determine the length of the panels in which a certain assumed rivet spacing, for example, 6-in., 4-in. or 3-in., shall take place.

In the girder diagram $A\,h\,B\,h$, Fig. 3, set off, from the point h, the desired rivet spacing, as $h\,p^3$, $h\,p^4$, $h\,p^6$.... From these lines draw lines to the point A, as a pole. Now, in the shear diagram, $C\,E\,F\,D$, set off from E the rivet value v, as $E\,v$. Through the point v draw parallel lines to the lines $A\,p^3$, $A\,p^4$, $A\,p^6$,.... as $v\,p^3$, $v\,p^4$, $v\,p^6$.... intersecting the end shear lines $C\,E$ and $E\,F$ in p^3, p^6.... etc. Through these points, p^3, p^4, p^6, in the shear diagram draw parallel lines to the base line $E\,F$. From the intersection points of these horizontal lines with the shear curve drop verticals to the base line $E\,F$. These ordinates will divide the girder into the desired panels, in each of which the respective assumed rivet spacing will obtain as in Fig. 3.

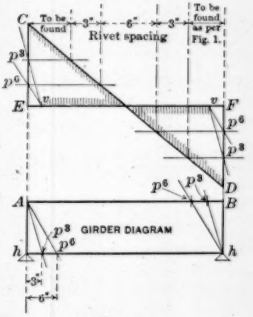


Fig. 3.

In like manner (see Fig. 4) we can find, say for the end of a girder, the rivet value v for a certain pitch by simply setting off the pitch $h\,p$ = say 3 ins. in the girder diagram, and proceeding in the reverse order of the operations carried out in Fig. 1, by drawing the line $A\,p$ and a line $C\,v$ parallel to it through the point C intersecting the base line of the shears in v, then $E\,v$ is the necessary rivet value. Scaling this value and inspecting the rivet table, the thickness of the web and the size of the rivet can be chosen with due regard to practical considerations.

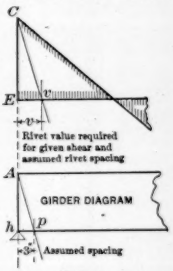


Fig. 4.

The method set forth, so far, gives all the data required for rivet spacing in ordinary practice and theory.

In railroad work, the computation must include, besides the horizontal component due to the main shear, the local vertical effect due to a maximum wheel load. This wheel load is usually assumed to be uniformly distributed over one or two sleeper spaces of 2 to 3½ ft.

As the foregoing forces act at right angles to one another, their resultant must be first ascertained before the proper rivet spacing can be determined.

Let S = main shear, acting horizontally;
h = depth of girder, in feet;
Q = maximum wheel load, acting vertically;
k = the sleeper space, in feet.

Then $\frac{S}{h}$ = shear per lineal foot, due to the main shear,

$\frac{Q}{k}$ = " " " " " " local shear.

The resultant of these two forces, to be taken up by n rivets per foot, will be :

$$r = \sqrt{\left(\frac{S}{h}\right)^2 + \left(\frac{Q}{k}\right)^2}$$

Expressing the sleeper space k in terms of the depth of girder h, by putting $k = \frac{1}{q}\,h$, or $q = \frac{h}{k}$, the above formula can be written

$$r = \frac{\sqrt{S^2 + \left(Q\,\frac{h}{k}\right)^2}}{h}$$

Further, let n = number of rivets per lineal foot;
v = the rivet value;
p = the rivet pitch;

also $n\,p = 1$ (ft.), $p = \frac{1}{n}$ and $n = \frac{1}{p}$, and considering that the condition $n\,v = r = \frac{v}{p}$ must also be fulfilled, we have

$$\frac{v}{p} = \frac{\sqrt{S^2 + \left(Q\,\frac{h}{k}\right)^2}}{h}$$

This formula is, as will be found by comparison, identical with the original rivet formula, presented at the beginning of this paper, and we will use it again as the basis in the determination of the rivet spacing in the present case by evaluating graphically the term under the root, which is the aggregate resultant of all the shearing forces acting upon the n rivets at section X.

This is readily done by forming the "triangle of forces," with the components S and $Q\,\frac{h}{k}$, acting at right angles to one another. The hypothenuse of this triangle will at once represent the term

$$\sqrt{S^2 + \left(Q\frac{h}{k}\right)^2}$$

Referring now to Fig. 5 and section X, let $C\,F$ represent the shear curve for the rolling load, $G\,D$ that for the uniform load, both in reference to the line $E\,F$. Then the ordinate $c\,u$ will be in magnitude equal to S, and will form one of the sides of the triangle of forces we desire to construct. Set off in the shear diagram horizontally from the point E the maximum wheel load Q, as $E\,q$ (in the scale of shears). Similarly, set off in the girder diagram the tie, or sleeper space k (in the scale used for laying off depth of girder), as $h\ K$. Draw the imaginary line $K\,A$ and parallel to it another, $q\,d$, through the point q in the shear diagram, piercing the end shear line in d; then $E\,d$ is equal to the magnitude of the local vertical shear component $Q\frac{h}{k}$, which must be joined at right angles to S.

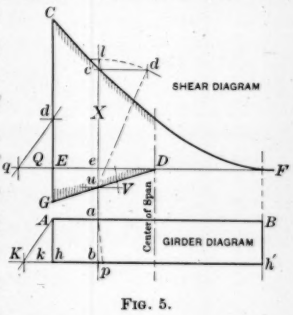


FIG. 5.

In the similar triangles $A\,h\,K$ and $d\,E\,q$, we have—

$$h\,K : A\,h :: E\,q : d\,E,$$

or,

$$d\,E = \frac{E\,q \times A\,h}{h\,K} = Q\frac{h}{k}$$

Now set off, horizontally, from point c in the shear curve $C\,E$, this value $E\,d$, as $c\,d$; draw the imaginary line $u\,d$ (produced in the diagram), then $u\,d$ is equal to the magnitude of the resultant referred to before. Bring this resultant into a vertical position with a compass, using the point u as a center for describing the arc $d\,l$. Set off, further, from the point u, horizontally, the value of the adopted rivet, as $u\,V$; draw the imaginary line $V\,l$, and parallel to it another, through the point a in the girder diagram, piercing the base line of the girder in the point p; then $b\,p$ is equal to the required rivet spacing at the section X, depending on the combined action of the several shearing stresses mentioned.

Practical Points: Web Thickness, Panel Lengths, Etc.—As stated, the web thickness is usually proportioned for the greatest shear and for buckling. For this case no stiffeners are required to assist the web. If, however, the web is calculated for simple shear only, or for

an intermediate condition between buckling and shear, which is also frequently done, then the method just shown for locating the stiffeners is to be used.

The rivet value of a rivet depends on the shearing capacity of the particular rivet and the web thickness in which it bears, etc., and the lesser of the two values, usually that for bearing, must be adopted to determine the pitch. If this pitch should be found to be less than the minimum spacing permissible, a larger rivet or a heavier web must be resorted to. To obtain, under certain circumstances, a sufficient rivet value as regards bearing, without changing either the diameter of the rivet, which may be a maximum, or the spacing, which may be a minimum, already, a deeper web must be selected.

This depth can be determined at once by means of the diagram, Fig. 4. In the case just stated the known quantities are: The total shear, the rivet value, consequently the direction of the line $C\ v$, to which draw a parallel line $p\ A$ through the point p, intersecting the abutment line of the girder in the point A; then $A\ h$ is equal to the depth of the girder which is sought.

The minimum spacings for rivets are; $2\frac{1}{4}$ ins. for a $\frac{3}{4}$-in. rivet, $2\frac{5}{8}$ ins. for a $\frac{7}{8}$-in., and 3 ins. for a 1-in. rivet.

The panels in which the rivet spacing changes, or should change, are, in the case of concentrated loading, given directly. For a uniform load, or a uniform and concentrated loading combined, suitable panel lengths may be assumed, mostly equal to the depth of the girder. The designer, with a little practice, however, will be able to settle this point without difficulty. The panel lengths also can be determined in such a manner as to obtain in them certain rivet spacings, as illustrated in Fig. 3.

Stiffeners are always provided over the supports, or where concentrated loading occurs, even if the web has been designed to take the greatest shearing stresses for buckling.

As this shearing stress is greatest at the ends, it is taken usually to determine the web thickness. Since, however, stiffeners are always provided at the ends, the writer would recommend, for the purpose of selecting a suitable web thickness, the use of the shear which obtains at a distance equal to the depth of the girder from the ends, or even at a still greater distance.

If the pitch for any panel point has been determined, it is con-

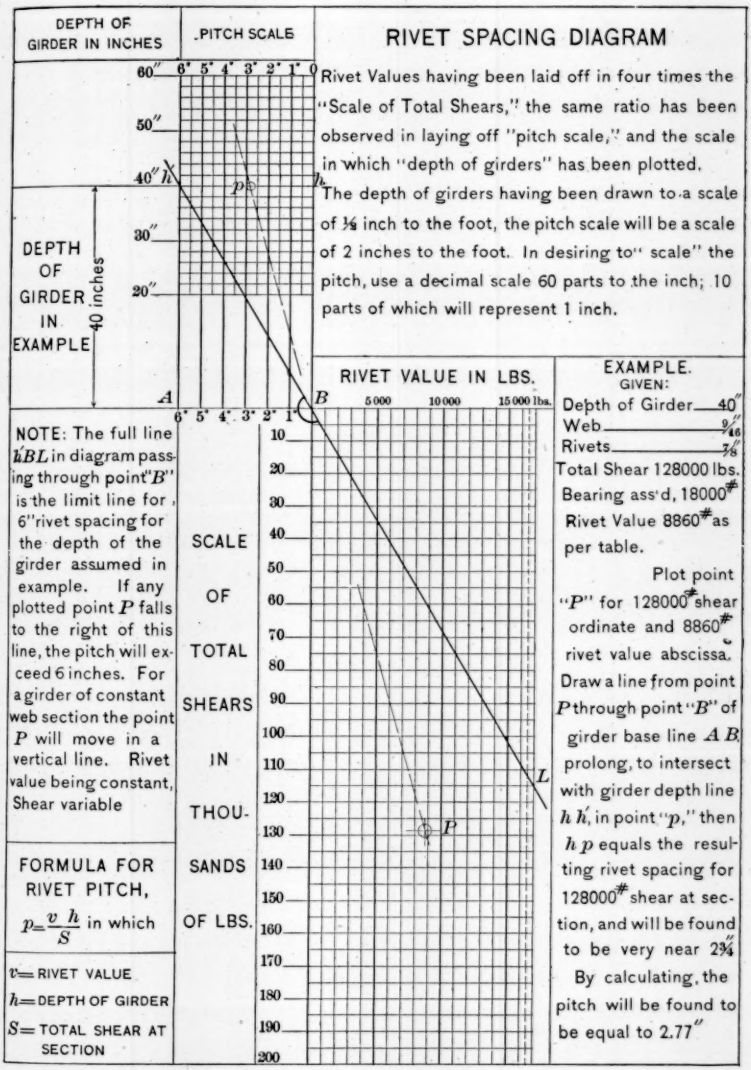

DEPTH OF GIRDER IN INCHES
PITCH SCALE
RIVET SPACING DIAGRAM
Rivet Values having been laid off in four times the "Scale of Total Shears," the same ratio has been observed in laying off "pitch scale," and the scale in which "depth of girders" has been plotted.
The depth of girders having been drawn to a scale of ⅛ inch to the foot, the pitch scale will be a scale of 2 inches to the foot. In desiring to "scale" the pitch, use a decimal scale 60 parts to the inch; 10 parts of which will represent 1 inch.
60″ 50″ 40″ 30″ 20″
6″ 5″ 4″ 3″ 2″ 1″ 0
DEPTH OF GIRDER IN EXAMPLE
40 inches
RIVET VALUE IN LBS.
B
5000 10000 15000 lbs.
EXAMPLE.
GIVEN:
Depth of Girder 40″
Web 9/16″
Rivets 7/8″
Total Shear 128000 lbs.
Bearing ass'd, 18000#
Rivet Value 8860# as per table.
Plot point "P" for 128000# shear ordinate and 8860# rivet value abscissa.
Draw a line from point P through point "B" of girder base line A B prolong, to intersect with girder depth line h h′, in point "p," then h p equals the resulting rivet spacing for 128000# shear at section, and will be found to be very near 2¾″
By calculating, the pitch will be found to be equal to 2.77″
NOTE: The full line ABL in diagram passing through point "B" is the limit line for 6" rivet spacing for the depth of the girder assumed in example. If any plotted point P falls to the right of this line, the pitch will exceed 6 inches. For a girder of constant web section the point P will move in a vertical line. Rivet value being constant, Shear variable
SCALE OF TOTAL SHEARS IN THOUSANDS OF LBS.
10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
FORMULA FOR RIVET PITCH,
$p = \frac{v\ h}{S}$ in which
v = RIVET VALUE
h = DEPTH OF GIRDER
S = TOTAL SHEAR AT SECTION
A
L
P
p
h
h′

FIG. 6.

tinued to the next panel point toward the center, for practical reasons. Theoretically, and as may be seen from the shear diagram and from the method by which the pitch is obtained, the distance from rivet to rivet changes gradually, or abruptly, as does the shearing stress. To realize such constantly varying spacing of rivets in girders would be very impractical and expensive unless performed by an automatically arranged punching machine. In any case, the rivets are never spaced more than 6 ins. apart, and, in case of girders carrying floors, the spacing is recommended not to exceed 4 ins.

Spacing-panels for rivets should always contain an even number of spaces. The usual spacing for rivets should be preferably given in full inches or full and half inches. Quarter-inch fractions, however, are frequently used.

The foregoing method of finding the rivet spacing in web members of plate girders has been compacted in the diagrams, Figs. 6 and 7. Auxiliary tables (Tables Nos. 1, 2 and 3) have been added to give all the necessary data to design the web plates of girders without having to inspect books or tables.

If the analytic method is used, the diagrams and tables will be of advantage in computing shear stresses. The necessary explanations of how to use the diagrams are noted at the proper places. An example is given for finding the rivet pitch for a given total shear of 128 000 lbs. for a $\frac{7}{8}$-in. rivet in a $\frac{9}{16}$-in. web, and for 18 000 lbs. bearing per square inch of rivet diameter.

It may be noted that the points P (see lower part of rivet spacing diagram), for a particular girder, with constant web section, will all be located in a vertical line, since the rivet value remains constant; the shearing stresses only varying along the length of the girder. The shear diagram, for finding the shear per square inch of web area of a given or assumed web, can be used in the reverse manner, that is, to find the necessary total web section for a given or assumed unit shear per square inch and a given total shear.

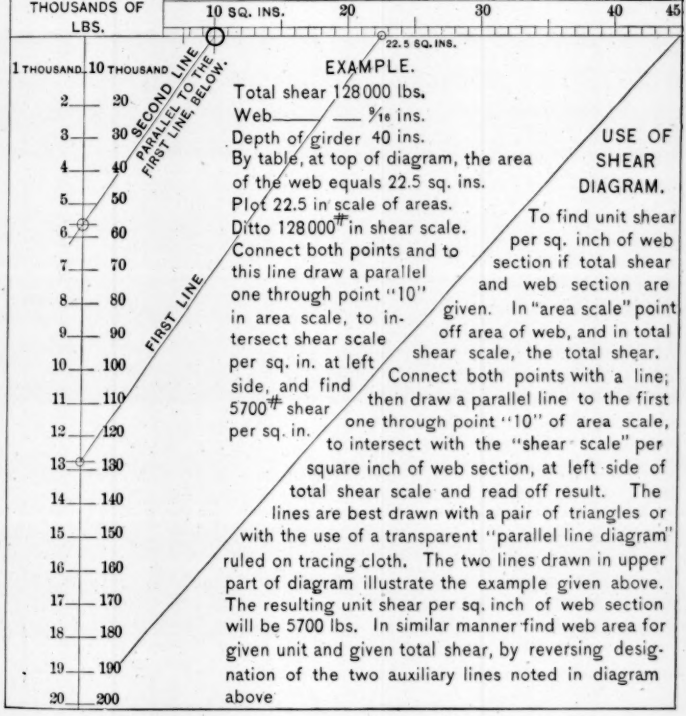
The manner of doing this is given indirectly in the explanation of how to use the shear diagram. By entering the table of web areas, the web thickness will be found with due regard to the assumed depth of the girder.

Allusion will have to be made to the term "depth of girder."

IN SHEAR SCALE BELOW ON THIS SIDE | THIS SIDE FIND RESULTING OR ASSUMED SHEAR PER SQ. IN. OF WEB SECTION | TOTAL SHEAR ON IN THOUSANDS OF LBS.

THICKNESS OF WEB IN INCHES	AREAS OF WEBS FOR DEPTHS OF										
	24	28	30	32	36	40	42	48	50	54	60″
5/16	7.5	8.75	9.37	10.0	11.25	12.5	13.12	15.0	15.6	16.87	18.75
3/8	9.0	10.5	11.25	12.0	13.5	15.0	15.75	18.0	18.75	20.25	22.5
7/16	10.5	12.25	13.12	14.0	15.75	17.5	18.38	21.0	21.87	23.62	26.25
1/2	12.0	14.0	15.0	16.0	18.0	20.0	21.0	24.0	25.0	27.0	30.0
9/16	13.5	15.75	16.87	18.0	19.75	22.5	23.62	27.0	28.12	30.37	33.75
5/8	15.0	17.5	18.75	20.0	22.5	25.0	26.24	30.0	31.25	33.75	37.5
11/16	16.5	19.25	20.62	22.0	24.75	27.5	28.86	33.0	34.38	37.12	41.25
3/4	18.0	21.0	22.5	24.0	27.0	30.0	31.5	36.0	37.5	40.5	45.0

DIAGRAM FOR SHEAR PER SQ. IN. OF WEB AREA FOR TOTAL SHEARS UP TO 200000 LBS. AND TOTAL WEB AREAS OF 45 SQ. INS.



EXAMPLE.

Total shear 128000 lbs.
Web ——— 9/16 ins.
Depth of girder 40 ins.
By table, at top of diagram, the area of the web equals 22.5 sq. ins.
Plot 22.5 in scale of areas.
Ditto 128000# in shear scale.
Connect both points and to this line draw a parallel one through point "10" in area scale, to intersect shear scale per sq. in. at left side, and find 5700# shear per sq. in.

USE OF SHEAR DIAGRAM.

To find unit shear per sq. inch of web section if total shear and web section are given. In "area scale" point off area of web, and in total shear scale, the total shear. Connect both points with a line; then draw a parallel line to the first one through point "10" of area scale, to intersect with the "shear scale" per square inch of web section, at left side of total shear scale and read off result. The lines are best drawn with a pair of triangles or with the use of a transparent "parallel line diagram" ruled on tracing cloth. The two lines drawn in upper part of diagram illustrate the example given above. The resulting unit shear per sq. inch of web section will be 5700 lbs. In similar manner find web area for given unit and given total shear, by reversing designation of the two auxiliary lines noted in diagram above

FIG. 7.

This term has several distinct and different meanings, relative and specific, all of which must be fully understood and correctly applied.

Generally, and as ordinarily understood, the depth of the girder is that fractional part of the span which constitutes its height, in common parlance, and in most cases is settled upon first and definitely after a thorough consideration of conditions, whether restrictions as to depth and width have to be observed, or whether the designer is at liberty to choose these, according to his best judgment. After the depth, etc., has been assumed in this general way, the other specific depths to be mentioned are derived therefrom as the design progresses, commencing with the flanges.

In ordinary building practice, for instance, if no restrictions are placed on the depth of girders, it is found (the result being in inches) by multiplying the span of the girder (expressed in feet) by six-tenths, and adding to the product the constant amount, 3 ins. The rate of depth of girder to length of span is, in this instance, one-twentieth plus the constant.

In heavier work the multiplier is eight-tenths, equal to a one-fifteenth rate. In bridge work the proportion of depth to span is increased to from one-twelfth to one-eighth, corresponding to multipliers of 1 and 1.5, respectively.

Having explained the first meaning of the term "depth of girder," the other meanings will now be referred to.

In determining the rivet spacing of the rivets connecting the web plate to the flange angles the depth of the girder is the vertical distance between the horizontal rivet lines located in the vertical legs of the top and bottom chord angles.

If the buckling value of the web plate is concerned, the depth of the girder is the clear distance between the inner edges of the vertical legs of the same angles.

The fourth subdivisable meaning of this term is involved when flange stresses are computed. If these are determined by the ordinary approximate method, the depth of the girder is taken from out to out of flange angles, if cover plates have to be provided to make up the required flange area. If none are needed, the depth is equal to the vertical distance between the centers of gravity of each pair of upper and lower flange angles.

If the exact method of moment of resistance, or, more correctly,

that of the section modulus, is selected to determine the cross-section of a plate girder, then we have to introduce into the computation for the depth (no flange plates) the distance "out to out" of angles, as before. With flange plates, take the depth over all the plates; the depth will change wherever a plate is dropped off.

There exists a sixth condition regarding this term, frequently overlooked by younger designers, and of which no mention is made in treatises on applied mechanics, in that the depth "out to out" of flange angles, as a rule, is greater by $\frac{1}{2}$ in. than the depth of the web plate over all. This allowance of $\frac{1}{4}$ in., at top and bottom, is made to avoid the projection of any part of the web beyond the flange angles. This allowance, therefore, does away with extra work, either shearing the web beforehand, or chipping it afterward, should the angles and web have been required to be flush.

If universal mill plates are specified for the web, some engineers do not demand this allowance to be made. Others insist on having it observed in every instance, as do most all of the mills in their shop practice. For special purposes, however, it may be required that the web shall project a certain amount beyond the backs of the angles; this is the exception to the rule.

Splicing in webs or angles in plate girders, in sizes ordinarily called for in architectural building practice, is not necessary, as universal mill plates for the web are rolled in dimensions from $\frac{3}{8}$ in. by 70 ft., up to $\frac{3}{4}$ in. by 65 ft., and angles on the average run up to a length of from 60 to 75 ft.

In conclusion, the attention of the reader is directed to two noteworthy papers on plate girders. The first, by J. C. Bland, M. Am. Soc. C. E., in the 1887 handbook of the Pottsville Iron and Steel Company. The second*, somewhat more extended in its scope, by Isami Hiroi, C. E., Professor of Civil Engineering in Sapporo College.

To fully complete this paper, and illustrate the graphical method for finding the rivet spacing in plate girders, the writer has chosen for exemplification the problem worked out by Mr. Hiroi, in his booklet, and in which, as regards the determination of the rivet pitch, he has used largely the analytic method of computation. In Fig. 8 will be found all the data requisite to a full understanding of the problem and the steps to be taken to solve it, and to form an opinion as to the

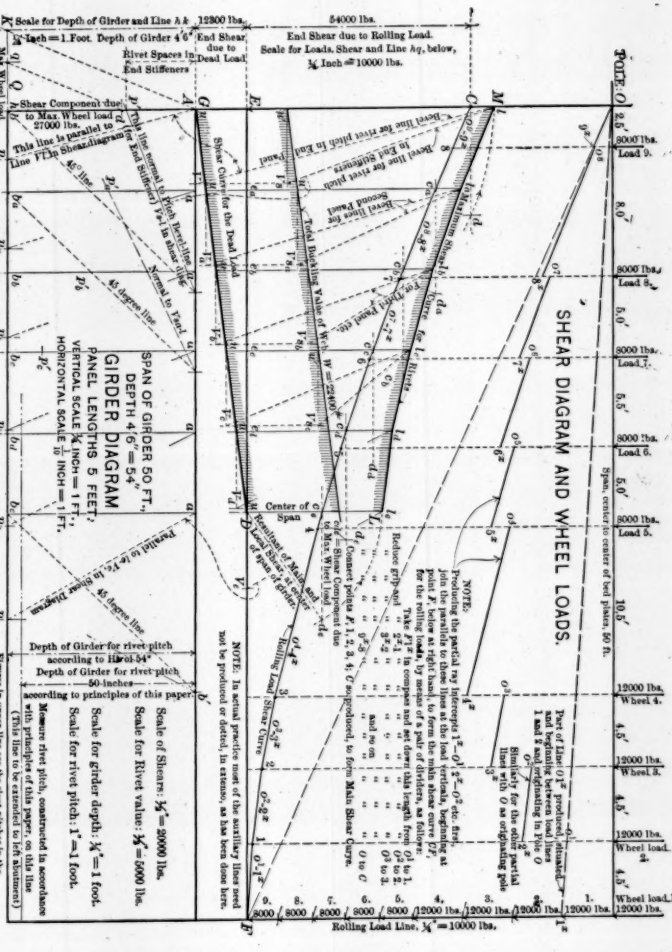
* Van Nostrand's Science Series, first edition, 1888; second, 1893, revised.

comprehensiveness and simplicity of the writer's method. The attention of teachers of structural mechanics is invited to the manner in which the polygon of maximum shear for the rolling or moving loads, has been formed, as fully explained in the "Programme of Example" accompanying the diagram, for determining whether it is not simpler and more readily executed than by the usual method. The difference in rivet spacing, due to assuming one or the other of the meanings of the term "depth of girder" has also been alluded to. It will be seen thereby how easily the rivet spacing can be adjusted to conform to any depth of girder chosen with this method, if otherwise the conditions of the external forces remain the same.

*Programme of Example.**—Single-track standard-gauge deck-girder, 50-ft. span between centers of bed plates, divided into 10 panels, 5 ft. each, for maximum stresses by bending and shears. Depth of girder, "out to out" of angles, 4 ft. 6½ ins. Depth assumed in computation for stresses, in Hiroi's book* and in this example, 4 ft. 6 ins. = 54 ins. Dead load, 490 lbs. per lineal foot. Rolling load, as per train diagram at top of Fig. 8. Maximum wheel load, 12 000 lbs., and, for local shear computation, assumed to be distributed over a sleeper space of 2 ft. End shear due to dead load, one-fourth of 490 x 50 x 2 = 12 300 lbs. Web thickness, $\frac{3}{8}$ in.; rivets, $\frac{7}{8}$ in. diameter. For bearing unit of 14 400 lbs., in $\frac{3}{8}$-in. web, rivet value 4 700 lbs.; shear unit for web, 6 400 lbs. Flange angles, 3½ ins. x 5 ins. x ½ in ; true depth for rivet spacing, 50 ins. (not adopted in this example).

Construct maximum main shear diagram (curve) for rolling load, *E F C*, as noted. Construct dead-load shear line, *G D*, in reference to *E F*. Construct local, vertical, shear-component, due to maximum wheel-load (see left lower corner of girder diagram). Construct total maximum shear curve, *M L*, as per Fig. 5, up to center of span. Construct girder diagram, using ¼-in. scale for the depth. Produce the flange-rivet bevel-lines $V\,l$, $V_a\,l_a$, $V_b\,l_b$, etc., in shear diagram, for each panel point, and draw the rivet-spacing bevel-lines $A\,p$, $a\,p_a$, etc., in girder diagram, and find the rivet spacings $b\,p$, $b_a\,p_a$, etc. Set off, parallel to *G D*, the web value $u\,u'$ in shear diagram, computed at eight-tenths of buckling value in Table No. 1, for a $\frac{3}{8}$ x 47-in. web (54 ins. — 2 x 3½ ins.) (see also page 63, Hiroi). Construct, as per Fig. 2,

* See "Plate-Girder Construction," by Isami Hiroi, C. E., Van Nostrand's Science Series, No. 95, second edition, 1893, pages 32, 38, 39, 62, 63, 79, 81, 83, etc.


SHEAR DIAGRAM AND WHEEL LOADS.
GIRDER DIAGRAM
SPAN OF GIRDER 50 FT.,
DEPTH 4'6" = 54"
PANEL LENGTHS 5 FEET.
VERTICAL SCALE ½ INCH = 1 FT.,
HORIZONTAL SCALE 1/10 INCH = 1 FT.
Scale for Depth of Girder and Line Ak
½ Inch = 1 Foot. Depth of Girder 4'6"
Scale for Shears: ½" = 20000 lbs.
Scale for Rivet value: ½" = 5000 lbs.
Scale for girder depth: ½" = 1 foot.
Scale for rivet pitch: 1" = 1 foot.
Rolling Load Line. ½" = 10000 lbs.
Span, center to center of bed plates, 50 ft.
Center of Span
Rolling Load Shear Curve
Shear Curve for the Dead Load
Total Dead Load Value of W = 54000 lbs.
Depth of Girder for rivet pitch according to Hool 54"
Depth of Girder for rivet pitch 50 inches according to principles of this paper.
Max Wheel load 18000 lbs.
Shorter Spaces 2 ft.
Max. Wheel load 12000 lbs.
Shorter Spaces 2 ft.
Wheel load. 4.
Wheel load. 3.
Wheel load. 2.
Wheel load. 1.
Load 9.
Load 8.
Load 7.
Load 6.
Load 5.
Load 4.
Load 3.
Load 2.
Load 1.
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12000 lbs.
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FIG. 8.

the stiffener-rivet bevel-lines $Vs\,l$, $Vs_a\,l_a$, etc., in shear diagram, and produce the corresponding normal lines to these, in girder diagram, giving the rivet spacing-points in stiffeners, as p', $p'a$, etc. The rivet values, uV, $u'V_s$, uV_a, $u'V_{sa}$, etc., having been laid off at four times the actual rivet value, of 4 700 lbs. (to obtain accuracy), rivet spacings must be measured in a scale having a four times larger denomination than the scale in which the depth of the girder was laid off.

TABLE No. 1.—Total Buckling Values of Webs, $w = \dfrac{10\,000\,d\,t}{1+\dfrac{d^2}{3\,000\,t^2}}$

Thickness of web, in inches.	Depth of Web, in Inches.											
	24	28	30	32	36	40	42	48	50	54	60	
5/16	25 300	23 750	23 100	22 300	19 700	19 200	18 650	17 200	16 500	15 350	13 850	lbs.
3/8	38 600	36 500	36 100	34 700	33 100	31 400	30 400	27 800	27 350	25 700	21 300	"
7/16	52 500	51 750	51 500	50 750	47 600	47 000	45 300	42 000	40 750	39 200	36 000	"
1/2	68 250	68 500	68 250	67 500	66 000	63 800	63 400	58 800	58 000	53 800	51 000	"
9/16	85 500	85 600	85 250	85 400	83 750	83 200	82 800	77 250	74 750	74 750	70 500	"
5/8	103 000	103 800	106 000	106 700	107 000	106 700	104 500	102 000	99 500	97 000	92 400	"
11/16	105 500	124 750	126 200	128 500	130 050	130 000	129 500	126 500	125 500	121 500	117 000	"
3/4	136 000	142 750	147 000	149 500	152 750	154 000	157 800	152 000	151 000	148 500	143 500	"

TABLE No. 2.—Total Buckling Values of Webs, $w = \dfrac{13\,000\,d\,t}{1+\dfrac{d^2}{3\,000\,t^2}}$

Thickness of web, in Inches.	Depth of Web, in Inches.											
	24	28	30	32	36	40	42	48	50	54	60	
5/16	32 800	30 600	30 000	29 000	25 600	25 000	24 300	22 200	21 500	20 250	18 000	lbs.
3/8	49 500	47 500	47 000	45 100	43 100	40 750	39 500	36 200	35 600	33 500	27 700	"
7/16	68 350	67 300	67 000	66 000	62 000	61 250	58 800	54 750	51 300	51 000	46 800	"
1/2	88 750	89 000	88 650	87 900	85 800	83 000	82 500	76 600	75 500	70 000	61 250	"
9/16	111 000	112 250	112 900	112 000	111 000	109 500	107 500	102 500	101 200	97 250	91 800	"
5/8	132 000	135 000	138 000	138 500	139 000	137 500	136 000	130 250	130 000	126 000	120 000	"
11/16	157 000	162 000	164 700	167 000	169 500	169 000	168 500	164 500	163 000	158 500	152 000	"
3/4	176 750	186 500	191 000	194 000	198 500	200 000	205 000	197 500	196 500	193 000	186 500	"

TABLE No. 3.—Shearing and Bearing Values of Rivets.

Diameter and area of rivet.		Rivet Values.		Thickness of Web or Plate, in Inches.								
		Single shear.	Shear unit ½ bear r. =	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
				Rivet Values for Bearing.								
3/4	in	3 310	7 500	3 520	4 220	4 920	5 630	6 330			
7/8	deci-	4 510	and	4 100	4 920	5 740	6 560	7 380	8 200	9 030	
1	mals.	5 890	15 000	.....	5 620	6 560	7 500	8 440	9 380	10 310	11 250
3/4	0.75	3 980	9 000	4 220	5 060	5 900	6 760	7 600			
7/8	0.875	5 400	and	4 920	5 900	6 880	7 880	8 860	9 840	10 820	
1	1.00	7 070	18 000	5 610	6 750	7 880	9 000	10 500	11 220	12 400	13 450
3/4	area	4 429	10 600	4 690	5 630	6 570	7 500	8 440			
7/8	in	6 010	and	5 470	6 570	7 600	8 750	9 840	10 840	12 300	
1	sq.in.	7 850	20 000	.....	7 500	8 750	10 000	11 250	12 500	13 750	
3/4	0.4418	4 860	11 000	5 160	6 190	7 220	8 250	9 280			
7/8	0.6013	6 610	and	6 020	7 220	8 430	9 630	10 840	12 040	13 240	
1	0.7850	8 640	22 000	.....	8 250	9 630	11 000	12 380	13 750	15 130	16 500

Note.—Within the extent of the table: Connections with double-shear rivets are governed by the bearing values given; connections with single-shear rivets are governed by the single-shear values given.

DISCUSSION

Mr. Schaub. J. W. SCHAUB, M. Am. Soc. C. E. (by letter).—The writer wishes to call attention to the author's method of proportioning the intermediate stiffeners to carry the shear on the web of a girder. This is the usual method of proportioning for stiffeners, but it can be shown that, if, in some way, the load can be put into the web, no stiffeners whatever will be required, except, of course, the end stiffeners. This is contrary to all accepted theories of web stresses, but a little experiment can be made to demonstrate it.

The following experiment was made by the writer* before the Western Society of Engineers, October 10th, 1900:

Make a plate-girder model of drawing paper, about 15 ins. long and 3 ins. deep, with two flange angles at the top and bottom, and let the web of the girder project below the bottom flange angles, so that the loads can be suspended by means of hangers attached to the projecting web. Now support the girder at the ends and support the top flange against crippling sideways. It will be found that loads which can be suspended from the hangers with perfect safety cannot be placed on top of the girder without buckling the web. Moreover, the web will stiffen up at once, and it will be found that, although the web is already carrying the suspended loads, additional loads can now be placed on top of the girder and the web will not buckle. It follows, then, that after the load is put into the web, the web is actually stiffer than it was before; so that stiffeners need only be provided to get the load into the web, and not to carry the maximum shear at any point. Would it not be more rational, then, to proportion the intermediate stiffeners to carry only the maximum concentrated load at any point, rather than the maximum shear?

With this experiment in view, the writer has specified, in a specification just issued for a western railroad, as follows:

"Intermediate stiffeners shall be provided for, as struts carrying the maximum concentrated load at any point, including impact, whenever the clear width between flanges exceeds 50 times the thickness of the web, and they shall be spaced at intervals of about the depth of the girder. End stiffeners shall be provided to carry the maximum end shear, including impact."

This method is certainly very simple. For through plate-girders provision need be made for intermediate stiffeners only to attach the floor system to the web, and between the floor beams, or panel points, no intermediate stiffeners are required, no matter how long the panels are, or how thin the web may be; whereas, by the usual method of

* "Proposed Specifications for Steel Railroad Bridges," *Journal*, Western Society of Engineers, October, 1900.

proportioning, intermediate stiffeners are provided for through plate-girders, the same as for deck plate-girders. This leads frequently to some curious results. The writer has in mind a specification for a prominent southern road which required intermediate stiffeners, by a so-called stiffener formula, to be spaced at 2-ft. centers near the ends, and a number of through plate-girders were built accordingly, and are probably still being built according to this specification. Mr. Schaub.

BERNT BERGER, Assoc. M. Am. Soc. C. E.—This paper is a valuable exposition of a graphical method of finding the exact spacing of rivets in flanges of plate girders under assumed loads and stresses. It is well worth careful study as it gives a clear insight into the variations in values and conditions of forces and stresses. The speaker has great faith and confidence in graphic statics, and uses that method quite extensively, but must differ with the author on the question of the applicability of the graphical method to the ordinary, every-day work of designing plate girders. The graphical method can at times be used to advantage in computations of railroad bridges for engine diagrams; but, for most of such cases, and especially for plate girders, the speaker has found, in his own work, that the analytical method, applied with the aid of the well-known moment diagrams, is much shorter and easier, especially when a slide rule is also used. For plate-girder bridges, as in the present case, it is, of course, necessary to find the end shear in the first place, and the shear at a very few intermediate points besides. For practical reasons, the rivet spacing in the flanges is constant for certain distances, and for shorter plate girders, therefore, all that is needed is the end shear and the shear at the quarter point. The 6-in. limit of rivet pitch will cover the shear in the middle half, especially when the rule about closer pitch in the flange which carries the track is also taken into account. For longer girders, up to 80 or 100 ft., the shear should be found at a few more points; but, with the aid of a moment diagram, this is readily and quickly done, more so, the speaker has always found, than by any graphical method. Mr. Berger.

The thickness of the web is determined by the required section of metal for shear and the necessary number of rivets. Shallow girders must have a thicker web, deep girders may have a thinner web. Each end stiffener must be able to take all the end shear in compression, but not necessarily as a column of length equal to the depth of the girder, as the web through the rivets distributes the shear over the end stiffener through its whole depth. The intermediate stiffeners cannot, according to the speaker's views, act as columns for the distribution of the shear over the web, as most of that shear is already in the web; but they can so act under concentrated loads, which are only part of the shear, as when engine loading for railroad bridges is

Mr. Perger. considered. Thus, for girders which carry the track direct on one or the other of the flanges, the service which such intermediate stiffeners perform, consists then principally in holding the web in position—preventing it from buckling. But where, for instance, the stringers of a bridge rest on top of the floor beams, the stiffeners on the floor-beam web must of course serve to distribute the concentrated load over the web of the floor beam, and must have sufficient area for compression.

As to the axiom laid down by the author in the beginning of the paper, the speaker begs to say that, generally speaking, he thinks formulas little worth the effort of remembering. The thorough understanding of principles and methods, and their application, is of far greater importance, and to this end the paper is of great help.

Mr. Skinner. F. W. SKINNER, M. Am. Soc. C. E.—A particularly interesting, and perhaps extreme, case of the application of the web stiffener system to a plate girder, and one which the speaker believes is not widely known, came under his observation recently.

In Cincinnati, Ohio, there is a plate-girder span of 213 ft.—the longest of this kind of which the speaker has ever heard. The bridge is formed of two main girders, each 16 ft. deep, which carry a 30-ft. roadway and two 10-ft. sidewalks. The thickness of the web could not be ascertained, but is not more than $\frac{3}{8}$ in., and, possibly, is not more than $\frac{5}{16}$ in. The structure has been in use more than thirty years, and the iron is badly corroded. The bottom chord is made with two small angles and several 42-in. horizontal flange plates. The top-chord is a three-web box section 42 ins. wide. The web is divided into panels of about 10 ft., and in each panel there are two transverse stiffener plates, each 26 ins. wide, and each formed with a flange angle all around.

Calculations may be possible as to how much that web, 16 ft. high and $\frac{5}{16}$ in. thick, would have stood without the stiffeners, and also how much it would stand with the stiffeners.

One peculiar feature was the way in which each panel of the web was built up of five pieces, 8 ft. high, butt jointed, and spliced with two cover plates.

Another peculiar feature of the bridge was the extreme generosity of the bearings on the abutments, each of which consisted of two very thick cast-iron plates about 7 ft. long; the upper one sliding between longitudinal flanges on the edges of the lower plate.

After many years' service, two intermediate steel trestle bents, each about 50 ft. from the center of the span, were put in to reduce the vibrations.

The whole construction is a curiosity, if not a monstrosity, but is still in active service, and is remarkable for its dimensions.

H. A. La Chicotte, M. Am. Soc. C. E.—The author has suggested a graphical method of spacing rivets in plate girders, which is really a graphical solution of a well-known formula: Mr. LaChicotte.

$$p = \frac{v\,h}{s},$$

in which p = the pitch;
v = the rivet value;
h = the depth of girder between rivet lines;
and s = the shear, assumed as more or less uniformly distributed over the section.

Fig. 9 will indicate clearly the derivation of the formula by the method of moments.

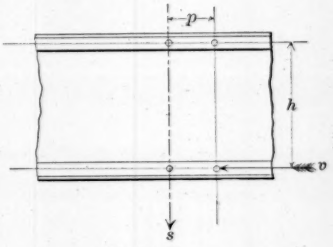

Fig. 9.

According to the common theory of the plate girder, the lines of stress in the web act at an angle of 45° with the vertical, so that the stress induced in the flanges by the web must, in a length of flange equal to the depth of the girder, equal the shear. In other words, there must be enough rivets in either flange, within a length equal to the depth of the girder, to equal the shear.

If, in addition to the horizontal stress in the flanges, the rivets are required to transmit to the web a local load, Q, more or less distributed and acting at right angles to the flange stress, the rivets must be spaced to resist the resultant of the two stresses.

If Q be distributed over a distance k (less than h), the diagonal lines of stress distribute it over an equal distance in the vertical section of the web, and, in order to distribute it with equal intensity over the whole section, Q must be first divided by k to find the unit intensity, and then multiplied by h, or, $\frac{Q\,h}{k}$. The formula for pitch now becomes:

$$p = \frac{v\,h}{\sqrt{s^2 + \left(\frac{Q\,h}{k}\right)^2}}.$$

If the load Q be distributed over a distance equal to h,

$$p = \frac{v\,h}{\sqrt{s^2 + Q^2}}.$$

The application of the formula $p = \frac{v\,h}{s}$ is very simple, after the shear has been computed at the requisite number of points; and, knowing the law of change in the shear, the desired results are quite as quickly obtained by the analytical as by the graphical method. For any particular girder, s being the only variable, the values of p, for

Mr. LaChicotte.

any number of points corresponding to known values of s, may all be read from a slide rule by a single setting.

If the stresses in the girders are found graphically, the author's method of spacing rivets is a valuable adjunct, but if these stresses are determined analytically, the time and labor of constructing the diagrams for spacing the rivets graphically would be much greater than in the analytical solution of the problem.

If any of the foregoing equations be transformed and solved for v, the resulting value for Maximum $s \times$ Minimum p determines the necessary thickness of web when reference is made to a table of rivet values for any particular size of rivet.

As to rivet spacing in stiffeners, these formulas may or may not apply. If they apply, it is necessary to assume that the stiffeners act as columns, and this, at least, is a moot question. To illustrate, imagine a number of planks piled on edge one upon another; unless supported laterally, they would not stand alone, much less carry a load imposed upon them; but, if posts directly opposite each other be placed on each side, in contact with the planks but not attached to them, a considerable load would be sustained by the planks, although receiving little assistance from the posts. This seems to show that the office of the stiffener is not to take stress, but to enable or compel the web to do so; and, for that reason, only a few rivets are needed in the stiffeners to properly hold them together. As usually constructed, there is no doubt that the stiffeners act with the web and serve to disturb and complicate the theoretical lines of stress, but there is no reason for assuming that because they take some stress they must take all over and above the buckling value of the web.

The foregoing reasoning applies specially to intermediate stiffeners, which do not serve to convey any external load to the web.

In the case of concentrated loads brought to the web from transverse floor beams, or more or less distributed on either flange of the girder, the stiffeners must be proportioned to convey these loads to the web, but in each of these cases practical considerations of construction will usually determine the size of stiffeners.

At the ends of the girder the lines of stress being cut, the web has little buckling value, and the stiffeners must be proportioned to convey practically the whole shear to the support. The stress in the end stiffeners due to shear varies practically from zero at the top to about the full amount of the shear at the bottom, and there must be rivets enough to convey this shear from the web to the stiffeners. A girder properly stiffened at the ends and at points of local concentrated loads will usually need very little intermediate stiffening.

The author suggests figuring stiffeners as flat-ended columns, and the questions naturally arise: Upon what will these columns rest? If on the bottom flange, how can rivets enough be grouped in the

flange at the base of the stiffeners to convey the reaction of the column from the flange to the web? And why should the shear follow the roundabout path through stiffeners and flanges? Mr. LaChicotte.

The author refers to a well-known formula for finding the buckling values of webs, and advocates the spacing of stiffeners about the depth of the girder apart whenever the shear in the web equals or exceeds the value of w in the formula

$$w = \frac{10\,000,}{1 + \frac{h^2}{3\,000\,t^2}}$$

This method of spacing stiffeners, while quite common, is not consistent, for the reason that this formula pretends to give the buckling value of the web, but does not express the relation of this value to the shear.

If the web needs stiffening at all, why not in some proportion to the shear? If the foregoing formula be applicable at all, it should be made to express the actual relation between load and column length by substituting "shear" for "w," and solving for h, whence results for the distance between stiffeners or flange angles,

$$h = \sqrt{\frac{30\,000\,000\,t^2}{\text{shear}} - 3\,000\,t^2};$$

When h is more than the depth of girder, no stiffeners are required, except for local concentrated loads.

The common practice of stiffening plate girders at distances apart of about the depth of the girders, is probably far on the side of safety; but it would, nevertheless, be unwise to depart materially therefrom, until a more rational method has been devised.

In regard to determining the size of stiffeners necessary to prevent the web from buckling, it may be said that no rational method of figuring the stresses in the webs of plate girders has yet been devised, and therefore no exact determination of the size of stiffeners is possible.

It is useless to say that if the web maintained its plane form, no stiffeners would be required; but the webs are not perfect planes, having many bulges or buckles of more or less magnitude as they come from the mills, and which buckles cannot always be detected or removed in the process of manufacture of the girder.

It may safely be said that stiffening angles large enough to accommodate the rivets and conform to the usual shop practice will prevent the web from buckling. Of course, the end stiffeners, as well as those at points of local concentrated loads, may be larger than elsewhere; and, perhaps, the most that can be said is that good judgment based on present practice will usually be a safe guide in special cases.

Mr. Nichols. O. F. NICHOLS, M. Am. Soc. C. E.—The speaker admits with candor, and with pride, that so far as he has had anything to do with plate girders, he has followed the teachings and experience of Theodore Cooper, M. Am. Soc. C. E., who taught that in order to get more rivets safely into the flange angles, and to devolop fully the efficiency of these angles, their depth should be increased so as to carry at least two rows of rivets wherever many rivets were required. This has a double advantage: The rivets need not be spaced dangerously near together, and the deeper angles confine all the elemental portions of the web, so that, considering them as columns, they are practically fixed at the ends, and, consequently, are better able to sustain the loads imposed.

The Pennsylvania practice has been very largely followed in this country and elsewhere. A good deal of the misunderstanding about stiffeners in plate girders is probably caused by following the Pennsylvania standard too closely. An interesting experiment was made by one of the Pennsylvania engineers: He constructed a model girder, from paper, then, after cutting the web on diagonal lines at 45° in one direction between the stiffeners, spaced at equal intervals about equal to the depth of the girder, the girder was put under stress. He found that the stiffeners took the vertical load, and that the elements of the web buckled under compression and remained straight under tension. Of course, cutting the web in that manner converted the model girder into a sort of Pratt truss, and the conclusion drawn, that stiffeners were necessary at intervals equal to the depth of the girder, was quite a logical sequence from the premises.

The more rational method, and that more closely followed in recent times, is to consider plate girders as acting very much as lattice girders act. Take a case in which the girder web is not stiffened; then, if it is regarded as a lattice girder, there is an infinite number of intersections, and if it be assumed that the stresses pass in diagonal lines through the web, in both directions, while the compression would tend to buckle the web, the tension along the other lines at right angles would tend to keep it straight. As a matter of fact, in all tests of plate girders, it is found that there is a remarkable and an unaccountable degree of stiffness in the unsupported web.

Mr. LaChicotte is right in stating that, if there are rivets enough in the stiffeners to prevent them from buckling out, it is all that can be required. Fitting them against the flanges is not necessary, unless, as he has said, the top of the girder is to sustain the load; in which case, of course, there must be a close fit, at least against the top angles.

There has been a great misuse of stiffeners. In some elevated railroad structures, for example, where the uniform loads are relatively low, and where the girder is often very shallow, a $\frac{3}{8}$-in. web is generally

used with a multitude of stiffeners. In quite ordinary practice the angle is used as a stiffener, and is often placed with its back toward the middle of the girder. At the middle of the girder, either for symmetry or from a desire to be strictly impartial between the two ends of the girder, two stiffeners are placed, back to back, where none is needed. These structures have generally been built by the pound, on designs by the contractors, and in such cases this excess of stiffeners has an advantage not easily overlooked. Some ten years ago, several miles of elevated structure were built in which, by deepening the standard girder about 15% and omitting stiffeners, the average strength was increased and the weight diminished nearly 20 per cent. Of this structure, a leading bridge expert, recently engaged by new owners to report on its strength, said: "The bold omission of these (stiffeners) has apparently worked all right in practice, and secured a substantial economy of metal." Mr. Nichols.

Of late years there has been a most exemplary attempt to avoid serious errors in the use of stiffeners; to rely, indeed, almost entirely on the web itself, unaided by stiffeners; and now it is the exception, rather than the rule, to find stiffeners in stringers on large bridges, although they carry very heavy loads and are subject to varying stresses, of impact, etc. In the Buffalo Trace Viaduct, designed by Edwin Thacher, M. Am. Soc. C. E., some ten or twelve years ago, the longer girders are about 60 ft. in length and approximate 5 ft. in depth. The viaduct carries a standard-gauge railroad, and no stiffeners whatever are used, but the webs of the girders are § in. thick.

Engineers are approaching a belief in simplicity of design, and that the fewer the actual number of members used, the better. If the metal in the web can be made to take the stresses directly, and without the aid of stiffeners, it is a great deal better, very largely because it is absolutely impossible to determine with the remotest degree of certainty how such stiffeners will act.

F. D. Rhodes, Jun. Am. Soc. C. E.—The speaker's experience in designing girders for buildings points conclusively to the advantage of using the graphical method for spacing rivets when the girders are subject to combinations of concentrated and distributed loads, or any kind of loading which gives an unsymmetrical moment diagram. Mr. Rhodes.

It has been contended that the author's method involves extra labor, as it necessitates the plotting of a shear diagram; but in cases of the kind just mentioned it is the speaker's belief that graphic statics will most quickly and accurately determine the length and position of the cover-plates, and that it requires but very little extra labor to plot the shear diagram directly off the load line of the force polygon.

Girders in buildings often carry walls or floor arches which are

Mr. Rhodes. supported directly by the flanges, for which cases the speaker's practice, in determining the shear values for the various pitches, differs from the author's, as is shown in Fig. 10, and the accompanying table. The example is taken from the design of a 36-in. girder, web $\frac{11}{16}$ in. thick, rivets $\frac{7}{8}$ in., with the top flange carrying a load of 9 tons per lineal foot.

RIVET DIAGRAM.

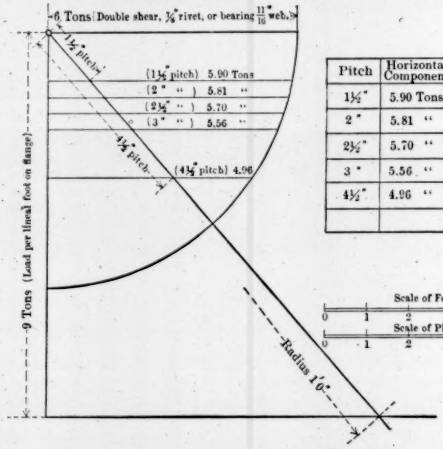

TABLE.

Pitch	Horizontal Component	Shear	Calculation
1½"	5.90 Tons	114 Tons	$\frac{5.90 \times 29}{1.5}$
2"	5.81 "	84.2 "	$\frac{5.81 \times 29}{2}$
2½"	5.70 "	66.1 "	$\frac{5.70 \times 29}{2.5}$
3"	5.56 "	53.7 "	$\frac{5.56 \times 29}{3}$
4½"	4.96 "	32.0 "	$\frac{4.96 \times 29}{4.5}$

$S = \frac{r' \times h}{p}$

$h = 36" - \frac{7}{8} = 29"$

FIG. 10.

The horizontal components are scaled directly off the rivet diagram, the calculations are made with a slide rule, and the shears are then plotted on the shear diagram.

Mr. Jonson. ERNST F. JONSON, Assoc. M. Am. Soc. C. E. (by letter).—The author's method of spacing rivets in plate girders is undoubtedly the most convenient one yet proposed. There are, however, some points in the paper which require further consideration. In order that the problem of the plate girder may be better understood, it is necessary to find the actual stresses in the web.

These stresses may be found by the following graphical method:

Let s = total vertical shear;

m = unit shear at neutral axis;

t = increase of extreme fiber stress per unite of length of girder;

a = area of girder section;

w = thickness of web;

r = radius of gyration of girder section;

d = distance from neutral axis to extreme fiber;

b = distance from neutral axis to center of gravity of half girder section.

Mr. Jonson.

We know that $s = \frac{a\,r^2\,t}{d}$, or $t = \frac{s\,d}{a\,r^2}$.

We also know that $w\,m = \frac{a}{2}\frac{t\,b}{d}$.

Hence, $m = \frac{b\,s}{2\,r^2\,w}$ ..... *I*

and $s = \frac{2\,m\,r^2\,w}{b}$ ..... *II*

Draw a sectional view of the girder, Fig. 11, and construct a Mohr diagram of the square of the radius of gyration. This is done in the following way: Divide the cross-section of the girder into a number of parts (twelve in the figure). Take half the cross-section area as pole distance *C D*. Lay off the parts of one-half of the cross-section area on the line *C E*. Draw the moment diagram *B C G* as if the parts of the cross-section area were forces acting in the center of gravity of each part. The horizontal ordinates of the curve *C G* multiplied by the pole distance *C D* give then the moment of the part of the cross-section area outside the point in which the ordinate is taken. Hence, *B G* is the distance from the neutral axis to the center of gravity of the half cross-section; $B\,G = b$.

Since the horizontal bases of the triangles of which the area *B C G* is composed represent the moments of the various elements of the cross-section divided by half that area, the triangles themselves must represent half the moment of inertia of these elements divided by half the cross-section area. Hence, the area *B C G* is equal to half the square of the radius of gyration of the cross-section area with reference to the neutral axis; $B\,C\,G = \frac{r^2}{2}$.

Since the shear at any point in the web is proportional to the cross-section area outside of that point multiplied by the average bending stress acting in that area, *i. e.*, proportional to the moment of the area with reference to the neutral axis, the horizontal ordinates of the curve *A G* are proportional to the shear.

The area *A B G* is twice as large as the area *B C G*, because the trapezoids of which it is made up are twice as large as the corresponding triangles of which the area *B C G* is composed; $A\,B\,G = r^2$.

If we now take the pole distance, $A\,F = b$, and draw the moment diagram $A_1\,C_1\,C_2$ with reference to *C*, we get $C_1\,C_2 \times A\,F = 2 \times A\,B\,G$, or $C_1\,C_2 = \frac{2\,r^2}{b}$. We also get the curve $A_1\,C_2$, the ordinates of which toward the axis $A_2\,C_2$ are proportional to the part of the double area *A B G* which lies below the point in which the ordinate is taken. Hence these ordinates are proportional to the compression produced by a load on top of the girder. If we then lay off the load per unit of length divided by the web thickness $A_2\,N$, and draw, by means of

Mr. Jonson. parallel lines, the curve $N\ C_2$, with ordinates proportional to those of $A_1\ C_2$, $A_2\ N\ C_2$ is the diagram of the perpendicular compressive unit stress due to the load.

Now, lay off $C_1\ K = 10$, $C_1\ K_1 = 10$, and $C_1\ L = 10w$, and draw the parallel lines $K_1\ L$ and $K\ L_1$, thus making $C_1\ L_1 = \frac{10}{w}$. Lay off $C_1\ M = s$, and draw the parallel lines $C_2\ L_1$ and $M\ M_1$, making $C_1\ M_1 = 10\ m$. For $C_1\ M$ and $C_1\ M_1$, a smaller scale must be used. Lay off $B\ G_1 = m$, and draw, by means of parallel lines, the curve $A\ G_1$ with ordinates proportional to those of curve $A\ G$. These ordinates then represent the unit shear at the various points of the web.

Multiply $C_1\ C_2$ with $\frac{a\ b}{2}$ and lay off the result $A_2\ H = a\ r^2$. Also, lay off the bending moment $A_2\ T$, both to a smaller scale than that of the rest of the diagram. Draw the parallel lines $H\ T$ and $A_3\ C_3$. The triangles $A_2\ A_3\ B_2$ and $B_2\ C_2\ C_3$ are then the diagrams of the unit stresses due to bending; the ordinates of $A_3\ B_2$ representing the compression and those of $B_2\ C_3$ the tension, for the unit stress is to the distance from the neutral axis as the bending moment is to $a\ r^2$.

In order to find the resulting principal stresses* lay off the unit shearing stress on a vertical line from U to R. Take the point Q, dividing the distance $O\ U$ in two equal parts, as a center, and draw the circle $P_1\ R\ P_2$ through R. The two principal stresses are then the tension $P\ P_1$ acting in the direction $P_2\ R$, or $P\ S$, and the compression $P\ P_2$ acting in the direction $P_1\ R$, or $P\ S_1$. Hence, the ordinates of the curve $I_1\ J_1$ represent the tensile stresses, and those of the curve $I_2\ J_2$ the compression stresses.

From the diagram it will be seen that unless the unit shear is very small (in the present case less than one-third of the extreme fiber stress) the maximum stress is not at the extreme fiber, as generally supposed, but in the web at the upper and lower pitch lines.

The shear diagram further shows that the h, in the rivet formula quoted by the author, should be taken as the distance between pitch lines only when the formula is used for rivets in stiffeners supporting concentrated loads or reactions. When used for rivets in flange angles it is quite on the safe side to make h equal to the distance from back to back of angles. In the present case it is even greater than the total height of the girder.

The author holds (page 553) that when stiffeners are used, the web takes only a part of the shear, leaving the remainder to the stiffeners. This is clearly impossible, unless the stiffeners are arranged so as to form a complete lattice system. If, in the present case, a stiffener were inserted along the line $A\ C$ it would greatly reduce the vertical compressive stresses $A_2\ C_2\ N$, but would not in any way affect the shearing stresses. It would, however, increase the resistance of the

* Rankine, "Applied Mechanics," Ch. V., Sec. 3.

Mr. Jonson.

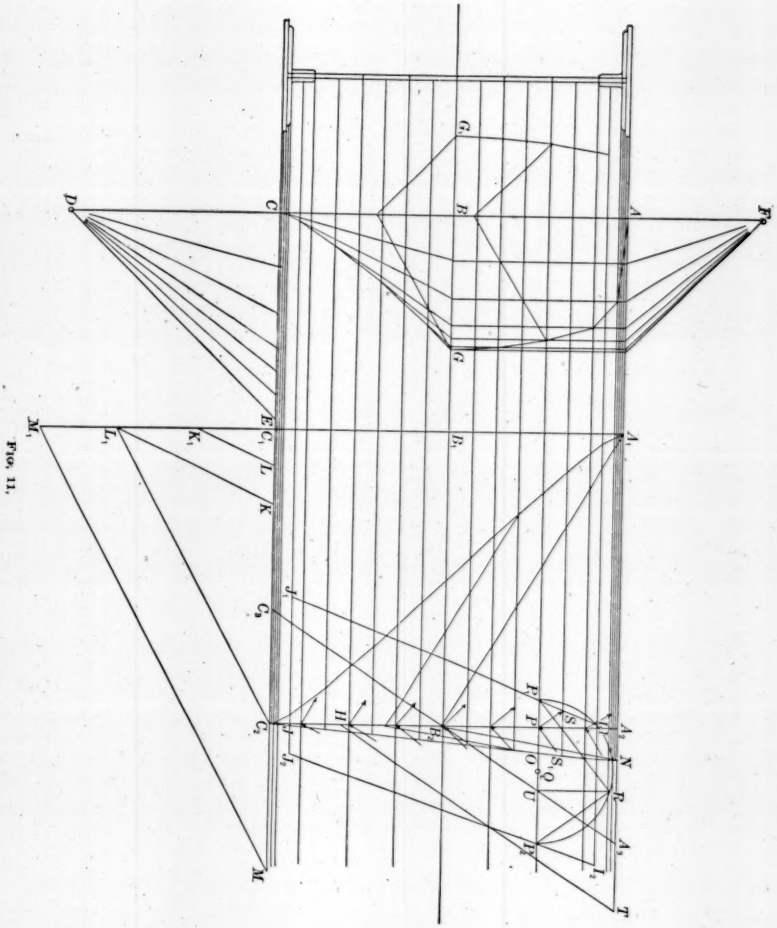

FIG. 11.

Mr. Jonson. central part of the web by preventing buckling or wrinkling. Unfortunately the theory of wrinkling has not yet been developed, so that the dimensions and spacing of stiffeners and their rivets cannot be calculated. The writer refers to stiffeners proper. Uprights, inserted to transmit a concentrated load or reaction to the web, should be regarded as columns loaded along their whole length and, therefore, subject to a stress proportional to the horizontal ordinates of the curve $A_1 C_2$. Such uprights may, therefore, as the author says, be calculated for the allowed unit stress. The length, however, should not be more than 120 times the radius of gyration perpendicular to the web, or the length beyond which a column is good for less than one-half of the allowable unit stress of the material, because, in such uprights, the load at the center is one-half of the load at the end.

If the load were applied at the bottom instead of at the top the present diagram would have to be turned upside down. The curve $I_1 J_1$ would then represent the compressive stress, and the curve $I_2 J_2$ the tensile stress.

Mr. Wing. C. B. WING, Assoc. M. Am. Soc. C. E. (by letter).—Methods of reducing the labor of computations, in designing bridge and other structures, are usually of most value to the person evolving the method. Therefore, it is with hesitation that the writer presents other methods for solving the problems, solutions of which have been presented so clearly in Mr. Schmitt's paper.

In the following graphical diagrams use is made of the principle that any equation involving three variable quantities may be readily solved for a large range of values of the variables by the use of co-ordinate paper with two sets of diagonal lines constructed to correspond to assumed values of two of the variables.

Thus, the diagram, Plate XI, Fig. 1, is used to solve the equation

$$v = t\,d\,C$$

in which d, t, and C are variables. v is the bearing strength of a rivet, of diameter d, bearing on a plate, of thickness t, and capable of resisting safely a bearing stress of C per unit of area.* The equation may be thrown into the form

$$C\,d = \frac{v}{t} = X.$$

The co-ordinate paper is now used to find the product $C\,d = X$, and then, from this product, the value of v is found for the given value of t. Thus, on the left of the diagram, values of C from 9 000 to 26 000 lbs. per square inch, covering the range of ordinary practice, are laid off to scale as ordinates. Similarly, at the bottom of the diagram, values of

*v is the quantity termed "rivet value" on page 552. In what follows, the writer has adhered to the notation of the paper as closely as possible.

SAFE LOADS FOR RIVETS IN BEARING. $v = t\,d\,C$

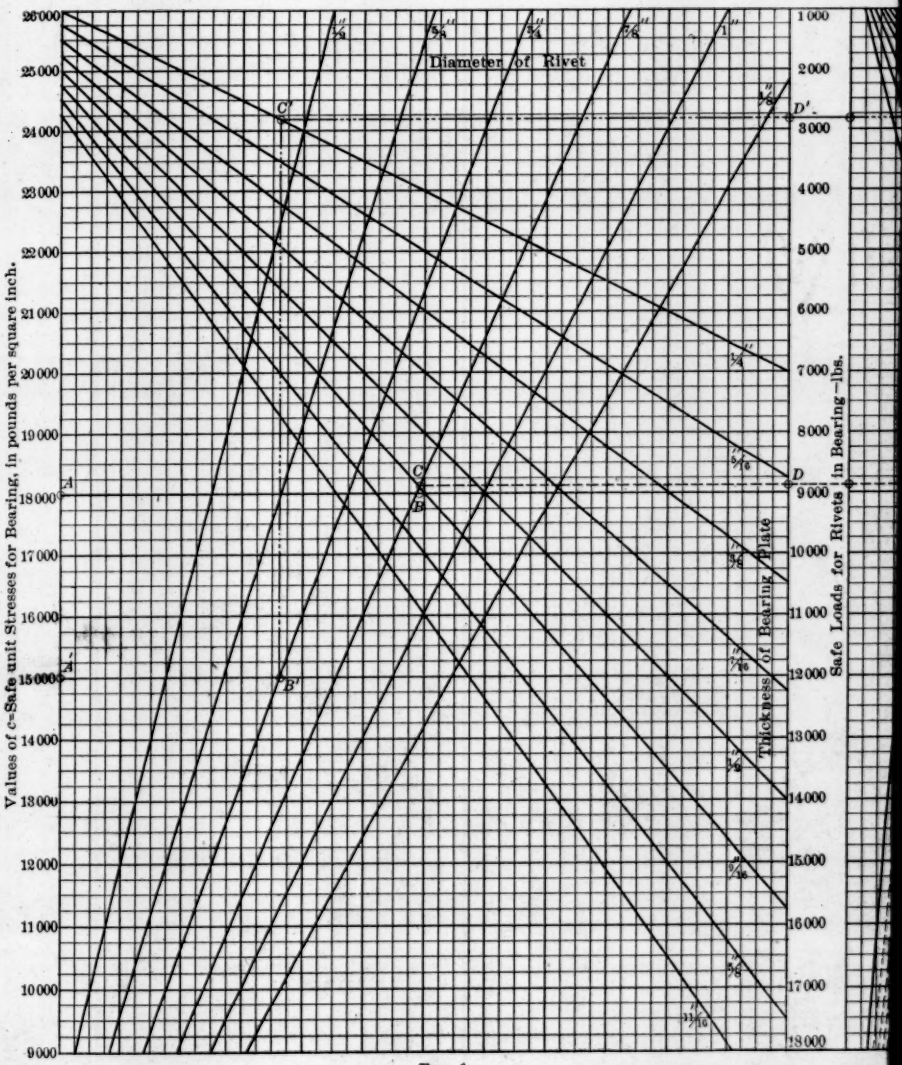

FIG. 1.

PLATE XI,
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WING ON RIVET SPACING.

SPACING FOR RIVETS IN GIRDER FLANGES $p=\frac{v\,h}{S}$

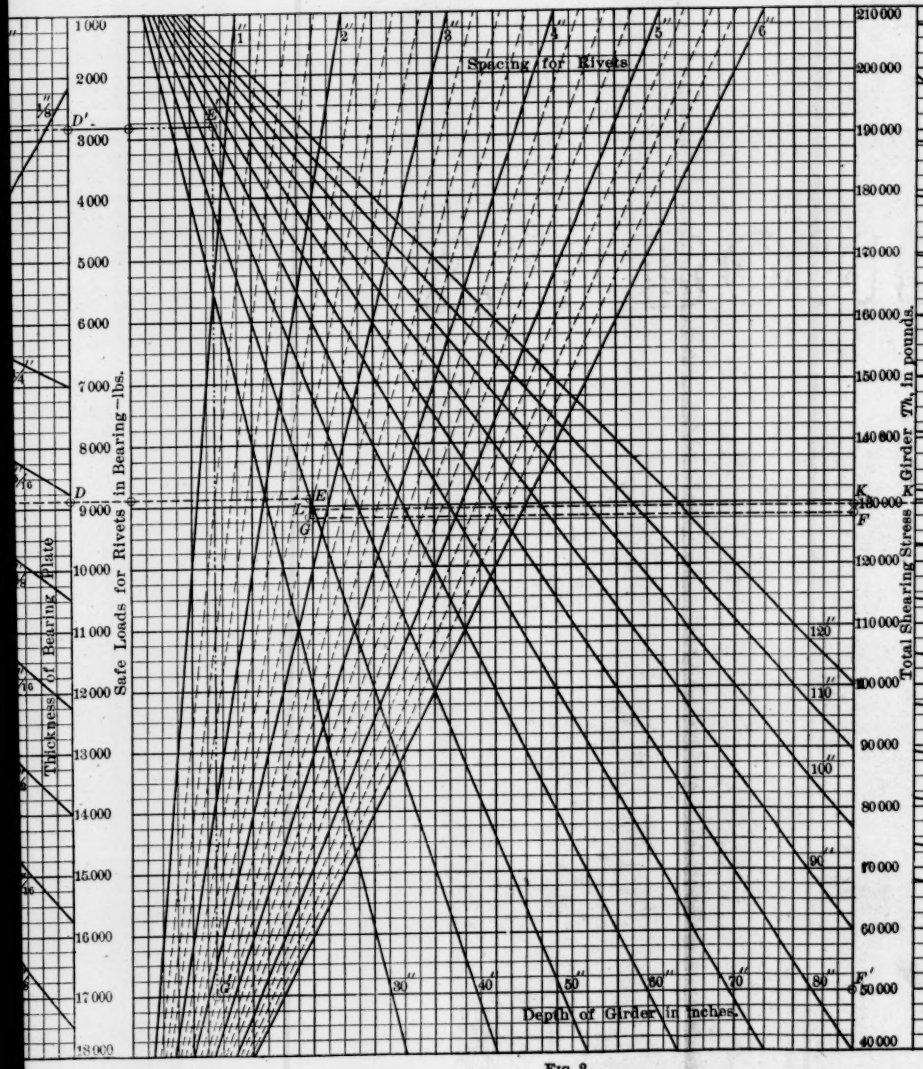

FIG. 2.

SOLUTION OF EQUATION $S'=\sqrt{S^2+w^2h^2}$

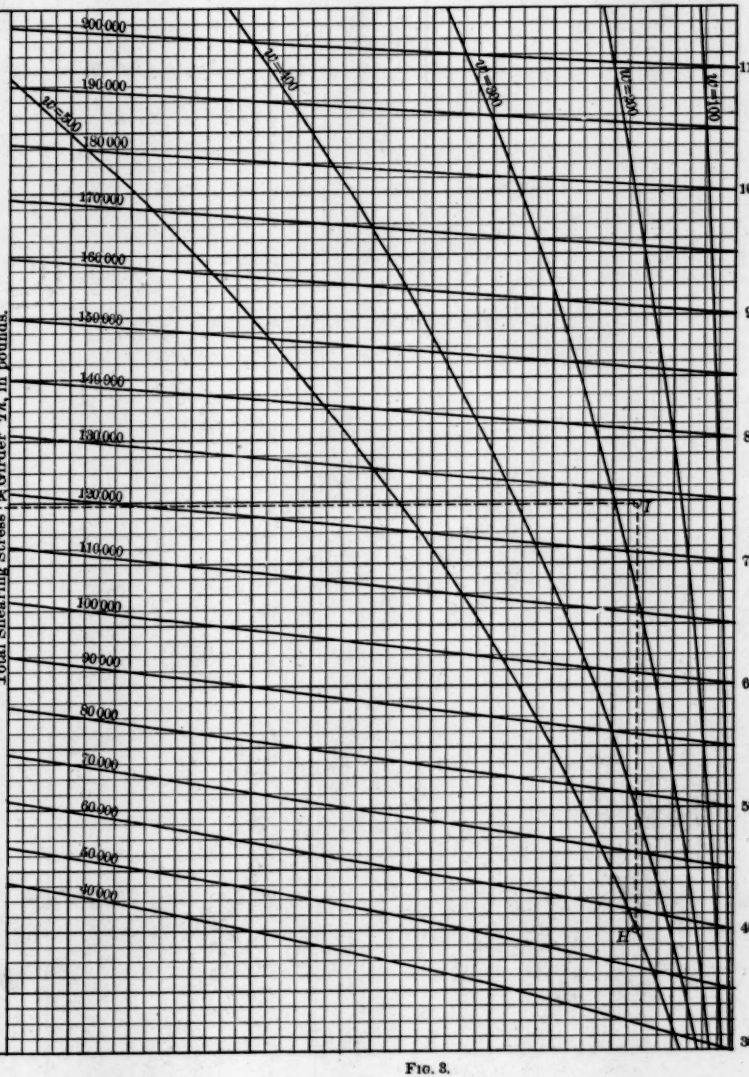

FIG. 3.

X, covering the range of values of the product $C\,d$, are laid off as abscissas to some suitable scale.* Then diagonal lines are drawn corresponding to values of d ranging from $\frac{1}{2}$ in. to $1\frac{1}{8}$ ins. With C and X as co-ordinates, the equation of any such diagonal, for a given value of d, is

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$$C\,d = X.$$

As the quantities in this equation are all of the first power, these diagonals are straight lines radiating from the intersection of the vertical and horizontal through the zero values of X and C, respectively; for, when $C = 0$, $X = 0$ for all values of d. Therefore, one other point on each diagonal is all that is required to determine its position on the diagram. To locate such points assume $C = 24\,000$. The following values of X result for the given values of d:

d	X
$\frac{1}{2}$	12 000
$\frac{5}{8}$	15 000
$\frac{3}{4}$	18 000
$\frac{7}{8}$	21 000
1	24 000
$1\frac{1}{8}$	27 000

On the horizontal through $C = 24\,000$ plot the foregoing values of X, and from these points draw straight lines to the point of intersection of a vertical through the zero of the scale of X with a horizontal through the zero of the scale of C. Points where these diagonal lines intersect horizontals through the scales of C will be vertically above the value $X = C\,d$ on the scale of X. Thus, to find the product $C\,d$ for any values of C and d within the range of the scales, follow the horizontal line through C to the right until it intersects the diagonal line corresponding to the value of d. The required product $C\,d = X$ will be found on the bottom scale vertically beneath this point.

In a similar manner, laying off on the right side of the diagram values of v, diagonal lines corresponding to values of t are drawn radiating from the intersection of a vertical and horizontal through the zero points of the scales of X and v, respectively. As before, one other point on each diagonal line is found from the equations of such lines, $X = \frac{v}{t}$, by assuming $X = 24\,000$ and plotting values of v corresponding to assumed values of t. Thus, to find the value of v corresponding to values of X and t, follow the vertical through the value of X until it intersects the diagonal corresponding to the value of t; the required value of v will be found on the scale of values of v horizontally to the right of this point.

Finally, with the diagram as constructed, to find the value of v corresponding to any values of t, d and C, within the limits of the scales, proceed as follows: Enter the diagram at the point on the scale at the left,

* The numerical values of X are not shown on the diagram, as they are not of practical value. The scale ranges from 4 000 on the left to 28 000 on the right.

Mr. Wing. corresponding to the value of C, move horizontally to the right to the point of intersection with the diagonal corresponding to the value of d,* from this point move vertically to the point of intersection with the diagonal corresponding to the value of t; the required value of v will be found on the scale of values of v horizontally to the right of this point.

Example.— $C = 18\,000$ lbs. per square inch, $d = \frac{7}{8}$ in., $t = \frac{9}{16}$ in. Required, the value of v. The diagram, Fig. 1, Plate XI, is entered at A, and, following the dotted lines through B and C, the value $v = 8\,860$ lbs. is found at D on the right-hand scale of values of v; or with $C = 15\,000$ lbs. per square inch, $d = \frac{3}{4}$ in. and $t = \frac{1}{2}$ in., entering the diagram at A', and following through B' and C' to D', the value $v = 2\,800$ lbs. is found.

The diagrams, Figs. 2 and 3, Plate XI, for the solution of the equations $p = \frac{v\,h}{S}$ and $S' = \sqrt{S^2 + w^2\,h^2}$ have been constructed in a similar manner.†

To facilitate this method of determining the spacing of rivets in girders, the equation given by the author on page 556, for the case in which the load carried by the girder is distributed uniformly to the flange angles, has been changed to the form

$$p = \frac{v\,h}{\sqrt{S^2 + w^2\,h^2}}$$

by the substitution of $w = \frac{Q}{k}$, in which w is the load on the flange per unit of length.‡

The diagram, Fig. 3, Plate XI, gives a solution of the quantity $S' = \sqrt{S^2 + w^2\,h^2}$. With this value, the value of p can be determined by Fig. 2, Plate XI.

The three diagrams on Plate XI, arranged in the order shown, give a ready means of solving graphically the problems ordinarily arising in practice concerning the pitch of rivets in girders. Thus, to determine the pitch of rivets in the flanges of a girder, at a point where the shear $S = 128\,000$ lbs., with $h = 40$ ins., $d = \frac{7}{8}$ in., $t = \frac{9}{16}$ in., and $C = 18\,000$ lbs. per square inch, Fig. 1, Plate XI, is entered at A on the left, as before, and the dotted lines followed to E, Fig. 2, which is the point of intersection of the horizontal through the rivet value v, with the diagonal corresponding

* Note that the vertical through this point gives the value of $Cd = X$, and that this value of X is to be used in finding the value of $v = \frac{X}{t}$.

† Note that in Fig. 3, Plate XI, the variable quantities in the expression $\sqrt{S^2 + w^2 h^2}$ are of the second power; therefore the diagonal lines of the diagram are curves instead of straight lines, and several points on each curve must be plotted in order to construct it on the diagram. Except for this additional labor, the method of constructing the diagram is the same as for Fig. 1, Plate XI.

‡ If the inch and pound are used as units, w must be expressed in pounds per lineal inch.

to a value of $h = 40$ ins. If the load carried by the girder is not distributed to the flange angles, the required pitch of rivets, p, is found by following the vertical through E down to the point G horizontally to the left of the point F, which point corresponds to a value of $S = 128\,000$ on the scale on the right of Fig. 2, Plate XI. This point G is found to lie very near the diagonal line for values of $p = 2\frac{3}{4}$ ins.; therefore, $2\frac{3}{4}$ ins. is the pitch required. Mr. Wing.

If the load carried by the girder is distributed to the flanges, with $w = 500$ lbs. per lineal inch, Fig. 3, Plate XI, is entered at H, the point of intersection of the diagonal corresponding to $w = 500$, and the horizontal through $h = 40$ on the scale on the right of the diagram; the vertical through this point is followed to I, a point corresponding to $S = 128\,000$, as shown by the transverse diagonals. $S' = 129\,500$ is found horizontally to the left of this point at K. The required pitch, p, is determined by the location of the point L, and is found to be 2.7 ins. Practically, spacings varying by less than $\frac{1}{4}$-in. are seldom used; therefore, $p = 2\frac{1}{2}$ ins. is the required spacing of the rivets in the flanges of a girder under these conditions.

Similarly, for $C = 15\,000$, $d = \frac{8}{4}$, $t = \frac{1}{4}$, $h = 60$, $S = 50\,000$, and $w = 100$, the required pitch is found to be $3\frac{1}{4}$ ins., by entering the diagram at A', and following the dotted lines through B', C', D', E', to G'. Referring to Fig. 3. Plate XI, at the point of intersection of the horizontal through $h = 60$ and the diagonal $w = 100$, it is seen at once that the fact that the load is uniformly distributed to the flange angles will not appreciably change the spacing of the rivets.

Fig. 3, Plate XI, shows clearly that, in many of the cases arising in practice, neglecting to consider the distributed load would cause but a slight error. It may be well to note, in this connection, that the formula

$$p = \frac{v\,h}{\sqrt{S^2 + w^2\,h^2}}$$

is only approximately correct. Thus, considering a section of a girder between two rivets as a free body in space in equilibrium under the forces acting upon it: On the assumption that the moment is carried by the flanges at the line of the pitch of rivets, the reactions of the portions of the girder removed will be as shown by the arrows in Fig. 12. The body being in equilibrium, the sum of the moments about any point, as the lower right-hand rivet, must equal zero, *i. e.*,

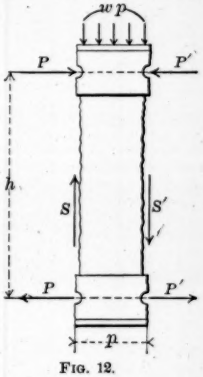


FIG. 12.

$$[P' - P]\,h + w\,p \times \frac{p}{2} - S\,p = 0 \ldots\ldots\ldots\ldots (1)$$

Mr. Wing. Next, considering the forces acting on one of the top rivets, Fig. 13, the following results:

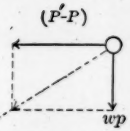


FIG. 13.

$$[P' - P]^2 + w^2 p^2 = v^2$$

$$\text{or } [P' - P] = \sqrt{v^2 - w^2 p^2}.$$

Substituting this value in Equation (1), the value of the required pitch of rivets is found to be

$$p = \frac{h v}{\sqrt{[S - \frac{1}{2} w p]^2 + w^2 h^2}}$$

$$\text{instead of } p = \frac{h v}{\sqrt{S^2 + w^2 h^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The approximate value given by Equation (2) is on the side of safety, and practically is sufficiently exact. In fact, as before stated, the error will be slight, in most cases, if the distributed load is entirely neglected.

Figs. 14, 15 and 16 have been constructed by the same methods used in constructing Plate XI. Fig. 14 replaces the author's Tables Nos. 1 and 2 and gives a solution of the equation

$$P = \frac{C h t}{1 + \frac{1 h^2}{3\,000 t^2}}$$

in which C has values varying from 10 000 to 18 000;

h = depth of girder;

t = thickness of web plate;

P = safe shearing load for a web plate without stiffeners.*

Nine tables similar to Tables Nos. 1 and 2 would be required to cover the range of values of C given in Fig. 14. Results can easily be read to the nearest 500 lbs., which practically is as close as required. Thus, if $h = 40$, $t = \frac{9}{16}$, and $C = 10\,000$, the diagram is entered on the left at A, with the value $h = 40$ as an ordinate, the horizontal line through this point is followed to the right until it intersects the diagonal line corresponding to a value of $t = \frac{9}{16}$, at B; from this point the vertical line is followed to its intersection at C with the diagonal corresponding to a value of $C = 10\,000$. The value of the safe total shear, 84 000 lbs., is found horizontally to the right of this point at D on the scale of values of P. Similarly, for $h = 60$, $t = \frac{3}{8}$, and $C = 10\,000$, the value of $P = 24\,000$ lbs. is found.

If the total shear on the girder is greater than the safe total shear given above, stiffeners will have to be used, and the safe total shear, or the required thickness of web plate, determined by Fig. 15.

The method of deriving the formula solved by Fig. 14, will be found in Church's "Mechanics of Materials," page 383. In spite of testimony against the necessity of using stiffeners, the writer still believes that the best results will be obtained by their use when required by the

*The author's notation has been changed in this instance, in order that it may more closely correspond to the notation used in the preceding discussion.

Mr. Wing.

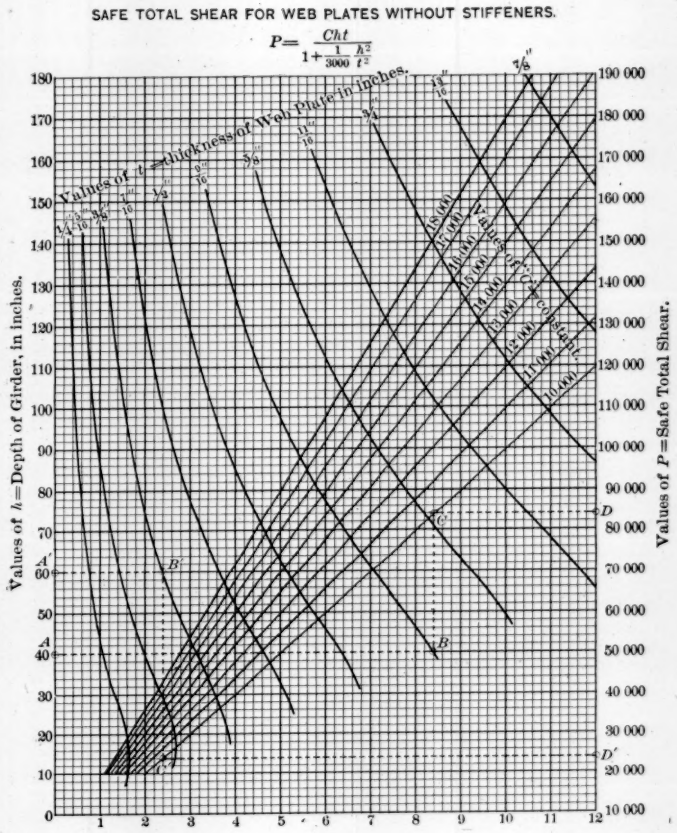


FIG. 14.

Mr. Wing.

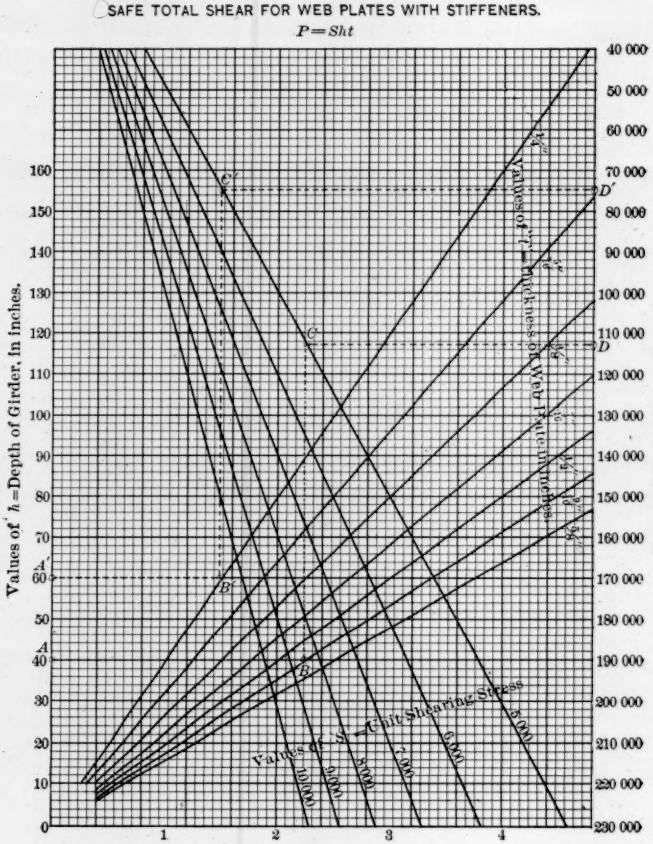


FIG. 15.

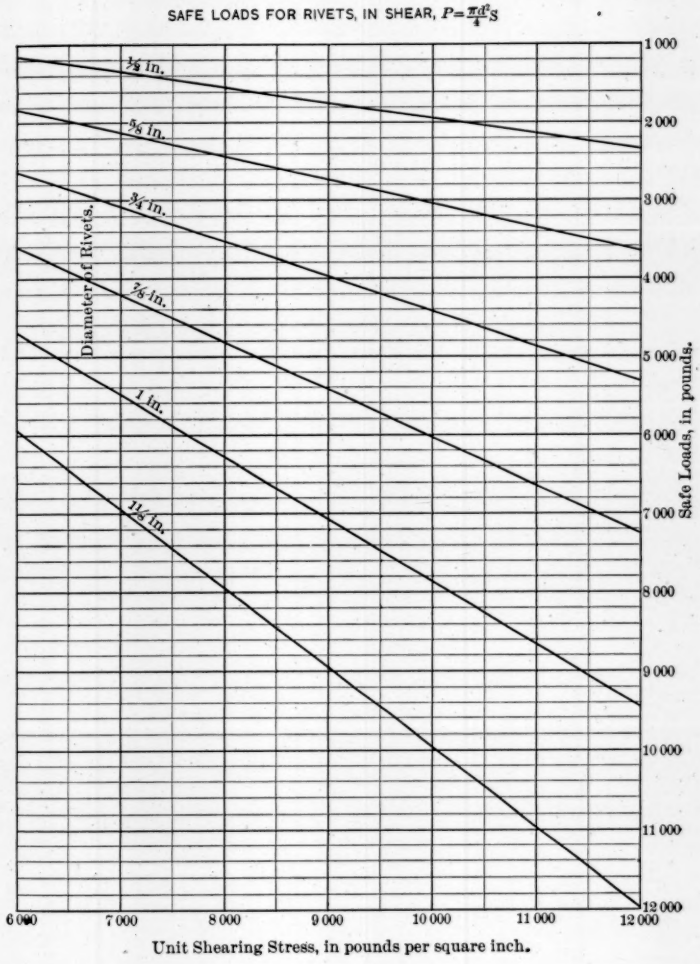

SAFE LOADS FOR RIVETS, IN SHEAR, $P=\frac{\pi d^2}{4}S$
½ in.
⅝ in.
¾ in.
⅞ in.
1 in.
1⅛ in.
Diameter of Rivets.
1 000
2 000
3 000
4 000
5 000
6 000
7 000
8 000
9 000
10 000
11 000
12 000
Safe Loads, in pounds.
6 000
7 000
8 000
9 000
10 000
11 000
12 000
Unit Shearing Stress, in pounds per square inch.

FIG. 16.

Mr. Wing. foregoing formula. When so used, they should be spaced at distances apart equal to the depth of the girder, and riveted securely to the web plate. The size of stiffeners and the number of rivets used in fastening them to the web plate should depend upon the judgment of the designer, and not upon calculation. If calculation is resorted to, it would be on the side of safety to design the two stiffeners with a reasonable amount of web plate as carrying the same stress as the post of a Pratt truss, similarly placed; that is, the total shear at that section should be carried by the stiffeners and a portion of the web plate acting as a column.

Fig. 15, giving safe shearing loads for web plates with stiffeners, and Fig. 16, giving the shearing values of rivets, have been constructed in a similar manner to the diagrams already described.

The foregoing methods for the graphical solution of equations are of wide application, and, in practice, are of great value as labor-saving devices. In general, a single graphical diagram will give a greater range of values than several pages of tables. The results, however, with ordinary scales, are only approximately correct to three places. The required accuracy of results, therefore, will usually determine whether the more convenient graphical diagram can be used as a substitute for a set of tables.

On page 562 the author gives an empirical rule for determining the depth of girders. If, as the author assumes, no restrictions are placed on the depth of a girder for a given purpose, it is desirable to use a depth which will carry the given load with a minimum weight of material in the girder. This "economic depth" is very closely given by the rule that the area of the flanges equals the area of the web plates, or

$$h = \sqrt{\frac{2M}{Tt}}$$

in which h = depth of girder;
M = maximum moment carried by the flanges;
T = safe unit stress for flanges;
t = thickness of web plates.

This formula, while theoretically correct in form, is based on the assumption that there is no variation in the cross-section of the girder, and does not make allowance for the weight of details which do not change with the depth of girder.

In practice, a depth about 10% less than that given by the formula will be found to give the least weight for a girder capable of carrying a given load.

It seems to the writer that the rule given by the author must result in girders which are uneconomical and which have excessive deflections.

* See Johnson and Turneaure's "Modern Framed Structures," and an article by Henry Szlapka in *The Engineering Record*, March 27th, 1897, for more elaborate discussions of this formula.

E. SCHMITT, Assoc. M. Am. Soc. C. E. (by letter).—Mr. Schaub's statement, deduced from experiment, concerning the erratic behavior of the webs in through and deck-plate girders is of much interest. Mr. Schmitt.

In what follows, the writer endeavors to approach the propositions (at least one of them) numerically. An example, arbitrarily chosen, will serve for the investigation, in which the following assumptions have been made:

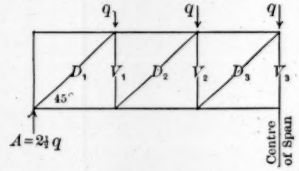
1. The sum of the web stresses in a panel acts along the center lines of imaginary web members of a square-paneled truss, with single diagonals in the panels, the length of each panel being equal to the depth of the truss.*

2. The loads (shear stresses) transmit themselves by the shortest route from the center of the span to the abutments.

3. The same factor of safety is to be maintained at all points in the web. Web members under compressive shear are to be safe against buckling. Compressive and tensile shears are to be considered as compression and tension stresses.

4. The web is to have the same thickness in the two cases.

Beginning with the case of the deck bridge, the imaginary truss will take the form represented in Fig. 17; the position of the diagonals



$D_1 = -A\sqrt{2}$ $V_1 = A - q$

$D_2 = -(A-q)\sqrt{2}$ $V_2 = A - 2q$

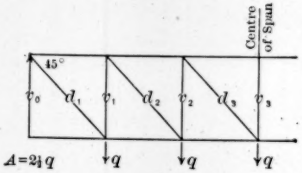
$D_3 = -(A-2q)\sqrt{2}$ $V_3 = A - 2\frac{1}{2}q = 0$

FIG. 17.

enabling the loads to be transmitted "by the shortest route." The stresses obtaining in the web members are noted at the right of the diagram.

The diagonals, only, being in compression, the web will have to be proportioned for the stress obtaining in them.

In the case of the through bridge (or, perhaps, better expressed "trough bridge"), the imaginary truss will be represented by Fig. 18,



$d_1 = +A\sqrt{2}$ $v_1 = -(A-q)$

$d_2 = +(A-q)\sqrt{2}$ $v_2 = -(A-2q)$

$d_3 = +(A-2q)\sqrt{2}$ $v_3 = -(A-2\frac{1}{2}q)$

FIG. 18.

* With the principal stresses acting at 45°, according to the theory of the plate girder, and with the loads concentrated at panel lengths, as assumed, and the further assumption that compressive web stresses will follow the line of least resistance, the arrangement of the direction of the forces, as shown in Fig. 17, suggests itself first. Fig. 18 follows by reversing Fig. 17.

Mr. Schmitt. permitting, in a similar manner to that in the first case, the loads to travel by the most direct route from the center of the span toward the abutments. The stresses obtaining in the web members are noted at the right of the diagram. They are of the same magnitude as in the corresponding members of the first case, but opposite in character. The diagonals being in tension and the verticals in compression, the stresses obtaining in the latter will, therefore, govern the dimensioning of the web for buckling.

Before comparing the compressive stresses per unit of length along the respective web members of the two classes of girders, first determine the ratio of the permissible buckling stress in the diagonals (in compression), of the deck bridge, with the one taking place in the verticals (also in compression), of the through bridge. For this purpose select the empirical formula given by Rankine.

Calling the reduced stress for buckling,

for the diagonals of the deck bridge, P^2..(length of diagonals $= h\sqrt{2}$)

" verticals " through " , p,..(" " verticals $= h$),

we have, $$P = \frac{k}{1 + c\left(\frac{h\sqrt{2}}{w}\right)^2} \text{ and } p = \frac{k}{1 + c\left(\frac{h}{w}\right)^2},$$

in which,

k = simple compression unit;

h = depth of girder;

w = thickness of web;

$c = \frac{1}{5\,000}$.

According to the specification, cited by Mr. Schaub, to avoid the use of intermediate stiffeners, assume the web to have a thickness of $w = \frac{1}{50}$ of the depth of the girder.

Introducing the given quantities in the equations for P and p, we obtain, by extending and simplifying, the denominators only of the two fractions,

$$P = \frac{2\,500\ k\ w^2}{2\ h^2};\quad p = \frac{5\,000\ k\ w^2}{3\ h^2},$$

and dividing the two values, the ratio is found to be:

$$\frac{P}{p} = \frac{2\,500 \times 3\ h^2}{5\,000 \times 2\ h^2} = \frac{3}{4}$$

or, the permissible compressive unit web stress in the deck bridge should be but three-fourths of the stress in the through bridge, due to the difference in length of the members.

Since the web is of the same thickness in the two girders, the foregoing statement would mean that the web in the deck bridge is stressed four-thirds as much as the web of the through bridge, for the same load, and only as far as buckling is concerned.

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Comparing now the ratio of the actual stresses, per unit of length, in the respective representative web members of the two bridges, we will first ascertain, which diagonal (stress) of the deck bridge should be compared with the corresponding vertical (stress) in the through bridge for any of the panels.

Referring to Fig. 17, it is evident that for the panel at the center the compressive stress D_3 will be the governing one, for the whole panel; its influence will extend from the center of the span to the vertical V_3.

In the case of the through bridge, Fig. 18, however, the computation of the web for buckling will begin at the vertical v_2; v_2 being in compression. Reasoning similarly as regards the operation of stresses in the other, related, panels, we can say:

TABLE No. 4.

In the deck bridge, in.....	1st....	2d.....	3d panel.		
compute web for comp. stress....	D_1...	D_2.....	D_3		(length of members, $h\sqrt{\ }$)
In the through bridge, at vertical	1	2	3		
compute web for comp. stress...	v_1....	v_2.....	$v_3 = 0$ (or d_3, tension)		(length of members, h.)

The sphere of action of the vertical, compressive stresses in the through bridge is represented in Fig. 19. Any ordinate in any of the triangles represents the columnar lengths of the web, subject, in turn, to the stresses v_0, v_1, v_2.

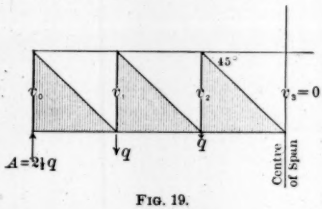

FIG. 19.

It will be seen that at v_0, v_3 and v_0, the critical points exist for web computation, and that it is permissible to compare the panel stresses, as given in Table No. 4.

Commencing with the stresses of the second panels of Table No. 4, the magnitudes of which are noted in Figs. 17 and 18, we have,

Simple comp. stress, per unit of length, along web, or diagonal D_2 $\Big\}$ $D_{u2} = -\dfrac{(A-q)\sqrt{2}}{h\sqrt{2}}$

Simple comp. stress, per unit of length along web, or vertical v_2 $\Big\}$ $v_{u2} = -\dfrac{(A-2q)}{h}$

and $$\frac{D_{u2}}{v_{u2}} = \frac{A-q}{A-2q}; \quad A = 2\tfrac{1}{2}\,q,$$

we find, $$\frac{D_{u2}}{v_{u2}} = 3.$$

Previously, it was found that, for equal thickness of the webs in the two bridges, the stress in the web of the deck bridge was four-

Mr. Schmitt. thirds as much as in the web of the through bridge, and that, therefore, the actual ratio of the stresses per unit of length of D_3 and v_2 will be

$$\frac{D_{u2}}{v_{u2}} = 3 \times \frac{4}{3} = 4.$$

Similarly, it is found, in the first panel, at the abutment,

$$D_{u1} = -\frac{A\sqrt{2}}{h\sqrt{2}}.$$

$$v_{u1} = -\frac{(A-q)}{h},$$

and the simple stress ratio $\frac{D_{u1}}{v_{u1}} = \frac{A}{A-q} = \frac{5}{3}$,

and the actual stress ratio $\frac{D_{u1}}{v_{u1}} = \frac{5}{3} \times \frac{4}{3} = 2\frac{2}{9}$.

In the third panel, or that at the center, compare D_3 (compression), with d_1 (tension). Both stresses are of the same intensity, and both members are of the same length.

Referring to the formula for permissible unit stresses in the diagonals, we will have for D_3, introducing for w, $\frac{1}{50}\,h$,

$$P = \frac{2\,500\ k\ w^2}{2\,h^2}.$$

$$P = \frac{k}{2}.$$

Assuming the permissible unit stress for tension, also, as k (no reduction being necessary to be made for this kind of stress), it is evident that, for equal thickness of web, in the two girders (loads being the same), the web in the panel at the center of the deck bridge will be stressed twice as much as the web at the end of the same panel of the through bridge.

Mr. Schaub's deductions from his experiments, concerning the contrary behavior of the webs in the two classes of plate-girder bridges, seem, therefore, to be verified in a measure, and it could not, perhaps, be otherwise. It occurs to the writer, however, that what Mr. Schaub desires to prove by experiment can be explained by saying that, until now, the "theory of bridges" has failed to recognize the two classes of bridges (deck and through) in plate girders, although having distinguished these in trusses long ago.

The exceptions, made by the several discussors, principally those regarding the problem of the stiffeners, were, to a degree, anticipated.

The (theoretical) solution of this problem, it seems, could be effected in two ways: One would be, to figure the web for shear, then space the stiffeners according to the formula presented by Mr. La Chicotte,* to

* This formula is not new; see notes by Professor Swain (Mass. Inst. of Technology), which the writer had opportunity to inspect, before writing the paper.

prevent the buckling of the web. The other method would be, to do away with intermediate stiffeners and to figure the web for buckling. The writer, however, will repeat, with Mr. La Chicotte, that, perhaps, the most that can be said about this matter, is that good judgment, based on present practice, will usually be a safe guide in special cases. Enough has been said regarding "present practice" in the discussion; a repetition seems superfluous. Mr. Schmitt.

Mr. Nichols' remark, that it would be better to use flange angles so as to carry two rows of rivets, instead of one row, in which case the depths of the girder would have to be increased (under certain previously mentioned circumstances), is to be noted by designers as another serviceable constructive expedient, of advantage in more than one direction, and the writer has made use of it.

It conflicts, however, with the desirable proposition to obtain a maximum section modulus with the least amount of material. The material composing the flange area should be placed as far from the neutral axis as practical considerations will allow.

Mr. Jonson's demonstration of the internal stresses is very interesting and instructive. The opinion, however, that stiffeners should be arranged so as to form a complete lattice system, differs from that of the writer, who has always held that stiffeners need be used in a vertical position only, but, if required, should be spaced uniformly along the web. This arrangement converts the plate girder into a truss, the web performing the duty of the diagonals, subject to either compressive or tensile shear, as the case may be.

The writer's object was, not to present a complete or a new theory of the plate girder, but to exhibit a comprehensive, uniform and practical method of spacing the rivets, and of determining the point, from which on, toward the abutments, stiffeners are needed; including the reciprocal operations connected with the problem.

The method shown seems to him to be simply a direct property of the shear curve, in a developed form, and should find a good application in the presentation of the theory of the plate girder in connection with its construction.

The use of special methods and diagrams for working out the same problem is a personal matter. It is a privilege and a source of information to become acquainted with other roads than one may have been accustomed to travel. In this respect, the writer acknowledges that he has profited by the discussion.*

Mr. Wing's graphical tables for solving the problems connected with the design of the web of plate girders cover, very likely, all the cases that may arise in present-day practice.

The writer believes that when the numerical tables, Nos. 1 and 2, giving the total buckling values of webs, for 10 000 and 13 000 lbs.

* The foregoing remarks were written before Mr. Wing's discussion was received.

Mr. Schmitt. unit shearing stresses per square inch, are properly used, no urgent need exists to add seven more similar tables, to cover the range of values given in Fig. 14.

Let it be required, for instance, to find the total buckling values for the unit shears of 11 000, 14 000 and 18 000 lbs.; the procedure would be as follows:

For 11 000 lbs. unit shear, add to values of Table No. 1, one-tenth of the same;
For 14 000 " " " " No. 2, " values of No. 1;
For 18 000 " " " " No. 2, one-half " " " etc.

Proceed similarly with the rivet tables.

The writer will not enter upon the question of the rapidity and ease with which the path of the several lines leading to final results can be followed through the different diagrams Mr. Wing has presented, and in the course of which a number of turning points occur. He would claim, however, that with his method the two main points, noted in the heading of the paper, are determined *in loco;* that is, the rivet pitch and the stiffener point are produced upon the girder itself, when once the shear curves for the loading have been plotted. This the writer considers an achievement to be appreciated for its clearness and directness.

The inexactitude of the rivet formula for compound shearing stresses is properly dismissed by Mr Wing as unimportant; yet it must be stated that this formula, as well as the simple rivet formula, are both incorrect. This is because no account has been taken of the fact that the web takes up part of the bending moment, so that the stress to be carried by the rivets connecting the flange to the web is less than the two formulas assume. The number of rivets required, therefore, is also less, and the pitch is increased.*

To complete the paper, both sets of formulas for rivet pitch are given here:

For simple shear stress, the usual formula for pitch is,

$$p = \frac{v\,h}{S}$$

For simple shear stress, the correct formula for pitch is,

$$p = \frac{v\,h}{S}\,\frac{F + \frac{A}{8}}{F}$$

For compound shear stress, the usual formula for pitch is,

$$p = \frac{v\,h}{\sqrt{S^2 + \left(Q\,\frac{h}{k}\right)^2}}$$

* See Johnson and Turneaure's "Modern Framed Structures," pages 298, 305 and 306.

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For compound shear stress, the correct formula for pitch is,

$$p = \frac{v\,h}{\sqrt{\left(S\,\frac{F}{F+\frac{A}{8}}\right)^2 + \left(Q\,\frac{h}{k}\right)^2}}$$

In which F = area of one flange section;
A = " the web "

The correct formulas can be somewhat simplified by expressing the area of the web in terms of the area of the flanges. According to the "Working rule for economic depth,"* the sum of the areas of the flanges ought to be from $1\frac{1}{2}$ to 2 times the area of the web plate.

Now, when $2F = \frac{3}{4}A$, we have $\frac{A}{8} = \frac{1}{3}F$

and " $2F = A$, " $\frac{A}{8} = \frac{1}{4}F$, and for:

Simple shear, correct formula for pitch,

$$p = \frac{4}{3} \text{ to } \frac{5}{4}\,\frac{v\,h}{S}$$

Compound shear, correct formula for pitch,

$$p = \frac{v\,h}{\sqrt{\left(\frac{3}{4} \text{ to } \frac{4}{5}\,S\right)^2 + \left(Q\,\frac{h}{k}\right)^2}}$$

In Figs. 1, 2, 3, 4 and 5, therefore, it is only necessary to lay off, instead of h, $\frac{F+\frac{A}{8}}{F}\,h$, for the case for simple shear; and to decrease, graphically, the ordinates of the main shear curves, in the ratio $\frac{F}{F+\frac{A}{8}} \times S$, for the case of compound shearing stresses, to obtain more exact results than those with which the ordinary theory is satisfied. The writer abstained from mentioning the correct formulas for rivet pitch, at the time, because he did not wish to overburden the presentation of his method, which is not affected thereby, and, further, for the reason that their acceptance in actual practice is problematical. The approximations all tend toward greater safety.

Mr. Wing's apprehension that the empirical rule† given by the writer would lead to uneconomical girders, having an excessive deflection, is too general an assertion, and should not be accepted unreservedly.

* See Johnson and Turneaure, page 300, Section 282.

† Mr. Wing evidently refers to the first one, which is, $h = 6\,\frac{L}{10} + 3$ ins (h in inches, L in feet), page 562.

Mr. Schmitt. This rule is to be used, as was stated in the paper, in ordinary building practice only; that is to say, in lighter structural work, and not in heavier, and railroad work.

The expression, "if no restrictions are placed on the depth of the girders," must not be interpreted too literally; for, however contradictory this may sound, restrictions to depths of girders in buildings will always exist. Interferences with door and window openings have to be guarded against constantly, and the projection of girders beyond the ceiling line into rooms below has to be looked out for, also, with the same care.

Mr. Wing is, perhaps, unaware of the results, on the other hand, to which the formula he cites for economic depths will lead, if used in "ordinary" building practice.

The formula for economic depth cited by Mr. Wing

$$h = \sqrt{\frac{2\,M^*}{T\,t}}$$

presumes for its application a "practical" thickness of web plate, say $\frac{1}{4}$ in. for building work and $\frac{3}{8}$ in. for bridge work.

In heavier work, when indeed no restrictions whatever hinder the designer, this formula will give proper results, and the economic depth found thereby, for a particular case, should be aimed to be realized, if possible.

Concerning Mr. Szlapka's formulas† for economic depths, it should be noted that they are based on different assumptions, and derived in another manner.

The depths he finds are about the same as those mentioned by the writer. (Page 562.)

Mr. Szlapka's formulas cover four cases:

Case I.—One-ninth of web allowed as flange area; no cover plates, $h = \frac{1}{8} L$;

" II.—No allowance made for web; no cover plates, $h = \frac{1}{7} L$;

" III.—One-ninth of web allowed as flange area; girder with cover plates, $h = \frac{1}{9} L$;

" IV.—No allowance made for web; girder with cover plates, $h = \frac{1}{8} L$.

As to the excessive deflections presumed by Mr. Wing to take place in girders, the depths of which are derived by the formula,

$$h = 0.6\,L + 3 \text{ ins.},$$

the writer will state that they are well within the safe limit established by the standard rule governing them in building practice.

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* See page 588. It may be mentioned that Winkler finds $h = \sqrt{\frac{3\,M}{T\,t}}$. See his "Theory of Elasticity and Strength," Prague, 1867, page 220.

† *The Engineering Record*, March 27th, 1897.

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This rule states that the deflection of a beam or girder shall not exceed $\frac{1}{360}$ of the length of the span, or $\frac{1}{30}$ in. per foot of span.*

This rule, in algebraic form, reads,

$$h = 0.497\ L; \text{ or, better, } h = 0.5\ L,$$

in which h is in inches and L in feet. Although this rule is looked upon as a thoroughly reliable and safe one, being, in fact, a standard in building practice, and recommended as such to architects as a safe guide; it will be found, nevertheless, that in their beam tables all the mills differ from it in excess.† This leads to the adoption of deeper and heavier beams than circumstances require. It is evident that the excess of material put into floor beams in this manner will be a considerable item, particularly in the modern type of tall buildings.

The derivation of this formula is very simple. From applied mechanics it is known that a girder supported at the ends, uniformly loaded, and with constant cross-section, has a deflection (in inches) $= \frac{5}{384} \frac{q l^2}{E I}$;

wherein, E = modulus of elasticity = 29 000 000 lbs. per square inch;
I = moment of inertia of the cross-section;
q = load per inch of length;
l = span of beam or girder, in inches.

It is known, further, that the bending moment and moment of inertia have the following relations:

$$M = \frac{2\ I\ T}{h}, \text{ or } I = \frac{M\ h}{2\ T}$$

also, $$M = \frac{q\ l^2}{8}, \text{ whence } I = \frac{q\ l^2\ h}{16\ T}.$$

Introducing this last term in the formula for deflection,

$$\text{deflection} = \frac{5\ T\ l^2}{24\ E\ h}.$$

As stated before, this deflection should be $\leq \frac{1}{360}\ l$; equating now, we obtain

$$\frac{l}{360} = \frac{5\ T\ l^2}{24\ E\ h},$$

introducing in this equation the known quantities, T = 16 000 lbs., E = 29 000 000 lbs.,

we obtain, $h = \frac{12}{290}\ l$ .....(h and l both in inches)

* This rule is mentioned in all mill handbooks and treatises on building construction, and the writer thinks Tredgold was its originator. No rule, limiting the deflection of railroad bridges, seems to exist.

† Except the tables in the "Handbook of the Cambria Steel Company."

Mr. Schmitt. If it is desired to derive the depth of the girder in inches, when the span L is given in feet, then, put $l = 12\ L$;

whence, $$h = \frac{144}{290}\ L = 0.497\ L\ ;$$

or, better, $$h = 0.5\ L.$$

In words, one-half of the number which expresses the span of the girder in feet gives the depth in inches for a beam which will not deflect more than $\frac{1}{360}$ of its span.

Dividing, now, the exact depth required by the depth determined by the empirical rule, their ratio taken inversely will be the ratio of the deflections for the two cases.

We have (all dimensions in inches),

exact depth, $$h = 0.5\ \frac{l}{12},$$

empirical " $$h = 0.6\ \frac{l}{12} + 3,$$

and $$\frac{\text{Empirical deflection}}{\text{Exact deflection}} = \frac{0.5\ \frac{l}{12}}{0.6\ \frac{l}{12} + 3},$$

or reduced, $$\frac{\text{Empirical deflection}}{\text{Exact deflection}} = \frac{0.5\ l}{0.6\ l + 36}$$

Assuming, now, for example, $L = 30$ ft., $= 360$ ins., the foregoing ratio would be $= \frac{5}{7}$, and the limiting deflection, in terms of the length of the span, would be,

by the exact formula, $$= \frac{1}{360}\ l,$$

" empirical " $$= \frac{5}{7} \times \frac{1}{360}\ l = \frac{1}{504}\ l.$$

Even if the constant 36 is neglected, in the general formula for ratio of deflections, the last fraction would yet be $= \frac{1}{432}\ l$, as against $\frac{1}{360}\ l$.

The foregoing will prove, therefore, that the empirical rule given for depths of girders, in ordinary building practice, ought not to be objected to.

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No. 897.

ADDRESS AT THE ANNUAL CONVENTION AT NIAGARA FALLS, N. Y., JUNE 25TH, 1901.

A CENTURY OF CIVIL ENGINEERING.

By J. James R. Croes, President, Am. Soc. C. E.

The century which has just passed, the nineteenth of the Christian era, is distinguished from any of the preceding hundred-year periods in the world's history by the advances made in the co-operation of investigators in numerous branches of science in the formulation of doctrines regarding the nature and co-ordination of natural phenomena, which stand the test of experiment and calculation, thus leading to a nearer approximation to the understanding of the laws which govern such phenomena and so to the development into a profession of the "Art of directing the great sources of Power in Nature for the use and convenience of Man," which Art is entitled Civil Engineering. This definition is itself one of the most noteworthy products of the Nineteenth Century, and a study of the sequence of events and reasoning which led to its formulation is not without interest.

Ever since man became endowed with consciousness and the power of reasoning, he has been striving to solve the problems of the physical world around him in which he perceived matter in motion, which was

evidenced to his senses by sight and touch, by sound and taste and smell, but which was devoid of sentience, so far as he could discover. He observed at once that by its different manifestations his physical comfort was materially affected, and it did not take long for him to learn that certain sequences of sensation, of one sort or another, followed certain manifestations occurring singly or in combination, and then that the order of many such manifestations could be controlled by him at will, while that of multitudes of others could not be so controlled at first, their methods and causes not being appreciable by his unassisted senses.

But until about three hundred years ago there does not seem to have been any systematic and well-directed effort to investigate the reasons why material changes occurred naturally, or how certain changes could be artificially produced with certainty.

To discover the sequence of natural events is first of all an empirical task: facts must be observed systematically and recorded and then their combinations reasoned upon. Hypotheses can then be formed as to the probable effects of slightly different collocations and sequences of events, and these subjected to experiment and computation. An hypothesis cannot be established as a scientific fact until it has been verified by observation and also proven to be in accordance with mathematical laws. Tennyson's apothegm, that "knowledge comes, but wisdom lingers," is profoundly true.

Now, up to the beginning of the seventeenth century of the Christian era, there had been no organized physical experimentation, on a comprehensive scale, and intelligent reasoning therefrom. The speculations as to the laws of Nature which had been made from time to time had been purely efforts of the imagination, and were unsustained by either practical demonstration or analytical reasoning. Various hypotheses had been framed, on insufficient or incorrect premises, and some of them had been so near the truth as to actually delay the progress of the truth, by the appearance of exactness in the reasoning from them, up to a certain point.

Really, the civil engineer had been practicing his art and directing the forces of Nature for the use and convenience of man, but without any conception of what those forces were, or how they acted, or of why he did anything, or what the result of it would be, unless he had done the same thing before in the same way. He did not know that

the earth moved, and he had no idea why or at what rate a stone fell to the earth, or water ran down hill. He had no measure of heat or light, and used no power but that which the muscles of an animal produced. And yet he had built the Pyramids, the Parthenon and the Pantheon; had constructed aqueducts, canals and sewers; had regulated and maintained the rivers of China for thousands of years; and had just been recognized, on account of his labors in protecting the low lands of Holland and the shores of Italian rivers from the encroachments of the water, as holding a distinct rank among the workers of the world.

Of the application of the forces of Nature to aiding his labors, he seems to have been ignorant, except by the use of a flowing stream to turn a wheel. The earliest recorded application of this mode of producing power in England was in 1582 when Peter Moryss, a Hollander, procured from the City of London a franchise for five hundred years for supplying water to the city by pumping from the Thames, using a wheel driven by the ebb and flow of the tide under London Bridge.

With the beginning of the seventeenth century, the world entered on a new era of science, theoretical, and applied. The casual observation by a little child, of the curious optical effect obtained by looking through two pieces of glass, led to the invention of the telescope and the microscope, which disclosed to man objects beyond the reach of his unassisted vision. The swinging of a hanging lamp suggested to a thoughtful man the idea of an unseen force, and Galileo, by experiment and reasoning, discovered the law of terrestrial gravitation and first grasped the idea of force as a mechanical agent.

Turning his newly-invented telescope upon the heavens, he was the first man in the world to witness the actual motion of the planets and their satellites, and to prove that they and the earth revolved about the sun, as Pythagoras had imagined two thousand years before, Copernicus had asserted a hundred years before, and his own contemporary, Kepler, had reasoned from the imperfect data then possessed and had actually formulated the laws of their motion in forms which the science of our day confirms exactly. He was the first to conceive the theory of transverse strains in solids, but the facility of experimentation with fluids diverted his attention from more rigid bodies, and the fundamental principles of hydraulic science were established

by him and by his pupils and immediate successors in the fascinating studies he had introduced.

To Galileo, also, is due the invention of the thermometer, which enabled definite measurement to be made of the mysterious phenomenon of heat which his great contemporary philosopher in England, Lord Bacon, conceived to be "an expansive undulatory motion in the particles of a body whereby they tend with some rapidity toward the circumference, and also a little upward." But Bacon was two hundred years in advance of the physicists, and the century was occupied almost exclusively in the elucidation of the laws of gravitation as exemplified in the action of fluids. For the study of liquids and their action demonstrated that they were, under certain conditions of temperature, transformed into invisible and elastic substances governed in general by the same laws, and also that there were all around other similar substances which could not be condensed into liquid inelastic form, but could be weighed by the barometer which Torricelli invented in 1643. And so the laws of gases came to be investigated and formulated by Mariotte, who, to aid him in his researches, invented the rain gauge in 1677, and measured the liquids condensed from the atmosphere. The value of the data thus obtained, to the hydraulic engineer, was appreciated by the French engineers at once, and ever since 1681 records of the rainfall have been kept continuously at Paris, and the practice has gradually extended over the whole world. But it is worthy of note that fifty years after the invention of the rain gauge, Belidor, in his magnificent treatise on Hydraulic Architecture (1728), the first compendium of engineering theory and practice, in treating of the sources of water supply for domestic use, did not mention the rain gauge or the amount of rainfall, but dwelt on the divining rod as the recognized means of discovering subterranean streams of water.

The close of this century saw the final establishment of the law of gravitation by Newton's proof of its governing the whole material universe. And any review of the progress made during the century toward the understanding of the laws of Nature would be incomplete without allusion to the two great steps taken in it toward the facilitating of mathematical computations, the invention of logarithms at the beginning of the century and of the calculus at its close.

That heat produced dynamic effects had been recognized for ages.

That it destroyed some solids, that it converted others into different forms possessing entirely different properties, that it caused the dissipation and disappearance of liquids, were facts well established. That by its application to certain combinations of materials an explosive effect could be produced which overcame the force of gravity had been discovered three or four hundred years before when gunpowder was invented, but that a mechanical effect could be produced by the use of heat was not understood, and the nature of the phenomenon itself was not comprehended. It was looked upon by physicists as an "element" or primary form of matter, and in 1690 Stahl conceived the idea of "phlogiston," an elementary substance, invisible and inappreciable by the senses, which entered into the composition of combustible substances, and which, by the process of combustion, was separated from them and passed off in the form of corpuscles which, striking the sensory nerves, were perceived as heat. Sir Isaac Newton espoused this phlogiston hypothesis and also conceived that light was the product of certain corpuscles which were perceived by the optical nerves. The discussion of these hypotheses occupied the attention of the philosophers for the whole of the eighteenth century, and, in the meantime, the physicists were busy experimenting on the methods of utilizing the vapor into which heat converted water. By degrees, the steam engine was developed into a practical machine, capable of doing work which before could only be accomplished by animal labor, and the engineer availed himself of it in the handling of materials. But although the chemists had been striving for centuries to learn the composition of matter and the means of transforming and combining its several natural conditions, they had not, at the beginning of the nineteenth century, learned how to produce either heat or light, except by the aboriginal method of striking flint and steel together. In the lack of knowledge of the properties of these phenomena and the fact that they could be utilized on a large scale, there was no occasion for the devotion of any special class of men to their production and development. There were two other classes of phenomena which seemed as if they ought to be controlled by man, but the laws of which had so far eluded discovery, electricity and magnetism. So that, really, all that the civil engineer had to deal with was the force of gravity acting on such materials as the earth yielded him, in their natural state or as they could be modified by heat and manual labor.

But he had been making progress. In Italy, where the first application of science to construction had been made, the study of the laws of hydraulic science had been constantly pursued, and those laws applied to the regulation of the rivers; in France, ever ready to grasp new ideas and to pursue their application to practical results, the principles of hydraulics had been studied and applied to the construction of great canals, and to the supply of water to Paris, where, in 1671, water was pumped by a water wheel driven by the current of the Seine and distributed through cast-iron pipes. In England, Hugh Myddelton had supplied London with water, and, in 1638, Nicholas Vermuyden had been called in from Holland to protect the lands along the River Ouse from overflow, a task so well accomplished that his work stood for a hundred years and only failed then from lack of proper maintenance, the English having by that time apparently concluded that, as Dr. Franklin wrote in 1772, "rivers were unmanageable things," and, inspired by the success of the public waterways of France, turned their attention to the construction of canals and the improvement of harbors. Many important works of this class were built in England during the last half of the eighteenth century. The men by whom these works were constructed were not educated men or men experienced in scientific research. They certainly were men of great natural ability and good judgment, and capable of conceiving and executing great projects. One of the greatest of them, John Smeaton, who was the first Englishman to call himself a civil engineer, thus expressed his conception of the profession which he adorned:

"Civil engineers are a self-created set of men whose profession owes its origin not to power or influence, but to the best of all protection, the encouragement of a great and powerful nation, a nation become so from the industry and steadiness of its manufacturing workmen and their superior knowledge in practical chemistry, mechanics, natural philosophy and other useful accomplishments."

Smeaton was himself an investigator, but he is the only one of the civil engineers of Great Britain during the eighteenth century who strove to discover the laws which governed the operations of Nature.

The most eminent civil engineer in England in the year 1800 was Thomas Telford, who was born in 1757. Beginning life as a mason, he developed an extraordinary faculty of generalization, combined with an intimate acquaintance with the details of workmanship in all the methods of construction known in those days. In the building

of canals, highways, harbors, bridges and docks he displayed great grasp of the subject of the improvement of transportation facilities, as then existing, and great boldness of design and ingenuity in construction.

But it must be borne in mind that at that time the canal was considered the only possible mode of increasing facilities of transportation and reducing cost, no motive power except animal force was known, the metals were but little used in construction, and a framed structure adapted to bear heavy loads was unknown. As "an eminent mathematician," quoted in the *Edinburgh Review* in 1805, remarked:

"While we give ourselves infinite trouble to pursue investigations relating to the motions and masses of bodies which move at immeasurable distances from our planet, we have never thought of determining the forces necessary to prevent the roofs of our houses from falling on our heads."

It is related of Telford that when on one occasion he was consulted by a young man as to the advisability of his engaging in civil engineering, he said to him: "I have made all the canals and all the roads and all the harbors. I don't see what there is that you can expect to do."

His ideas regarding the training of the civil engineer are given at some length in his Personal Memoirs prepared shortly before his death.

"Youths of respectability and competent education who contemplate Civil Engineering as a profession, are seldom aware how far they ought to descend in order to found the basis of future elevation. Not only are the natural senses of seeing and feeling requisite in the examination of materials, but also the practised eye, and the hand which has experience of the kind and qualities of stone, of lime, of iron, of timber, and even of earth, and of the effects of human ingenuity in applying and combining all these substances, is necessary for arriving at mastery in the profession. For how can a man give judicious directions unless he possesses personal knowledge of the details requisite to effect his ultimate purpose in the best and cheapest manner?"

"It has happened to me more than once, when taking opportunities of being useful to a young man of merit, that I have experienced opposition in taking him from his books and his drawings and placing a mallet, chisel or trowel in his hand, till rendered confident by the solid knowledge which experience only can bestow, he was qualified to insist on the due performance of workmanship and to judge

of merit in the lower as well as the higher departments of a profession in which no kind or degree of practical knowledge is superfluous."

This is doubtless good, sound doctrine, but it does not betoken any very lofty conception of the aims and ends of the profession. But during the first quarter of the nineteenth century Telford stood at the head of the profession of civil engineering, and when, in 1820, the recently formed association of its practitioners for mutual advancement in science, which was termed The Institution of Civil Engineers, desired a prominent leader, he was chosen its President, and held that office until his death in 1834. He does not appear to have contributed to The Institution any papers or discussions on engineering subjects.

Among the members of The Institution at that time was a man thirty years the junior of the President, who, like him, had risen from humble origin, and by his own exertions attained a high rank in the profession, and who, rather oddly, had the same fore-name, and a surname of two syllables the initial and final letters of which and the vowel sounds of which were the same as Telford's. This similarity of name has led to some confusion and sometimes to the attributing to one of these men the sayings and doings of the other.

Thomas Tredgold, born in 1788, began life as a carpenter, but soon devoted himself to the study of engineering science and its practice in the office of the Chief Engineer of the Ordnance Bureau. He early recognized the deficiency of the knowledge then existing as to the nature and strength of the materials used in construction, and he studied, experimented and reasoned systematically, and published the results of his labors. His Treatise on Carpentry, in 1820, was the first published attempt to determine scientifically and practically the data of resistance of beams to transverse flexure. During the next seven years he contributed to the *Transactions* of the Institution of Civil Engineers papers showing the mode of application of science to engineering problems, and he also published treatises on Warming and Ventilation, on Steam Navigation, on Railroads and Carriages, and on the Steam Engine. It was to him that the Institution turned when it wished to apply for a Royal Charter, in 1828, and requested him to prepare a definition of Civil Engineering.

As we look back upon the history of science, theoretical and applied, during the first quarter of the nineteenth century, we can

see how a new definition of the profession of Civil Engineering was needed at that date.

At the very beginning of the century there had occurred a marvelous revolution in the conception of the nature and operation of the laws governing matter and its motion. Lavoisier had revolutionized chemical science and Dalton had propounded a theory of atomic constitution of matter which has been sustained by observation and reason. All matter is composed of a few primal elements in an atomic or minutely subdivided form. These atoms have varying chemical affinity for each other, and, combining in certain proportions, form molecules of matter of various kinds. The study of these combinations has been the business of chemists for the last hundred years, and the laws of combination have been so successfully elucidated that many forms of matter which before were found only in a state of Nature can now be artificially produced, and many other forms have been produced which are never found in Nature and which are useful for purposes and under conditions where no natural product can be used to advantage. The impetus given to chemical research by the formulation of Dalton's theory was sufficient to establish the fact, early in the century, that chemical affinity was a source of power which could be directed by man intelligently and with prospect of advantage.

Just with the incoming of the century, too, came Rumford's demonstration of the fact that heat was not a material substance, but only a mode of motion. Almost simultaneously was propounded the theory of Thomas Young, that light, too, was not material, but was simply due to vibratory motion in an all-pervading medium to which he gave the name of the luminiferous ether.

It was in 1800, too, that Volta demonstrated that an electric current could be artificially produced. How it could be controlled and applied to practical use did not yet appear, but a new direction had been given to the minds of those engaged in physical research.

As it became manifest that chemical affinity and heat and light could be controlled and directed and converted into Energy, as Young termed it, the men who had been trained in utilizing the force of gravity turned their attention to the development of these newly understood sources of power. Fitch and Fulton, with the aim of reducing the cost of water transportation, succeeded in applying

the steam engine to the propulsion of boats, and Trevithick made successful application of steam propulsion to vehicles on land. Murdock had proved that illuminating gas could be produced and distributed to consumers. The civil engineers of the day had seized on all these inventions and discoveries, and in both Europe and America were designing and constructing works to render them useful to the greatest number of people.

Reviewing then what had been accomplished during the first quarter of the century, Tredgold could not but perceive that civil engineering was something broader and more comprehensive than the mere construction of harbors, breakwaters and canals, and he presented on January 4th, 1828, in response to the request of the Institution, this ever memorable definition of Civil Engineering:

*"Civil Engineering is the art of directing the great sources of power in Nature for the use and convenience of man; being that practical application of the most important principles of natural philosophy which has, in a considerable degree, realized the anticipations of Bacon, and changed the aspect and state of affairs in the whole world."

After a brief sketch of the objects of Civil Engineering, he added: "The real extent to which it may be applied is limited only by the progress of science; its scope and utility will be increased with every discovery in philosophy, and its resources with every invention in mechanical or chemical art, since its bounds are unlimited, and equally so must be the researches of its professors."*

A more concise and comprehensive definition of a great truth can hardly be conceived. From a physical and intellectual standpoint, a nobler aim for the exercise of the mental powers cannot be imagined than the direction of the great sources of power in Nature for the use and convenience of man. Psychology deals with mind alone, Physics considers the nature and the laws of matter, but Civil Engineering treats of the intelligent direction of the laws governing matter so as to produce effects which will reduce to a minimum the time and physical labor required to supply all the demands of the body of man and leave more opportunity for the exercise of the mental and spiritual faculties. Philosophy, Physics and Civil Engineering must work hand in hand. The philosopher must imagine, the physicist prove by experiment and mathematical computation, and the engineer apply to practice the laws of matter. Each must keep himself informed of the progress made

* *Minutes of Proceedings*, Institution of Civil Engineers, Vol. xxvii, p. 181.

by the others and must aid them by suggestions as to the lines on which research needs to be carried forward. The civil engineer, in attempting to solve some problem of construction, finds that he needs a material which shall possess a certain quality which he cannot discover that any natural product possesses. He calls the chemist to his aid, and he, from a study of the combinations of existing forms of matter which most nearly approach the desired ideal, reasons that some special combination of elements will entirely fulfil the conditions, and he experiments to find whether such combination can be made. Sometimes he is successful in his first attempt and sometimes not. But, whatever the result, he has added to his knowledge of the laws of combinations and has furnished to the philosopher fresh data for his generalizations and to the engineer a new material for his use.

It not infrequently occurs that, from investigations made for a specific purpose, results are obtained of the greatest usefulness for an entirely different purpose. Thus, in 1855, when Henry Bessemer, inspired solely by that desire "to kill something" which is alike the ruling passion of the rudest savage and the most highly civilized man, bent all his energies to the production of a metallic combination which should be able to resist the force of the highest explosive and so enable a cannon ball to be projected from a gun to a greater distance than ever before, he discovered a method of expelling all foreign substances from iron and then adding a minute quantity of another element, carbon, in such proportions that the original mass was materially changed in character and made more ductile, stronger and stiffer. The product was exactly what the railroad engineer wanted at that time for the bearing surface of his roadway, and the material which had been sought for destructive purposes became a most important factor in facilitating the transportation of men and goods with certainty and safety at high speeds.

As the knowledge of the nature of steel and the precise methods in which it can be manufactured have progressed, the engineer has gradually come to know just what he wants and how it can be produced, and, in his specifications, requires that the particular material of this class which he desires shall be of a certain chemical composition and also possess certain characteristics. The same is the case with almost every material which enters into the construction of engineering works of the present day. Matter in its original state is rarely used. Its

chemical condition must be transformed before the engineer can utilize it with any confidence. That almost any desired transformation can be affected was not realized until late in the century. Starting with the atom, the ultimate particle of matter so far comprehended by us, the chemist found that several different kinds of atoms could be identified, and that these would combine in certain ways according to laws which could be formulated. But in the application of these laws and the tabulation of the results gaps were found to exist which could not be filled without the supposition that other elements existed than those already known. The existence of such elemental substances was confirmed by the revelations of the spectrum analysis, and, later on, several of such elements have been actually identified by the use of the electric current in creating vibrations in the ether. The limit is probably not reached yet, but as each new element is discovered its affinities are sought by the chemist, its sensibility to various forms of vibratory motion are investigated by the dynamist, as we may term the physicist who is seeking the laws of either heat or light or electricity, and then it is the function of the civil engineer to study how it can best be applied to the use and convenience of man. For, ever since the beginning of the nineteenth century, the evidence has been cumulative that matter in motion accounts for all physical phenomena, that motion produces energy, that energy is never wasted but is simply transformed, and that it manifests itself to the senses of man in various modes which are appreciable by the several organs of sense.

What our senses recognize as chemical affinity, heat, light and electricity are simply conditions of matter induced by vibrations or quivers or waves or strains, whatever we may call them, of different kinds and at different velocities. Neither matter nor motion can be originated by man, but, by a careful study of the sequence of events, control can be acquired of their modes of interaction, and natural phenomena can be artificially reproduced and other phenomena be produced. The intelligent application and direction of such means of control is the function of the civil engineer.

With the advance of science, the scope of civil engineering widened and advanced. The study of the action of forces induced analytical investigation of the means by which forces could be resisted and the best results obtained from proper distribution and arrangement of materials of different kinds. Steamships and locomotive engines were

constructed by which the products of the earth and the manufactures of man, by machines and methods not before conceived, could be transported across the seas and overland by artificial highways and across bridges of previously unimagined span; and light and heat and electricity and water could be delivered in the apartment of every person to be used at will.

It was in the carrying out of the delivery of pure and wholesome water and the removal of its unused surplus that the civil engineer first was called on to deal with organic life. That minute organisms affected the comfort and the health of man had been recognized for hundreds of years. In the middle of the seventeenth century Leeuwenhoek, a Dutch maker of microscopes, discovered and described bacteria, and Nicholas Andry, a pathologist, ascribed to them the causation of disease. But later scientists discarded the idea, and it was not until 1831 that any real advances were made in the study of these micro-organisms, and it is only within the last twelve years that it has become thoroughly recognized that the regulation of the growth of living organisms in air and water and sewage is necessary and practicable, and comes within the domain of civil engineering.

Indirectly, however, biological research has been one of the most important factors in the progress of engineering science, by calling the attention of students of physics to the fact that advance and not retrogression is one of the fundamental laws of Nature. For the first half of the century, the old ideas of cosmogony, based on an hypothesis unsupported by proof, were prevalent everywhere. It was assumed that the world, in all its details, had been created perfect and had since been simply deteriorating and tending to a final dissolution. "Change and decay, in all around I see" was the dogma of the theologian, the philosopher and the scientist alike. While it had come to be recognized that the forms of inorganic matter could be changed by man, and that, by the exercise of man's intelligence, certain characteristics of organic matter and the vital forces with which it was imbued could be modified and perpetuated, it was not considered possible that the superior intelligence which controlled everything could modify or transform such characteristics in any special form of matter.

But in 1859, Charles Darwin, after twenty years of study of the sequence of events in biological phenomena, demonstrated that there was an intelligence beyond that of man, which was constantly acting

to change and modify the forms, the habits and the mode of life of animals and plants, and that such action resulted in the perpetuation of the fittest type of organism. The proof was irrefragable, and the effect of his wonderfully clear exposition of the processes by which his conclusions had been reached was marvellous in inducing a co-ordination of thought and a co-operation in methods of procedure in intelligent research in the investigation of all natural phenomena, whether relating to organic or inorganic matter.

In considering the means of directing the great sources of power, the psychological element must not be forgotten. A mere intellectual application of the laws discovered by physical research is not enough to make a civil engineer. Breadth of view, the faculty of analyzing what has been done so as to discover how and why some enterprises have been successful and others have not, and the ability to forecast the future, are essential. These qualities are largely natural, but may be cultivated to a great extent by study and experience. That there has been a wonderful advance in this direction during the nineteenth century is shown by the great number of civil engineers who hold positions of prominence in the management and control of large enterprises which require the exercise of faculties which cannot be acquired in any other way than by experience in the designing, construction and management of engineering works.

A prominent factor in causing this advance in engineering science which has occurred simultaneously on the Continent of Europe, in Great Britain and in America, has been the collaboration of scientists. Early in the century it became evident that the multiplication of lines of research demanded a differentiation of the labor of their prosecution and a close co-operation of the workers in any special line, and various associations of specialists were formed to promote various branches of scientific research. By the middle of the century it had become apparent that Civil Engineering was not the prosecution of a specialty, but was the co-ordination and direction of the work of all specialties in Science and its applications. And so in 1852, James Laurie and his associates, following the example of their English brethren, founded The American Society of Civil Engineers, the first and only National Organization devoted to "the professional improvement of its members, the encouragement of social intercourse among men of practical science, the advancement of engineering in its

several branches and the establishment of a central point of reference and union for its members." To the privileges of its membership may be admitted not only every "professional engineer," but also "any person who by scientific acquirements or practical experience, has attained a position in his special pursuit qualifying him to co-operate with engineers in the advancement of professional knowledge and practice." This meeting of that Society, which now has 2 500 names upon its rolls of membership, and owns a commodious society house with a reference library of some 40 000 titles, is sufficient proof of the wisdom of its founders.

Recognizing, then, that progress is a law of Nature, the acceleration of progress is the aim of Civil Engineering. It strives to simulate the results of the slow processes of Nature, by causing the sources of power to act rapidly in any desired direction. Appreciating, too, the fact that there is constant progress, and that what now seems admirably adapted to our needs may in a short time require to be superseded by improved structures and processes, the tendency of the time is toward the production of works which will have a definite term of life, rather than toward the construction of everlasting monuments. We see that in the old nations, where the effort to build for eternity was made, time has outstripped the intent of the builders and what is antiquated is useless, and we see the same thing in our own streets to-day. The idea of building a monumental structure which will hand one's name down to future ages is a fascinating one, but it is simply a survival of the engineering of the Pharaohs.

The most thorough exemplar of the condition of civil engineering at the beginning of the twentieth century is the modern office building in a great city. One hundred years ago, the man of enterprise who resided 50 miles from a large city and wished to consult an engineer regarding a project for a new canal, arose before daylight, struck a spark from his flint and steel, which falling on a scrap of tinder was blown by him into flame and from that a tallow dip was lighted. In the same primitive manner, the wood fire was kindled on the kitchen hearth and his breakfast was cooked in a pot and kettle suspended from the iron crane in the fireplace. Entering the cumbrous stage coach, hung on leather springs, which passed his door, he was driven over muddy roads, crossing the narrow streams on wooden trestle bridges and the navigable rivers on a ferry boat, the paddle wheels of

which were turned by a mule on a treadmill. At last he was landed in the city where he walked through dirty streets paved with cobble stones until he reached his destination, a plain three-story brick building founded on sand, with a damp cellar and a cesspool in the back yard. Entering a dark hall he climbed a wooden staircase and was ushered into a neat room, rag-carpeted, warmed by a wood fire on the open hearth and lighted by a sperm oil lamp with one wick, for it was dark by this time. No wonder that before proceeding to business he was glad to take a good stiff noggin of New England rum.

To-day, his grandson, living at the old homestead, while comfortably eating his breakfast, which has been cooked over a gas range, reads in his morning paper that the high dam of the irrigation reservoir in Arizona, in which he is interested, sprang a leak the day before, and he telegraphs to his engineer in the city that he will meet him at his office at noon. Then, striking a match, he lights the lamp of his automobile which is fed by petroleum brought 200 miles underground in pipes from the wells, rolls over macadamized roads to the railroad station, where he boards a luxuriously appointed train, by which he is carried above all highways, through tunnels, under rivers, or across them on long-span steel bridges, and in an hour is deposited in the heart of the city, where he has his choice of proceeding to his destination through clean and asphalt-paved streets in electric surface cars at 9 miles an hour, elevated steam cars at 12 miles an hour, or through well-lighted and ventilated tunnels at 15 miles an hour. Reaching the spot his grandfather had visited, he finds there a huge and highly decorated building, twenty or more stories high. Founded on the primeval rock, far below the surface of the natural ground, the superjacent strata of compressible material having been penetrated by caissons of sheet metal sunk by the use of air, compressed by powerful pumps driven by steam or electricity generated at a power station half a mile or more away, and these caissons filled with a manufactured rock such as the ordinary processes of Nature would require millions of years to produce, there is erected a cage of steel, the composition of which has been specified, and the form and mode of construction of which have been so computed that the force of the elements cannot overthrow the structure or even cause it to sway perceptibly. Towering above the courts of Law, the temples of Religion and the palaces of the Arts, the meshes of this mighty cage are filled

with products of the earth, the mine and the forest, transformed so as to be strong and light and incombustible, and all interwoven with pipes and wires, each in its proper place and noted on the plans. In one set of these pipes there is pure water, which has been collected from a mountain area of igneous geological formation, depopulated and free from swamps, on which a record of the daily rainfall is kept, and in which impounding reservoirs have been constructed by masonry dams across its valleys. From these reservoirs, the water, after filtration through clean sand, is conveyed 30 or 40 miles through steel or masonry conduits to covered reservoirs whence it is drawn as needed through cast-iron pipes to the building where it is to be used, and there distributed to all parts of it, chilled nearly to the freezing point through one system of pipes or heated nearly to the boiling point, through another system. Another set of pipes carries steam which, passing through radiators, keeps the temperature of the air throughout the building at the proper standard for comfort. Sanitary conveniences are provided everywhere, and all wastes are consumed within the building by the surplus heat generated, leaving only ashes to be removed. Wires convey electric currents to all points, so that the occupant of a room, sitting at his desk, can by the touch of a button ventilate his apartment, illuminate it, call a messenger, be kept informed of every fluctuation in the markets, converse with anybody who is not "busy" within 40 miles of where he sits and if entirely "up to date" can require his autograph and portrait to be reproduced before his eyes for identification. He dictates his correspondence and his memoranda, and "takes his pen in hand" only to sign his name. He need not leave his seat except to consult the photograph hanging on his wall, which shows to him the latest condition of the mine, the railroad, the arid lands irrigated, the swamps reclaimed, the bridge in progress, the steamship, the water-works, the tunnel or the railroad, the dam, the filter or the sewage works, the town, the machine, the power plant or the manufacturing establishment in which he is most interested.

Entering the brilliantly lighted hallway of this building, the air of which is kept in circulation by the plunging up and down of half a dozen elevators, the visitor is lifted at a speed of 500 ft. a minute, past floor after floor, crowded with the offices of financiers, managers and promoters of traffic and of trade, lawyers, chemists, contractors,

manufacturers, to the headquarters of the controlling genius of the whole organism, the Civil Engineer. For he it is to whom all the members of this microcosm must apply for aid and advice in the successful operation of their respective occupations. It is not his to mechanically transform elements into matter, or matter into other forms, or to show how energy may be produced, but to direct the application of energy to the various forms of matter, original or produced, in such way as to bring about the most satisfactory results in the most speedy and economical manner.

He has grown with the growth of the nineteenth century, and is, so far as the relations between man and matter are concerned, its most striking product. And so, while the definition given in the "American Edition of The Encyclopædia," which appeared at the beginning of the century, that: "Civil Engineers are a denomination which comprises an order or profession of persons highly respectable for their talents and scientific attainments and eminently useful under this appellation," is still true, it is hardly probable that the compiler of the Twentieth Century Encyclopædia will be content to let it stand without further explanation.

But the end is not yet: there are still many problems of Nature unsolved. The experience of every day shows that there are sources of power not yet fully developed, and we cannot but say with the great poet:

> "I doubt not through the ages one increasing purpose runs,
> And the thoughts of men are widened with the process of the suns."

MEMOIRS OF DECEASED MEMBERS.

JAMES COLWELL ALDRICH, M. Am. Soc. C. E.*

DIED APRIL 3D, 1900.

James Colwell Aldrich was born at Chopmist Hill, Scituate, Providence County, R. I., on July 24th, 1838. The land comprising Chopmist Hill lies on the west side of the seven-mile line included in the original ground purchase of Providence Plantation, which was laid out and conveyed by a committee of the early proprietors, consisting of Henry Harris, of Providence, and John Walton and Richard Steere, of Gloucester, the last named being Aldrich's ancestor and a Justice of the Court of Common Pleas for the County of Providence during many years of the reign of George II.

James C. Aldrich's father's name was James, and he belonged to the well-known New England family. His mother was Abigail Colwell, whose emigrant ancestor to America, Robert Colwell, was born in Warwickshire, England, in 1630, came to Providence in 1653, where he settled, and removed to Long Island in 1667, where he died. Abigail Colwell's forefathers were educated in the art of war and the kindred sciences. Her grandfather's brother, Colonel John Colwell, commanded the Third Regiment, Rhode Island Colonial Brigade, and her grandfather, William Colwell, married Lucia Arnold, one of the three daughters of Caleb Arnold, the patriot known as the father of the Revolutionary family, consisting of eight sons and three sons-in-law, who served in the Continental Army. One son, Othonial Arnold, received his death wound at the battle of Bunker Hill, and Caleb Arnold, the patriot, was sent as Deputy to the Rhode Island General Assembly from 1773 to 1778, was paid State bounties in 1778, and served on the War Committee and the Committee for Recruits in 1780. This family of Arnolds is of great antiquity, having their origin among the ancient Princes of Wales, and, according to a pedigree recorded in the College of Arms, they trace from the second son of Cadwalader, King of the Britons.

James C. Aldrich was graduated from the Scientific Department, Dartmouth College, in the Class of 1859, and immediately secured a position on an engineering party commanded by Colonel Silvester Mowry, fitting out at Providence, to journey overland from St. Louis to Arizona. Indian hostilities caused delays, and, tired of waiting, he sailed on the ill-fated steamship *North Star*, *via* the Isthmus, for California.

* Memoir prepared by Miss D. Aldrich.

After an eventful passage of disaster and danger, he arrived in San Francisco, where he served for a while in the Land Department of the Law Office of Giles H. Gray.

He taught school at the Mission of San José, of which C. L. Beard, who, with others, afterward founded and built Washington College, near that locality, was a Trustee. While thus engaged, Colonel Fred. Lander tendered him a position on the overland wagon road surveys,* in which Colonel Lander took command of all available forces. He changed the programme, however, and, later, came by the old Butterfield Route, Los Angeles and Ft. Yuma, up the Gila River, through Tucson, Apache Pass, El Paso, Ft. Davis and Ft. Chadbourn, to the vicinity of old Ft. Graham, Texas.

This was after the John Brown raid, and civil war was imminent. He then pushed on through the Indian Territory and Ft. Smith to the terminus of the Pacific Railroad, then westward only 30 miles from St. Louis.

He next taught in a Military School on the Hudson, and worked at intervals for Chickering and Mapes, Civil Engineers, at 18 Wall Street, New York City. While there he made some of the earlier preliminary surveys, under charter for a railway up the west shore of the Hudson River; but his active life commenced on forming the acquaintance of George P. Hopkins, a civil engineer, by whom he was introduced to Israel Smith, Engineer of the New Jersey Railroad and Transportation Company, under whom he was assigned a position on Hopkins' work, the construction of the Bergen Point Railway.

He next went with the company's experienced Locating Engineer, Edward Everett, on the final location for the Rahway, Woodbridge and Perth Amboy Railway, and in the following spring aided the late I. C. Chesbrough, on the location of the Taunton, Somerset and Fall River Railway. He now became permanently connected with the Engineer Corps of the New Jersey Railroad and Transportation Company (United Division, Pennsylvania System), and for several years during the Civil War served as Assistant Engineer and Superintendent of Construction on new lines and terminals.

He was also employed on the earlier topographical surveys for the maps and plans, to go before the Legislature at Trenton, for the water-works at New Brunswick, N. J., of which E. Millard Smith was Acting Chief Engineer.

His next work was on the construction of the Flushing and Manhasset Railway extension on Long Island.

Mr. Aldrich then became associated with John Houston, Chief Engineer of the Long Dock and the Bergen Tunnel and Erie Railway; Houston acting as Consulting Engineer on all lines seeking connection

*See advices to the Interior Department, from Magraw's Section, Wagon Road Expedition, Indian War.

with the Erie and entrance to New York City by way of the Bergen Tunnel, and Aldrich performing professional duty on location on the rugged line from Hancock through Delaware County, and on the New York and Newburgh, known as the short cut, the Newark and Paterson and the New Jersey Southern, extending to Atlantic City and Penn's Grove, opposite Wilmington, on the Delaware.

On these different roads his professional lines were in conjunction with those of George Earl Church, M. Am. Soc. C. E., later of the Amazon River Improvement, Edward Everett, L. D. Bruyn and William S. Sneden, Chief Engineer and General Manager of the New Jersey Southern Railway.

It was during this era of a busy life that he was called to Brooklyn, N. Y., by the late Benjamin D. Frost, M. Am. Soc. C. E., afterward of Hoosac Tunnel fame, to push with diligence the topographical surveys to be used as a basis upon which Mr. Fred. Law Olmsted was to make a study and design to lay before the Legislature at Albany to advocate the interests of the Brooklyn Park bill.

Mr. Olmsted's plan proved a success, and for over four years, as Division Engineer, Mr. Aldrich gave to this great work his undivided attention.

Mr. Aldrich next went to Chicago, in the interest of Messrs. Olmsted and Vaux, Landscape Architects, on the layout of Riverside, a suburban city. On his return to New York he took charge of the construction of the Spuyten Duyvil and Port Morris Railway (Vanderbilt's Hudson River Railway entrance into Harlem); then he went with Mr. Edward Everett on the difficult location of the Rochester, Mt. Morris and Munda Railway, and in the same winter completed the topographical survey for a study and design for a cemetery at Middle Hope, on the Hudson, for the City of Newburgh. Following after came the nice surveys for the extension of the northern limits of New York City, and then, in company with the late John Houston, the examinations and triangulations of the Passaic River and its tributaries, in the interests of the United Societies of the State of New Jersey.

Mr. Aldrich now made a four-years' engagement to accompany Mr. Houston to Peru and Bolivia, but, on the eve of departure, was allowed to cancel it and accept a Government position, first under the late George H. Elliott, M. Am. Soc. C. E., Engineer Secretary of the Light-House Board, U. S. A., and later as Aide and Assistant Engineer to General O. E. Babcock (Engineer of Public Buildings and Grounds, Chief Engineer of the Washington Aqueduct and Private Secretary to President Grant).

Mr. Aldrich was elected a Member of the American Society of Civil Engineers on May 7th, 1873. In October, 1873, he was appointed

Assistant Landscape Engineer, Department of Public Parks, New York City.

After completing this engagement he went to Rhode Island and entered upon the task of tracing a double-track, medium-grade railway location from tide water to some point on the Connecticut River between Windsor Locks and Springfield, Mass., to be constructed in the combined interest of the Connecticut-Poughkeepsie Bridge System, Boston and Albany, and Vermont Central Railways. This was a project which had enlisted the services of the late I. C. Chesbrough, as early as 1863, but as there were no topographical data of the intervening territory, and as all previous profiles had ended high in the air, the route was generally considered impracticable. Aldrich, however, from his earlier acquaintance with its New England features, was prone to believe in its available possibilities, and associated with himself the veteran locator, Edward Everett, and A. D. Beach, who had already made such a success of stadia measurements on the canal surveys across the Isthmus. He entered upon an exhaustive examination of the three most probable lines, and finally established a favorable and scientific location. Afterward, he rendered material aid in securing the charters in the three States.

He next accepted a position on the working up of the maps, plans and contracts of the Riverside Avenue Improvement, Department of Public Parks, New York City, and was appointed Superintending Engineer, which position he retained for two years, until a change occurred in the political complexion of the Park Commission. After this (in the interest of Austin Corbin and Western bankers), he took charge of a heavy railway construction on the south side of Long Island.

He was now tendered two positions, one in an unhealthy portion of Mexico and the other on a mountainous line to connect with the Santa Fé System. But the effects of past exposure and the inroads of chills and fever on an extremely sensitive organization had begun, and his physician called a halt.

Ambitious and active, his work had required brave bodily toil, courageous persistence and studious thought. It cost him, too, what may go to hearten us, and what we are apt to forget—frets, worries and contentions—and he learned, in the school of many teachers, benign forbearance and self control. Perfect integrity was observed in all his affairs. His last feeble years were passed at the homestead where he was born, and which he took great pleasure in improving.

EDWARD ARNOLD CORREA, M. Am. Soc. C. E.*

DIED JUNE 24TH, 1900.

Edward Arnold Correa, only son of Gabriel A. and Sarah Woods Correa, was born in the Island of St. Thomas, Danish West Indies, on November 7th, 1855.

His father was a gentleman of culture, with marked musical taste, and, having had the benefit of a European education and of extensive travel, was a linguist of no mean ability, speaking five or six languages fluently.

Mr. Correa's mother was of English parentage, her father having graduated at Oxford College. He was a planter, owning estates in St. Croix, Danish West Indies, which were quite valuable until the emancipation of the slaves by the Danish Government and subsequent disorder destroyed their value.

When but seven years old Mr. Correa's father died, and his mother, considerably reduced in fortune, came with her young son and daughter to the United States, locating in Brooklyn, N. Y., where the subject of this memoir attended the public schools until his fifteenth year. This was practically all the schooling he received, as his mother then removed to Iowa; but he had already earned the reputation of being a bright pupil and a voracious reader. He was employed in a bookstore at Mt. Pleasant, Iowa, for a few years, and, continuing his studious habits, improved his mind by reading the best books.

He entered the offices of the Chicago, Burlington and Quincy Railroad in 1879, without preliminary training, and, by close application and study, advanced rapidly in his chosen profession, being at all times noted for the careful, painstaking manner in which he did any work entrusted to him. He was repeatedly employed by men under whom he had served before, and his going to Mexico was due to the offer of a situation by former employers.

He remained with the Chicago, Burlington and Quincy Railroad until 1881, serving as Rodman, Topographer, etc. In 1881 he was Topographer and Leveler on the Utah and Northern Branch of the Union Pacific Railroad.

In 1882 he commenced work for the Atchison, Topeka and Santa Fé Railroad Company, and, with the exception of one or two short intervals, he remained with that company until 1888. During these years he served as Leveler and Topographer on surveys, as Division Engineer on construction and as Assistant Engineer in the Building and Water-Supply Departments.

* Memoir prepared by the Secretary, from papers on file, and from information supplied by Lewis Kingman, M. Am. Soc. C. E.; J. T. Norton, M. Am. Soc. C. E., and Mrs. H. C. Jones.

In 1889 he served as Transitman on the location, and as Division Engineer on the construction, of the Fort Worth and Rio Grande Railway, from Granville to Stephenville, Texas.

In 1890 and 1891 he became Locating Engineer and Assistant Engineer in charge of the construction of the Mexican Pacific Railroad, a proposed transcontinental line from Tonala to Frontera. During this period he also made a reconnoissance of a line from Alvarado to San Andres Tuxtla, in Vera Cruz, Mexico, and located 20 kilometers of that line.

From April to December, 1891, he was one of the Assistant Engineers on the Tampico Harbor Works, under A. F. Wrotnowski, M. Am. Soc. C. E.

During 1892 and 1893 he was Locating and Resident Engineer on the Mexican National Railroad across the Isthmus of Tehuantepec. In reference to this piece of work the following is quoted from a letter from the Chief Engineer of that road, and found among Mr. Correa's effects:

"I wish to express to you my appreciation, not only of the thoroughly conscientious work you have done for me, but also as to your ability as an Engineer. You have had charge of the location of this line from the Sarabia to the Jumuapa Rivers, which I consider by far our most difficult piece of work. Owing to the configuration of the country, Ship's Pass being the only point at which you could cross the range between these two rivers; to get a line up to it from the Sarabia, and from it to the river Jumuapa again and not carry long supported grades; to avoid rock cuts, which would not only have been nearly impossible but impracticable, from their cost, and get a line as you have, both with such good alignment and so cheap, has, I think, been the greatest engineering feat accomplished here on the Isthmus, and well shows your ability. For your prompt attention to all instructions, for your earnest manner in performing your duties, allow me to thank you. Wishing you every success in your profession, and that you may reap those rewards which you so richly deserve, I remain,

"Yours very respectfully,

"S. B. BURTON, *Ch. Eng. and Gen. Manager.*

"COATZACOALCOS, February 4th, 1893."

During 1894 and 1895 Mr. Correa was engaged in private practice in Mexico, in various classes of work.

In July, 1895, he became Assistant Engineer on the Mexican Central Railroad, under Lewis Kingman, M. Am. Soc. C. E. He was engaged on reconnoissance surveys and reports of lines in various parts of Mexico until about July, 1897. He was then engaged on location, and had charge of various pieces of construction. He then became Division Engineer at Aguas Calientes in charge of Yards and Shops plant.

Shortly after this, Mr. Correa was sent to Parral, in Chihuahua, where he was engaged in supervising the construction of a branch of the Mexican Central Railroad. He was employed upon this work at

the time of his death, which occurred in camp, on June 24th, 1900, after an illness of a day or two.

Mr. Correa was elected a Member of the American Society of Civil Engineers on March 7th, 1900.

He never married, but supported his mother up to the time of her death, September 1st, 1898, and it was in fulfillment of her expressed wish that he sought membership in this Society.

Mr. Correa was a man utterly incapable of a mean or dishonest action, and, in an unostentatious way, one of the most charitable. He was a true friend, and was sincerity personified in all his dealings with his fellowmen. He was at the time of his death one of the ablest railroad engineers in Mexico. He was an indefatigable worker and was conscientious to a fault. His ideals were high, and his ambition was to do anything to which he set his hand, not as well, but better, than others could do it.

In his death the Society has lost a useful and honored member, and one who was universally respected, even by his enemies.

WILLIAM GIDDINGS CURTIS, M. Am. Soc. C. E.*

DIED JUNE 15TH, 1900.

Like many other natives of Connecticut, the subject of this memoir was of good colonial stock, tracing his lineage back hundreds of years in Essex County, England. The descendants of William Curtis, his ancestor, who came to the colonies in the ship *Lion*, in 1632, took a prominent part in the colonial life, and, when the final separation from the old country came, several of the Curtises figured in the great struggle for independence.

Mr. Curtis was born at Bridgeport, August 19th, 1849.

At the age of fifteen, he removed with his parents to San Diego Cal., thereby doubtless changing the whole trend of his life and fortunes.

At that period, there was a ready market on the Pacific Coast for probity and ability, even in the very young, and, but a year later, 1865, Mr. Curtis obtained employment with the Central Pacific Railroad Company, and began a career of success no more marked than deserved, and whose achievements only terminated with his life.

Most engineers commence the practice of their profession after a more or less complete course of study, but Mr. Curtis, beginning as a rodman at the age of sixteen, was obliged to acquire most of its theory during such leisure hours as were left him from active duties in the field and office. He passed through nearly the entire period of location and construction on the Central Pacific Railroad, an experience which profited him during his whole professional career.

He became successively Assistant Engineer, Resident Engineer, Assistant General Superintendent, and Superintendent of Track, and finally reached the position of Assistant to General Manager, and Engineer, Maintenance of Way, of the Pacific System of the Southern Pacific Company, comprising, at the time of his death, nearly 6 000 miles of railroad, which position he retained during the rest of his life.

Owing to the total lack of timber on most of their lines of railroad, the Southern Pacific Company early turned their attention to the subject of timber preservation, in which they have had marked success and have perhaps done more than any other railroad company in America, if not in the world. The achievements in this regard have rightfully been largely credited to Mr. Curtis, under whose able direction the work progressed until, at the present time, practically all the timber and ties used in or about the roadway, excepting redwood, are

* Memoir prepared by John D. Isaacs and Virgil G. Bogue, Members, Am. Soc. C. E.

treated with creosote and chloride of zinc, respectively. Mr. Curtis was an authority in this science, and one or more of his papers are among the best contributions that have appeared on this most important subject.

As Engineer, Maintenance of Way, Mr. Curtis kept himself well informed by observation, reading and discussion, and was fully abreast of the times. His enthusiasm, however, was tempered by that judgment which comes only from experience and attrition with men and the duties of life. His constant aim was to do the best possible under the conditions and circumstances presented.

His advice was frequently sought in relation to the business of other departments of the railroad, and there was scarcely a work or study related to the railroads of the Southern Pacific Company with which he did not have more or less to do, and always with credit and success. He was one of the controlling forces of that vast property.

From boyhood to his last day he remained faithful and industrious in the same employ, and it is fair to say that Mr. Curtis, more than any other, was the author of the present perfected methods and organization of the Maintenance of Way Department of the Southern Pacific Company. His duties covered thousands of miles of railroad and a wide range of climate, from the tropic heat of the Colorado Desert to the Arctic cold of winter in the Pequod Hills; from the restful scenes and orange groves of the Los Angeles Valley to the deep snows of the Sierra Nevada and Siskiyou Mountains, or from the sunny skies of the San Joaquin to the storms and floods of the Soledad and Sacramento Cañons, or the cloudbursts of the Arizona Plains. There was no time of the year when his presence might not be required on some distant portion of the property where inundation or snow impeded traffic or threatened disaster. But he was full of resources and always ready for any emergency, and in the process of time he devised expedients which have largely lessened the disastrous effects of storm and flood.

The busy life he led left him little time for recording the results of his experience, but on several occasions he prepared papers dealing with subjects to which he had devoted much thought and attention. These are most instructive, and models of brevity and careful statement.

Mr. Curtis had a wide range of knowledge and an accurate, comprehensive memory, which doubtless added to his success. It was said among his associates that he never forgot a figure.

On June 15th, 1875, at Stockton, California, he married Mary Elizabeth Burton. In her he found a fitting helpmeet. Both as son and husband he was exceptional, manifesting in his domestic life the same disposition which had made him so respected and liked by subordinates and associates. For all about him he had ever a genial

smile; for the ambitious, a word of encouragement, and for the unfortunate, one of sympathy. He never forgot the social side of life, and even on a wearisome business trip, was wont to entertain his companions with enlivening repartee and anecdote, when no affair of moment was under consideration.

Mr. Curtis was not only an engineer and manager of rare judgment, but also a clever man of affairs, outside of technical matters. His assistance and advice were often sought in other enterprises and organizations.

Two years before his death his health began to break, but with the tenacity of purpose and loyalty to his employers which had characterized his long period of service, he gave of his best until compelled to seek complete change and rest. The end came at Highland Springs, California, on the twenty-fifth anniversary of his marriage, 1900.

His loss, great as it has been to the corporation with which he was so prominently identified, was not alone a public one. It was also distinctly a private loss, and those intimately associated with him can only recall him with the poignant regret caused by the departure of one possessing all the traits of a kindly gentleman, a well-balanced citizen and a true friend.

Mr. Curtis was elected a Member of the American Society of Civil Engineers on May 3d, 1882, and served as a Director from January 15th, 1890, to January 21st, 1891. At the time of his death he was Second Vice-President of the American Railway Engineering and Maintenance of Way Association. He was also a member of the Geographical Society of California; of the San Francisco Microscopical Society; of the Technical Society of the Pacific Coast; and of the Bohemian Club of San Francisco. In the two latter societies he was formerly a Director. To all of these he gave the same careful, precise attention which he gave to his railroad duties.

JAMES FRANCIS, M. Am. Soc. C. E.*

DIED DECEMBER 1ST, 1898.

James Francis was the second son of the late James Bicheno Francis, Past-President, Am. Soc. C. E., and Sarah W. Francis, and was born in Lowell, Mass., March 30th, 1840. His education was obtained mainly in private schools in the vicinity of Lowell and Boston. At the age of 20 years he entered the Lowell Machine Shop as an apprentice, studying and working in the different departments. Upon the call for troops at the outbreak of the Civil War he enlisted in the Second Massachusetts Regiment, and served until August, 1865. He was with the Army of the Potomac in the winter of 1862–63, participated in the battles of Cedar Mountain, Chancellorsville, Antietam and Gettysburg, was with the troops sent to quell the draft riots in New York City, and afterward served along the line of the Chattanooga Railroad and in Sherman's forces on the march to the sea. Mr. Francis was commissioned a Second Lieutenant in May, 1861, and rose to the rank of Lieutenant-Colonel in July, 1865. He was wounded in his left hand in the battle of Antietam, and lost his little finger by amputation.

After his discharge from the volunteer forces, Mr. Francis entered the engineering department of the Hoosac Tunnel in the autumn of 1865, as an assistant, and was engaged, chiefly, in surveying and leveling. Returning to Lowell in April, 1866, he became an assistant engineer of the Proprietors of Locks and Canals on the Merrimac River. In this position he devoted himself to the study of hydraulic works and especially to the important undertakings with which his father's name is associated. Upon the retirement of his father from active service, Mr. Francis was appointed, in January, 1885, Agent of the Company, in which position he had the management of the water power of the Merrimac River, and the charge of all the company's property.

The engineering problems involved in the control of the water power of the Merrimac River had been thoroughly worked out by the elder Francis, and the son, on assuming control, followed, without radical departures, the plans of work and methods of management which his father had inaugurated.

A notable original work was his scheme for the regulation of the power of the Concord River at Whipple's Falls, Lowell. The conflicting interests had, for years, engaged in litigation over the matter,

* Memoir prepared by the Secretary from papers on file, and by abstract of a Memoir published in the *Journal* of the Association of Engineering Societies.

without approaching a settlement, when the Court referred the whole question to Mr. Francis. He designed a system for the regulation of the power, which has been in operation since 1894, to the satisfaction of the parties in interest.

Mr. Francis' dealings with corporations, as with individuals, were marked by absolute impartiality, and his honesty of thought and purpose were apparent in all his relations with his fellow-men. He was severely hurt during work on the Pawtucket Canal in 1888, both legs being broken. He recovered from these injuries, but a few months before his death he was thrown from a carriage. He never regained his former vigor after this accident, but lost strength steadily until his death.

He was married to Caroline, daughter of Franklin Forbes, of Clinton, Mass., on July 19th, 1873. Three children were the result of this union: Joseph S., Clara and Duncan F. The first two survive their father, but the latter died at an early age. Mrs. Francis died on August 13th, 1889. On July 12th, 1892, Mr. Francis married Mary C., daughter of Thomas A. Carew, of Cambridge, Mass., who survives him.

Mr. Francis served as a member of the Lowell Board of Aldermen in 1884 and 1885. He was a director in the Lowell Gas Light Company and in the Railroad National Bank, and a trustee and member of the Board of Investment of the Mechanics Savings Bank.

Mr. Francis was elected a Member of the American Society of Civil Engineers on January 4th, 1893.

He was also a Member of the American Society of Mechanical Engineers, the Boston Society of Civil Engineers, the Institution of Civil Engineers of Great Britain, the Loyal Legion, the Massachusetts Military Historical Society, the Victoria Institute, the Philosophical Society of Great Britain, and the Unitarian Society.

RANDELL HUNT, M. Am. Soc. C. E.*

DIED JANUARY 24TH, 1898.

Randell Hunt was born in New Orleans, October 30th, 1856. He was graduated at Yale with degrees in engineering and philosophy. After leaving college, his first practical experience was gained in connection with the New York Public Parks.

In 1878 he established an office in Dakota, and identified himself thoroughly with the principal engineering works of the then very active and prosperous West. As a specialty, he devoted himself to bridge structures and foundations, and all through his professional career these were the particular branches in which he excelled and with which his name became prominently connected. Until 1883 he remained in Dakota, with the exception of a period in 1879, when, as hydraulic engineer, he was attached to the Mississippi River Commission, in the investigations of that great waterway.

From 1883 until 1888 he was located at St. Paul, Minn., and employed as constructing engineer by the Chicago, Burlington and Northwestern Railway. He had charge of the bridgework, and, under his direct supervision, the noted Chippewa River Bridge was built.

In 1888 he went to San Francisco and began a general engineering practice in the line of his specialty. Many of the bridges of this State were designed by him and built under his direction. In 1889 he was made the Engineering Expert of the contracting firm of Antonelle & Doe, and his name became very prominent in the controversy regarding the concrete foundations for the extensive State work on the sea wall at the foot of Market Street, in San Francisco, which caused considerable interest at the time, he contending that the system of sinking the wall by means of open floating caissons possessed innumerable advantages over the method of constructing coffer-dams for this work. He devised means for an efficient and rapid execution in this direction, and defended his position so ably that he converted the authorities to his ideas. The Harbor Commissioners finally permitted the use of this system, and the work was successfully carried out. The total length of the wall to be constructed aggregated 450 ft.; seven caissons were used, six of them 70 ft. long and one 34 ft. They were sunk in depths of from 10 ft. at low water stage to 17 ft. at high water. Only one set of sides and ends was used for the six large caissons. The sides and ends were attachable and detachable to and from the grillage forming the bottom of the structure. These designs were very interesting, and all through this extensive work

* Memoir prepared by Otto Von Geldern, M. Am. Soc. C. E.

this method of operation gave great satisfaction, for no mishap of any serious consequence occurred.

With Mr. Hunt, engineering was more than the mere means of making a livelihood. With an inborn love for his profession, he gave to every case before him a careful and deliberate study; and if he advocated his designs and upheld them with all the vigor of his energetic character, it was because he was led by an honest conviction that he contended for that which he knew to be right, and that nothing should influence him to swerve from the path which he had laid out.

In 1890 the firm of Doe & Hunt, engineering contractors, was established, and it carried on a successful business for a number of years. Here, Mr. Hunt displayed great energy and activity, and the results of his labor are manifest to-day in a great variety of existing public works. His last work under the name of the firm was the very important construction of the headworks and canal of the Turlock Irrigation District. This extensive engineering operation would have been carried to a successful completion by him had it not been for the suspension of all irrigation enterprises, by reason of legal and financial difficulties which threatened disaster to almost every irrigation district in California, the problem of recovery therefrom not having been solved to this day.

In 1897 Mr. Hunt's services were engaged by the Commission which had been charged by the Government to decide as to the respective merits of Santa Monica and San Pedro Harbors for deep-water improvement and shelter to vessels. Here he became the expert in the work of making borings in two localities, in order to establish the composition of the materials of the respective harbor bottoms. Although then suffering from a malady which proved incurable, and being very ill at the time, he carried out this trying work to the end, at a season when the conditions of the weather made it a severe test for a man in health to have been so constantly in attendance as he was.

This indomitable energy, to carry out whatever he had undertaken, was one of the principal characteristics of his nature. Physical indisposition could not incapacitate him. As long as he could be about he performed his full share of the work—and this he did to the very end of his useful life.

Socially, he was the most estimable of men. His early training and education had opened his mind to all the ennobling influences of culture and art, and, while he devoted himself to the advancement of his profession, he never lost sight of the many other elements that make up human progress. Classic literature and the literature of the day were known to him as to few others, and he delighted to converse on these subjects with those about him who enjoyed his more intimate acquaintance. He was a most genial companion, alive to every interest, kind hearted, devoted to his family and friends, and always ready to assist others in giving aid and advice.

His father, the Hon. William H. Hunt, was a very prominent man in his day. Under the administration of President Garfield he was Secretary of the Navy; after Garfield's death he was sent to Russia as Ambassador to represent the United States. He died in St. Petersburg in 1885, full of honors and mourned by two great nations.

In Randell Hunt the American Society has lost one of its most prominent members. He will be remembered, not only as an engineer of high quality, but as a man of many virtues and broad principles, who made it the maxim of his life to maintain fearlessly an honest conviction under all circumstances.

He died January 24th, 1898, after a lingering illness of several weeks.

Mr. Hunt was elected a member of the American Society of Civil Engineers, May 2d, 1883.

NILES MERIWETHER, M. Am. Soc. C. E.*

DIED DECEMBER 28TH, 1900.

Niles Meriwether was born in Christian County, Ky., on January 26th, 1830, of parents who came of old Virginia stock, and who had gone to Kentucky in the early part of the century.

At the age of eighteen, both his parents being dead, he entered the engineering field, taking a position as a rodman on the Nashville and Chattanooga Railroad, then in process of location. He remained on this railroad, in various positions, for five years.

In 1853, he became First Assistant Engineer on the Mississippi and Tennessee Railroad (now a branch of the Illinois Central Railroad), his brother, Minor Meriwether, being Chief Engineer. In 1857, his brother resigned, and Niles became Chief Engineer, which position he held until 1862, when the occupation of Memphis by the Union Army made further railroad work impossible.

In 1865, this road being practically destroyed by the contending armies, he interested himself in securing the means to rebuild it, in which he was successful.

In 1868, he became Chief Engineer of the Memphis and Charleston Railroad (now a part of the Southern System), and held that position for eight years, during which time he built the Middleton and Ripley Branch and extended the Tullahma and McMinnville Branch to Jasper, Tenn. He also constructed most of the important bridges of this line which had been much injured during the war.

In 1875, as Chief Engineer, he took charge of the consolidated roads formerly known as the Southern and Mississippi Central, and extending from New Orleans to Cairo, Ill.

In 1877 he began the construction of the Natchez and Jackson Railroad, from Natchez to Jackson, Miss., but the yellow fever epidemic of 1878 so depleted the finances of that country that in the following winter the construction was suspended.

In February, 1879, Major Meriwether accepted the position of City Engineer of Memphis, and held this office for fourteen years, during which time he did more than any other man to restore an almost ruined city to its present prosperous condition.

In March, 1897, he took charge of the construction of a branch of the Mobile and Ohio Railroad, running from Montgomery, Ala., to Columbus, Miss., and completed the line in May, 1899. His last work was as Superintending Engineer of Elmwood Cemetery, in Memphis, which occupied his time until two weeks previous to his death.

* Memoir prepared by Mrs. M. M. Betts, of Memphis, Tenn.

During these fifty-two years of active work Major Meriwether was frequently called on to act as arbitrator or consulting engineer on many important pieces of engineering work all over the South.

He was elected a Member of the American Society of Civil Engineers on November 1st, 1871, and, at the time of his death was President of The Engineering Association of the South.

In 1855 he married Miss Lide Parker Smith, of Accomac Co., Va., and their union of forty-five years was a rarely happy one. His wife and two daughters, Dr. Lucy V. Davies, of New York, and Mrs. M. M. Betts, of Memphis, survive him and cherish his memory.

MARC JOHN REISEGER, M. Am. Soc. C. E.*

DIED MAY 26TH, 1900.

Marc J. Reiseger was born October 31st, 1861, near Groningen, The Netherlands, where he attended the public schools, going from the high school there to a college at Delft. After concluding his college career, he entered the employ of the Government as Engineer in Charge of Railroad Bridges, the railroads in that country being owned by the Government.

On May 1st, 1887, Mr. Reiseger left his native land for the United States. For a time he worked as a common laborer on the County Court House in Cheboygan, Mich., his object being to acquire a knowledge of American architecture, etc. He next went to Grand Rapids, Mich., and entered the office of a prominent architect there. In 1888 he left Grand Rapids for Muskegon, Mich., where he had been offered a similar position, which he subsequently resigned for the purpose of opening an office for himself in the same city. The business was eminently satisfactory. Among the more prominent buildings designed by him and erected under his supervision are the North Muskegon School House, the Fox Cracker Factory and the Chase Piano Factory.

In April, 1890, he took up his residence in Greenville, Mich., with the intention of engaging in manufacturing there, but in 1891 he returned to Muskegon. In 1893 he entered the United States Engineering Office in Grand Rapids, and in 1894 secured a position with Keepers and Thatcher Wynkoop, better known as the Detroit Bridge and Iron Works of Detroit, Mich., which firm subsequently became Keepers and Thatcher. While in their employ he was Engineer in charge of bridges built at Albion and Tecumseh, Mich., and Paterson, N. J. He was also for a time in the Company's office in Little Rock, Ark., on which occasion he prepared the plans for the bridge erected by them at Topeka. In April, 1899, Mr. Reiseger returned to Grand Rapids, and organized the Grand Rapids Bridge Company, of which he became President and General Manager on its incorporation, January 31st, 1900. In addition to many smaller bridges, the Grand Rapids Bridge Company built the metal-concrete bridge at Battle Creek, Mich.

Mr. Reiseger was elected a Member of the American Society of Civil Engineers on December 7th, 1898.

Early in 1900 Mr. Reiseger's health began to fail, as a result of too close application to business, and on May 26th, 1900, he died at his residence, 241 North Prospect Street, Grand Rapids, Mich., of tuberculosis-meningitis.

* Memoir prepared by Paul van Deinse, Esq.

WILLIAM HOWARD WHITE, M. Am. Soc. C. E.*

DIED DECEMBER 11TH, 1895.

William Howard White was born in Watertown, Mass., on February 21st, 1847. He was the son of William Abijah and Harriet Tilden Sturgis White. His father's sister, Maria, was the first wife of James Russell Lowell. Mr. White's father and mother died when he was a little boy, and he was taken into the family of and brought up by Mrs. Francis George Shaw, his mother's sister.

In July, 1865, Mr. White was graduated from the Lawrence Scientific School, Harvard University, in the course of civil engineering (*magna cum laude*), and then commenced practice as a civil engineer, in which profession he distinguished himself with great honor.

His first engineering work was in connection with the sewerage of New York City, from August 9th, 1865, to January 10th, 1867. He then became engineer in charge of the construction of seven miles of the Sullivan and Erie Railroad, in Pennsylvania, until March 14th, 1868.

From May 13th, 1868, to March 31st, 1870, he was an Assistant Engineer on the Burlington and Missouri River Railroad. From 1870 to 1873, as Assistant to Colonel Mendell, U. S. Engineers, he worked on the original survey of San Pedro Harbor. The following year he spent in Europe, studying foreign engineering and languages.

Mr. White afterward became Chief Engineer of the New York and New England Railroad System, covering about 450 miles. During 1879–1883 he was Assistant Chief Engineer of the Chicago, Burlington and Quincy Railroad, having charge of all the engineering work on the Iowa lines, embracing some 900 miles.

At the end of this period Mr. White again went to Europe, where he spent two years in studying foreign sanitary engineering and appliances.

For seven years following 1885 he was a Consulting Engineer in New York City with a branch office in Chicago. Some of the clients by whom he was consulted were Messrs. Brown Brothers & Company, of New York; Carey & Whitridge, attorneys for an English investment company, and Messrs. Morton, Bliss & Company, of New York. On the strength of reports made by him the Pillsbury & Washburn Mills, at Minneapolis, having an aggregate capacity of 20 000 bbls. of flour per day, were purchased, and action taken on properties in Kentucky aggregating $5 000 000 in value.

Mr. White was elected a Member of the American Society of Civil Engineers on March 5th, 1873, and served as a Director for the year 1886.

In 1878 Mr. White was married to Margaret Howard Parker, of New

* Memoir prepared by W. A. Nicholls, M. Am. Soc. C. E.

York City, and in the fall of 1892 they went to Redlands, Cal., from South Orange, N. J., with their three children, seeking a milder climate for the benefit of Mr. White's failing health. Some months previous to this Mr. White had visited Redlands and purchased 24 acres of orange lands, and here he built a home.

He was a man of great intellectual force, and always prominently identified himself with the best interests of the community in which he resided. He died in Redlands, December 11th, 1895.

His last professional work was the giving of expert testimony, from the bed from which he never was to arise, before a Commission, appointed for the purpose by the Superior Court, in a suit involving important questions in water development near Redlands.

Mr. White presented several valuable papers before the Society, and was a frequent contributor to the discussions.

DOMINIK LINDENTHAL, Assoc. M. Am. Soc. C. E.*

DIED JUNE 7TH, 1900.

The cabled news of the tragic death of Dominik Lindenthal, Assoc. M. Am. Soc. C. E., on June 7th, 1900, in Cuba, came to his family and friends like a thunderbolt from a clear sky. They knew that his health was affected, but they were not prepared for a fatal termination. A letter from him, received the day before his death, had announced to his family his intention to return to New York on July 1st, and that he had already engaged his home passage.

At the time of his death he was the Manager and Chief Engineer of the Cuban Steel Ore Company, opening up iron ore quarries, and building shipping docks and a railroad at Guama, about 35 miles east of Santiago de Cuba. It was not his first work in Cuba, he having been there several times before on the surveys and construction of different railroads.

He had, years before, successfully resisted an attack of yellow fever, and therefore was thought to be inured to the tropical climate. But in the beginning of 1900, he began to suffer from malarial fever, from which he obtained temporary relief during a business trip to New York and Philadelphia, in May, 1900. Foreseeing that, during the coming hot season, his weakened health would not endure the strain of the great responsibility of the important work entrusted to him, he had given notice, to the officers of the Company, of his resignation, which had been accepted very reluctantly. He returned to Cuba in the middle of May merely to prepare his work for transference to his successor. But immediately after his arrival there he had a relapse; the attacks of malarial fever became more severe, and his mind became affected. For four days he neither ate nor slept, and in a fit of insanity, he killed himself with a revolver, while unwatched.

He was then thirty-four years of age, in the fairest years of mature manhood, having been born in Bruenn, Austria, on the 10th of October, 1866. He was the youngest son of Dominik Lindenthal, a substantial burgher and master cabinet maker, who died in 1889. The Lindenthals belonged to that now mostly extinct type of German burgher families, in which the membership in the craft or guild had descended with the old home and traditions from father to son for generations.

Dominik, named after his father and grandfather, was at first intended to continue the business and family traditions of his ancestors, and to become an artistic cabinet maker. But his natural brightness

* Memoir prepared by Paul S. King, M. Am. Soc. C. E.

and intense desire for study led to his being sent to college (Staats Gymnasium), first in Bruenn and then in Triest, from which he was graduated near the head of the class. He was regularly matriculated at the University of Vienna in the year 1885 for the study of medicine, to the practice of which he then looked forward as his professional ambition. At the same time he complied with his military obligations as a one-year volunteer in the Austrian army, in which he had the rank of lieutenant.

Besides his classical studies, he had also obtained through his elder brother Ernst, Professor of Mathematics at that time in Triest, a thorough education in the physical sciences, and when, in the summer of 1887, he was invited by his eldest brother Gustav, then a civil engineer in Pittsburg, Pa., to come to the United States, he came over well prepared for an engineering career, which he then preferred.

His first experience was in the shops of the Bethlehem Iron Company, in Bethlehem, Pa., which he entered as a workman, to familiarize himself with the practical work in the machine shop from the bottom up; he passed on to building operations and the construction of machinery foundations.

From there he stepped into the position of leveler and draftsman on the Wheeling and Harrisburg Railroad, where he was under the direction of Paul S. King, M. Am. Soc. C. E., Engineer in Charge of Surveys.

His ability and love of hard work caused him to be selected by the Juragua Iron Company for the engineering staff, and sent to Cuba to survey and construct a railroad and iron ore mines in the rugged country near Santiago de Cuba, where he was employed until April, 1889.

Returning to the United States, he occupied the position of Assistant Engineer on the surveys for the extension, at that time contemplated, to the anthracite coal mines of the Maryland Central Railroad and the Baltimore Belt Railroad, which work was in charge of John B. Bott, M. Am. Soc. C. E., as Principal Assistant Engineer, under Gustav Lindenthal, M. Am. Soc. C. E., as Chief Engineer.

In 1890 he took charge of the construction of the foundations, masonry and superstructure for the McKeesport and Duquesne Highway Bridge across the Monongahela River, of which his brother Gustav was the Chief Engineer.

From this position, he stepped into a Division Engineership on the extension and double tracking of the Lehigh Valley Railroad, from Sayre, Pa., to Buffalo, N. Y., under his former chief and friend, P. S. King, as Chief Engineer. On this work he was employed until July, 1892.

In the early autumn of that year he went on his second trip to Cuba, as chief of a party of surveyors for the railroad to the man-

ganese mines, for the Ponupo Mining and Transportation Company, in Santiago de Cuba, on which work he remained until January, 1893.

Again returning to the United States, he became Accountant Engineer for Coleman, Ryan and Brown, contractors for the new Croton Dam, Croton-on-Hudson, where he remained until October, 1895.

His next position was as Accountant Engineer for H. H. Brown, contractor for the revision and improvements on the New York, New Haven and Hartford Railroad, at Roxbury, Mass.

In all these positions he had occasion to acquire great experience in the practical execution of work and in the handling of working forces on a large scale. He was highly regarded for his technical ability, sterling honesty and untiring industry, which made him to his employers a very valuable man.

In 1897 Dominik became office assistant to his brother Gustav, who had in the meantime moved his engineering office from Pittsburg to New York. His skill as a fine draftsman and designer found a ready field in his brother's work.

In the fall of 1899 he went the third time to Cuba, again with his former chief, Paul S. King. This was the work for the Cuban Steel Ore Company, mentioned at the beginning of this memoir, and from which Mr. King, through ill health, was compelled to resign in the autumn of 1899, leaving Dominik Lindenthal as his successor. He was in charge of the surveys and construction for the railroad, ore docks, and the opening of the iron ore quarries, when his untimely death occurred.

Of his immediate family, a widow and a daughter of four years survive.

By his employers the deceased was highly regarded for his integrity and manliness, and by his closer friends for his accomplishments as an educated man. He was a linguist, who had mastered—in addition to his native tongue of German—English, French, Italian, Spanish and Bohemian; and was an accomplished performer on the piano—a lover of music. His reticent manner did not invite easy friendship, but those friends whom he had loved him dearly, and mourn deeply his all too early death.

There was every prospect that, had he lived, he would have attained a high reputation in his chosen profession for executive ability and sound judgment in the direction and organization of engineering and construction work, which was more to his taste than the designing work in the office, in which he was also very proficient.

He was elected an Associate Member of the American Society of Civil Engineers on March 4th, 1896. He was also a Member of the German Technical Society of New York, in the reorganization and direction of which, as one of the officers, he had taken a conspicuous part.

JOSEPH RUGGLES RICHARDS, Assoc. Am. Soc. C. E.*

DIED SEPTEMBER 28TH, 1900.

Joseph Ruggles Richards, oldest child of Wyatt Richards (born at Hardwick, Mass., March 8th, 1802, died in Boston, February 9th, 1872), and Sarah Page Ruggles (born at Roxbury, Mass., June 9th, 1796, married April 5th, 1827, died in Boston, March 17th, 1878), was born at 63 Myrtle St., Boston, February 18th, 1828.

He obtained his education in the public schools of Boston, passing through the various grades of the Mayhew School, on Chardon St., and, finally, being graduated from the English High School in the year 1845, winning one of the six Franklin Medals for scholarship. He was especially good in mathematics, a taste which followed him all through his professional life; nothing pleasing him more than to work up a truss or difficult problem in construction, or to figure out the dimension figures on a set of plans, working for an hour at a time to find the missing inch to tie a column of figures. When he had finished, one was sure the figures were correct. His natural taste for drawing is shown in the many drawings he made when a boy.

His expense accounts, carefully kept from youth up, bear witness to his businesslike methods.

Under the date of October 6th, 1845, he records:

"I, J. R. Richards, this day entered the office of Mr. Gridley J. F. Bryant, 4 Court Street (Boston), for the purpose of serving my apprenticeship under him in the Architectural profession. Seventeen years old on the 18th day of February, 1845."

At the end of the first year he began to receive regular pay, in ever increasing amounts as the half years went by, turning also many an honest penny by over work, writing, translating French, and odd work on his own account.

The many embryo architects who worked with him under Mr. Bryant, then one of the most active and influential architects in the country, will bear witness to Mr. Richards' faithfulness and ability, and tell with admiration of the amount of work he could "throw off."

The systematic time cards or diaries he kept show where each hour was spent; and the number and variety of the works he undertook is remarkable.

On January 1st, 1853, he opened an office on his own account at 46 Court St., corner of Tremont St., Boston, in a building famous from the names of Washington and Lafayette associated with it, and famous later for having Daniel Webster as a tenant.

* Memoir prepared by William P. Richards, Esq., Boston, Mass.

In July, 1865, the firm of Richards and Park was formed by the admission of William S. Park, his brother-in-law. The firm was dissolved in 1872, Mr. Richards continuing the business alone.

On October 1st, 1880, he admitted his son William P. Richards into partnership with himself, ending it only with his death on September 28th, 1900.

His list of clients contains the names of many honored people, and his work, though principally confined to the neighboring States, extended to New York, Pennsylvania, Louisiana, Alabama, California, Wisconsin, Kansas, Florida and Minnesota.

The City of Boston early employed him, and for a number of years he designed for it schools, engine-houses, armories, stables and other structures.

The State of Massachusetts also appears early, as a client, and at intervals up to a recent date. At one time he made plans for work on the State House, and at another he was one of the Commissioners on extensive alterations to it, executed for them by Mr. G. J. F. Bryant.

The State Capitol at Montpelier, Vt., was erected under his supervision, the drawings of another architect being handed to him to carry out and finish, and in 1885–86 he designed and executed an annex to this building for the Supreme Court and State Library.

Harvard, Amherst Agricultural, Dartmouth, Williams and Hamilton Colleges, also Baker University, Kansas, appear on his books; and he erected important buildings for them.

The cities of Cambridge, Brookline, Lowell, Woburn, Blackstone, Mass., and Kenosha, Wis., employed him on schools and various public works.

The counties of Suffolk and Franklin each had their Court Houses enlarged and remodelled by him.

A stone church in Greenfield and a brick church in Taunton show his ability, as also brick and stone libraries in West Brookfield, Greenfield, Sudbury (the latter again enlarged in 1894), the Waverly House, in Charlestown, and the Crawford House, in Boston.

Numerous large stores and business blocks, and town, city and summer houses in Boston and neighboring towns were planned and carried out by him.

Among his more recent and larger works are the "Jarvis" and "Stanstead" apartment houses in Cambridge, "White" apartment house, Commonwealth Ave., Boston, Five Cents Savings Bank Building in Woburn, Royal Arcanum Building, Boston, and Colored Odd Fellows' Building, Boston.

Throughout his entire practice it had been his custom to lay out and do most of the important office and outside work himself; his draughtsmen, in consequence, few in number, doing what he could not accomplish in the busy days.

Of him *The American Architect and Building News* speaks, as one:

"Whose unfailing courtesy and interest in artistic matters had made him for many years a favorite in the profession. Although his works belong rather to the last generation than to this, he carried out in his younger days very many mercantile and other buildings, characterized by a quiet good taste which was then perhaps the best quality that such architecture could show."

Outdoor sketching in pencil and water color was a constant pleasure to him, and his leisure moments in the office were often taken up in working with his brush. A large collection of pictures and studies give evidence of a remarkable taste and talent for water-color painting.

As the years passed he gradually left more and more of the office work and outside duties to the younger partner, but, without fail, except when sickness prevented, and that rarely, would come to the office and attend to his duties.

At the close of office hours, except for rare occasions, home was the next object in view, and, arrived there, it would again be a rare occasion, as time went by, for him to spend the evening away, though he was peculiarly sociable in his temperament and enjoyed the company of his friends. Masonic cares occupied his time early in his career, but he gradually dropped the active cares and attendance at meetings. He was admitted to Amicable Lodge, Cambridgeport, in August, 1856; St. Paul's Royal Arch Chapter, in 1859, and Boston Commandery, Knights Templars, in 1860.

He was married to Mary A. Phillips of South Natick, Mass., daughter of Thomas and Clarissa Brackett Phillips, on August 27th, 1851, and commenced housekeeping in Cambridge, where he watched the gradual building up of residences, schools and library on the acres of vacant land near him. To reach his Boston office, he profited by the slow change from omnibus to horse car, and from horse to electric car, the first horse cars from Cambridge passing his door. The Fitchburg Railroad had a branch track almost to Harvard Square, now discontinued, which he used on first coming to Cambridge.

His wife and daughter and one of two sons survive him.

As a referee, he acted acceptably in a number of cases, and as executor or administrator he settled up several estates successfully and happily. As Trustee for the North End Savings Bank for a number of years, he rarely lost a meeting with his co-trustees, and his advice on investments in real estate was invaluable.

Though actively interested in the political life of the town and State, and never failing to go to the polls to vote, he never accepted public office.

On May 22d, 1867, he attended the first meeting called by the architects of Boston for the purpose of forming a Society of Architects, and on September 3d he was elected a Fellow Member, which membership he

kept up until his death. He was a loyal member to that Society and his fellow architects, was ever ready to help it or them, and his fairly regular presence at the monthly meetings of the Society will be missed.

He was elected an Associate of the American Society of Civil Engineers on February 4th, 1880, and, though distance from New York prevented him from taking an active interest in all the meetings, he looked forward with pleasure to the Annual Meetings and Annual Conventions which make up a large part of the social life of the Society, and, when he could, joined these gatherings.

JOHN MONTGOMERY TOUCEY, F. Am. Soc. C. E.*

DIED SEPTEMBER 29TH, 1898.

John Montgomery Toucey was born in Newtown, Conn., on July 30th, 1829. He prepared for Trinity College, but, finding that his tastes did not lie in the direction of the ministry, he took up the profession of teaching. After pursuing that profession for two years, he began railroad work as a station agent at Plymouth, now Thomaston, on the line of the Naugatuck Railroad, then in process of construction.

Soon after, he was placed in charge of the construction of the railroad between Waterbury and Winsted—heavy freshets having severely damaged it—and was highly commended for the rapidity with which the work was carried out under his direction. He was married in 1851, and secured employment with the Madison and Indianapolis Railroad, and, subsequently, as a freight agent on the Morris and Essex Railroad. In the spring of 1855 he entered the service of the Hudson River Railroad Company, as a passenger conductor, and afterward became its agent at East Albany. In 1862, Samuel Sloan, then President of the road, appointed him Trainmaster, and so well were his duties performed that he was appointed Assistant Superintendent in 1867.

In the same year he resigned, to accept the office of General Superintendent of the Delaware, Lackawanna and Western Railroad, but retained the office only two months, as he was recalled to the Hudson River Railroad and made its General Superintendent in full charge of the line. He continued in charge of this section of the road after the consolidation with the New York Central in 1869, and in 1881 his authority was extended over the lines west of Albany. In February, 1890, the office of General Manager was created for Mr. Toucey, and this office he held until it was abolished, eight years later, after the election of Mr. Callaway as President. A month later he presented his resignation, and was placed on the retired list at half-pay.

For forty-three years Mr. Toucey was a responsible officer of the New York Central and Hudson River Railroad. He was a man of great energy and resolution, his dominant trait being unwearying fidelity; and he had, in particular, the gift of managing trains and moving passengers and traffic expeditiously and at minimum cost. He was bold in assuming responsibility, but had a keen desire to learn what was best in the practice of others. In his personal relations with the employees of the railroad, he was genial and sympathetic, and the older men in the service were strongly attached to him

* Memoir prepared by the Secretary from papers on file at the House of the Society.

on account of his friendly interest in their associations, his unostentatious charity, and his management of the Christmas dinners provided for the men.

The railroads of the United States are indebted to Mr. Toucey for many improvements. He was first to introduce interlocking switches in the United States. These were placed in the Grand Central Yard. The present circular form of scoop for taking water for locomotives from a tank between the tracks is due to his experiments at Montrose in conjunction with Mr. Wm. Buchanan. He was Manager of the Grand Central Station at the time the entire system of tracks was changed at that locality, and proposed and urged the abolition of the crossings at grade.

At the World's Congress of Railways, in London in 1895, as a result of the profound impression made by Mr. Toucey's remarks, with regard to the speed of the Empire State Express, several representatives of the continental lines have inspected the track of the New York Central, and have made reports to their companies confirming Mr. Toucey's statements. At the Congress Mr. Toucey was accorded great respect as a representative American railroad man.

Mr. Toucey wrote little besides his personal and official correspondence, seldom spoke in public, and was reluctant to appear in conspicuous positions. His personality, therefore, was known to comparatively few, save those with whom he had business relations. He had taken little rest from active work since his first engagement in his boyhood, and he suffered from illness for several months before his death.

Mr. Toucey became a Fellow of the American Society of Civil Engineers on May 17th, 1870.